CS667 **Lecture 1: Radiometry** 29 January 2004

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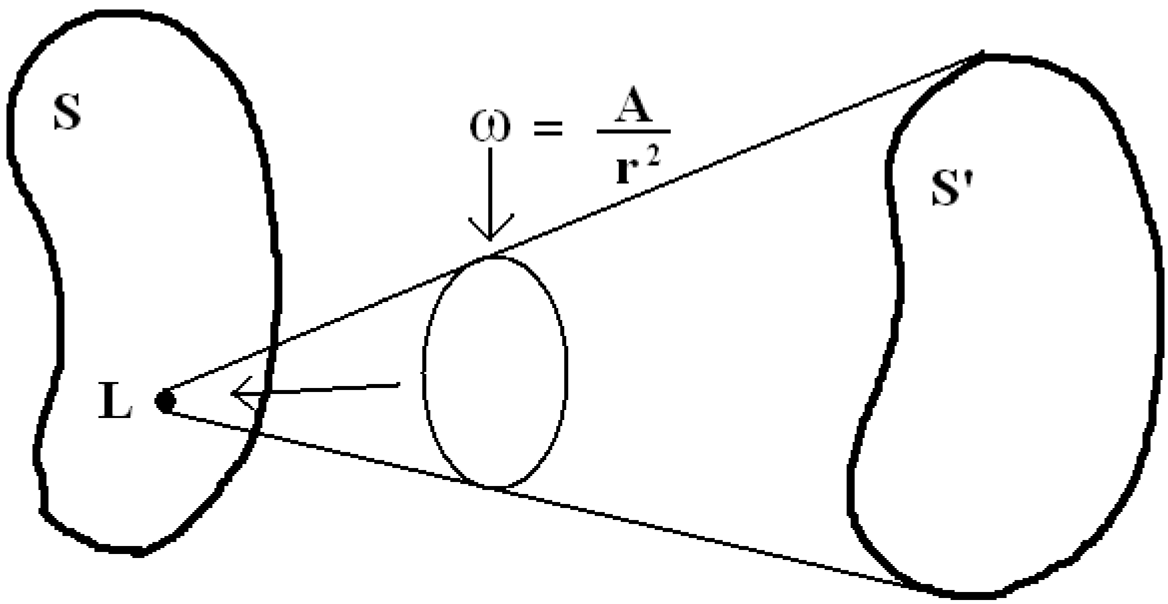
1 Radiometry (cont'd)

Big Fact: Radiance is invariant along a straight line! This assumes no interfering media (ie - a vacuum).

As a proof, look at radiance from a linespace point of view. Assume that S and S' are "small" and "far apart", so that we can use the $\omega = \frac{A}{r^2}$ approximation for solid angle. Light is travelling from left to right, passing through S, then S'. In the image below, S and S' are oriented so that their normals align.



Radiance is defined as the flux divided by the size of the set of lines it is travelling along. In this case, we will look only at the flux that first passes through S, then S'. The set of lines is then all lines that pass through both regions. We can measure the radiance at S, giving us:

$$L = \frac{\Phi^2}{A_S \frac{A_{S'}}{r^2}}$$

or at S', giving us:

$$L' = \frac{\Phi^2}{A_{S'} \frac{A_S}{r^2}}$$

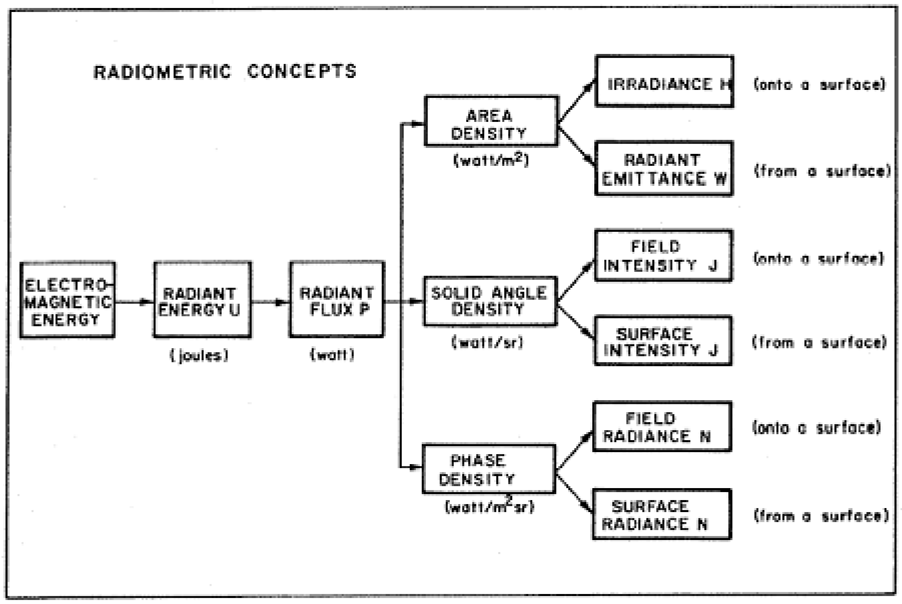
The above set of equations show that L and L' are computed symmetrically with respect to the A_S and $A_{S'}$. The value computed for the radiance measured at S is the same as the amount of radiance measured at S'.

This is an important fact because radiance is essentially what our eyes "see" when we look at a surface. Radiance does not change by bringing the object closer to you (increasing the solid angle), or by making it bigger (increasing the area). This is consistent with the fact that neither of these actions makes a surface brighter. Both of these action increase the amount of power, or flux, of the surface, but not the radiance.

Note: The meaning of $L(x \to \Theta)$ is a limit take as some small area dA approaches a point x, and some small solid angle ω approaches a single direction Θ. More formally:

$$L(x \to \Theta) = \lim_{dA \to x} \lim_{d\omega \to \Theta} \frac{d^2\Phi}{dA\, d\omega\, \cos(\Theta)}$$

Chart from the Preisendorfer paper displaying relationship among radiometric units:



2 Measure Theory view of Integration

Measures are a way of assigning "sizes" to a subset of a domain. The usual way in which this is done is to define the value of some basic unit of the domain (e.g. rectangles in the plane) and approximate any subset of interest with many of these units.

Integration is always done with respect to a measure, μ, and most of the time this is so obvious that we don't even think about it. Primary examples are integration over the real line with respect to the measure of "length", or on the plane with respect to the measure "area". The symbol dx is really a shorthand for the measure theoretically correct way of writing an integral:

$$F(x) = \int f(x)\, dx = \int f(x)\, d\mu(x)$$

$$F(x,y) = \int f(x,y)\, dA = \int f(x,y)\, d\mu(x,y)$$

Where μ is some arbitrary measure

Two commonly used measures in radiometry are: $\sigma(D)$ – the solid angle of D, and $\mu(D) \approx \sigma(D)\cos(\theta)$ – the projected solid angle of D.

Distributions, such as probability distributions, are also defined with respect to some measure.

- distributions vs. densities
- occurrence of Dirac δ functions when distributions don't have densities

Normal PDF's (probability distribution functions) prescribe an infinitely small chance that a variable will take any particular value. For instance, assuming a uniform distribution over some range $[0, L]$, the pdf is:

$$f(x) = \begin{cases} \frac{1}{L} & x \in [0, L] \\ 0 & \text{otherwise} \end{cases}$$

The chance that f takes on any value is not $\frac{1}{L}$, but rather 0, since there are infinitely many values that f may take. Integration will save us here though, as the chance that f will assume some value in the range $[a, b]$ is $\int_a^b f(x)\,dx$. This technique works fine until we encounter a distribution that *does* actually associate a non-zero probability with a single value. For instance, suppose that the probability g would be 2 is $\frac{1}{2}$, and the chance that g would be any other value in the range $[0, 4]$ is evenly distributed amongst the remaining values. Our PDF would look just like f, but with an infinite spike at 2. How do we define density for such a distribution?

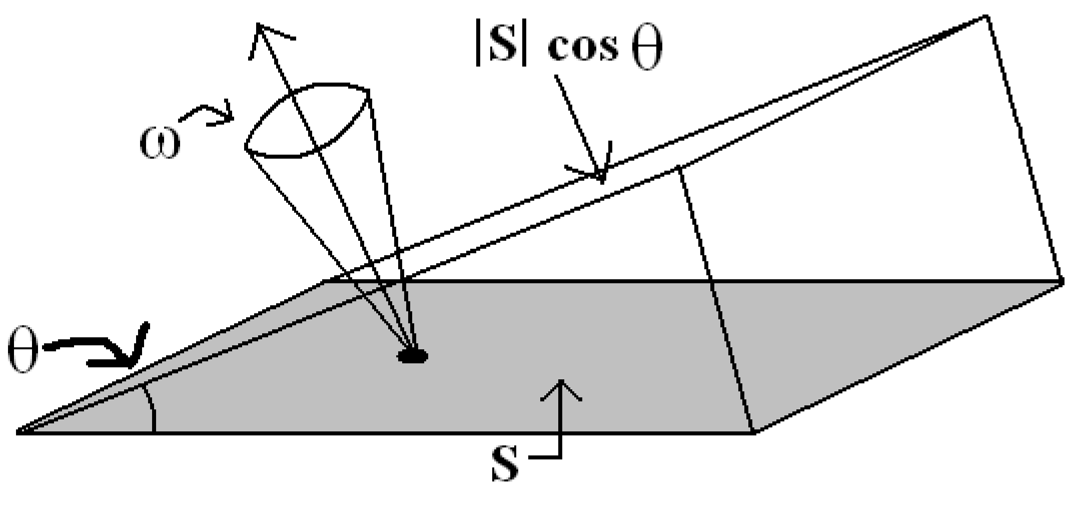
Rather than looking at the function as a PDF, we'll look at its integral, since the integral of g (rather than g itself) is what is well defined. This integrated version of g is called its cumulative distribution function, or CDF, and is defined as $G(x) = \int_{-\infty}^{x} g(x)\,dx$. This integral is computed using Dirac δ functions, which act like on/off flags for all "distribution spikes":

$$\int_D f(x)\,d\mu(x) = \delta_a + \int_{D \setminus a} f(x)\,d\mu(x)$$

3 Units from an Integral View

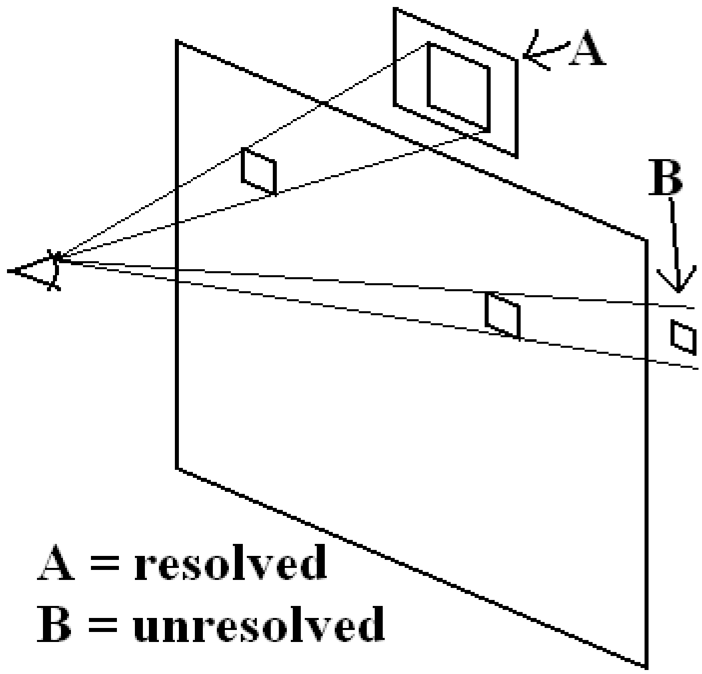
- Radiance $= \frac{W}{m^2\,sr} = L(\vec{x}, \vec{\omega})$.
- Irradiance $= \frac{W}{m^2} = E(\vec{x}) = \int_{H^2} L(\vec{x}, \vec{\omega})\,d\mu = \int_{H^2} L(\vec{x}, \vec{\omega})(\vec{n} \cdot \vec{\omega})\,d\sigma$
- Intensity $= \frac{W}{sr} = I(\vec{\omega}) = \int_A L(\vec{x}, \vec{\omega})\,dA^{\perp} = \int_A L(\vec{x}, \vec{\omega})(\vec{n} \cdot \vec{\omega})\,dA$
- Flux = Power $= W = \Phi = \int_{\Omega}\int_A L(\vec{x}, \vec{\omega})\,dA\,d\mu = \int_{\Omega}\int_A L(\vec{x}, \vec{\omega})(\vec{n} \cdot \vec{\omega})\,dA\,d\sigma$
- Note: S^2 = Sphere, H^2 = Hemisphere

4 Examples



In the image above, we see a lambertian emitter with constant radiance L_0 for all directions and locations. Suppose we'd like to treat this emitter as a point source. What is its intensity distribution? We can get the total intensity in some direction ω for the whole area by integrating the radiance (which is always the same) over the projection of that area normal to the direction ω. The intensity then is given by the simple integral:

$$I(\vec{\omega}) = \int_S L_0 \, dA^{\perp} = \int_S L_0(\vec{n} \cdot \vec{\omega}) \, dA = L_0 |S| (\vec{n} \cdot \vec{\omega})$$



In the above example, we see a problem associated with discretizing a radiometry problem. We have a camera looking through and image plane with boundary lines drawn to indicate the window of view that the camera would see if it were looking through the two indicated pixels on the plane. Behind the image plane, we have two surfaces A and B. A is positioned such that it does not fit entirely into the view of one pixel. B is oriented so that it does. The other thing to keep in mind is that this camera can essentially record only one value for each pixel (lets leave color out of the picture for now), representing the "brightness" of the pixel.

The two situations are marked as "resolved" and "unresolved". The "resolved" situation is the normal one, in that increasing the surface area does not result in a brighter pixel. This correctly simulates the fact that radiance does not increase as a result of increased surface area. The "unresolved" case is one where the solid angle of a pixel in the image fully encloses the surface area of the surface. This effectively makes B a point source. Increasing the size of B does not result in an increase in resulting pixel coverage in the image, but rather results in a brighter pixel at the same spot. This, this pixel measures the intensity.

Side note: it does not make sense to talk about the radiance of the point source, as a point source has no area, making its radiance infinite. Instead, an intensity is usually associated with a point source.

5 Other Related Points

There is a closely related field to Radiometry, which is Photometry. While radiometry is the study of light, photometry is the study of light detection by people.

Up until now, a dimension of information has been completely lacking from our discussion of light - wavelength, or color. All of the quantities (radiance, irradiance, etc) can also be divided up depending on the color of incident light. This only really becomes an issue when dealing with real measurement tools in a lab, since most have associated sensitivity curves.

The number given back by a receptor is really the integral of that receptors sensitivity curve against the incoming spectral distribution. The lecture slides give the sensitivity curve for the human eye, and we can use this curve as an example. Pretend you were completely color blind and could only see shades of light and dark. This sensitivity curve then would dictate the brightness of any particular direction. This curve is given by the equation $\bar{Y}(\lambda)$, where λ is the wavelength.

Given some spectral distribution g (in other words the amount of light for any particular wavelength), the brightness humans see is given by the integral:

$$\int \bar{Y}(\lambda) g(\lambda)\, d\lambda$$

The domain of the integral is technically all valid wavelengths, but that is an infinite domain, and the computation is usually done just over the support of the response function.

Although human eyes have their own response function, so does every other photometric device. This response function needs to be taken into account during calibration etc.