CS667 **Lecture 1: Radiometry** 27 January 2004

Adam Arbree Lecturer: Steve Marschner

1 Overview of Radiometry

Radiometry is a system for describing the flow of light energy in 3 dimensions. It is primarily a geometric description and is not specific only to light. In this description, light is modeled as particles, *photons*, that have specific properties: they travel in straight lines, there are enough in any given region of space that their distributions can be considered continuous, and they do not interact with each other (non-interference). Each photon has a color (wavelength) and carries some amount of energy (proportional to frequency). The different radiometric quantities that follow are a "photon counting game", describing the number of photons that move in a specified region in a specified time. Finally, this radiometric description makes certain assumptions about light. These are that:

- it is incoherent (possessing a random distribution of phase)
- it is unpolarized
- it is in the visible range
- the objects to be measured are macroscopic (much larger than the wavelength of light described)

2 Basic Radiometric Quantities

2.1 Radiant Energy (Q in Joules, J)

Radiant energy, a number of photons, is fundamentally the only truly measurable radiometric quantity. All other radiometric quantities are derived from radiant energy. These quantities will measure the amount of radiant energy (read: number of photons) in some region of space and/or time.

2.2 Radiant Flux (P or Φ in Watts, $W = \frac{J}{s}$)

Radiant flux (or power) is energy per unit time. In most cases, the world is in a steady state and radiant flux and radiant energy can be interchanged. When flux is measured by some detector, the detector records the average energy over some time period. As this time period is reduced, the detector produces a better and better estimate of the *instantaneous* flux. Power is a derivative:

$$\Phi = \frac{dQ}{dt} \quad \text{or} \quad dQ = \Phi\, dt$$

2.3 Area Distribution of Flux (E or M in $\frac{W}{m^2}$)

Area distribution is the amount of flux per unit of area, or:

$$E = \frac{d\Phi}{dA}$$

The area distribution of flux has two names depending on whether it describes the flux leaving a surface, *radiant exitance*, M (aka. *radiosity*, B), or falling on a surface, *irradiance*, E.

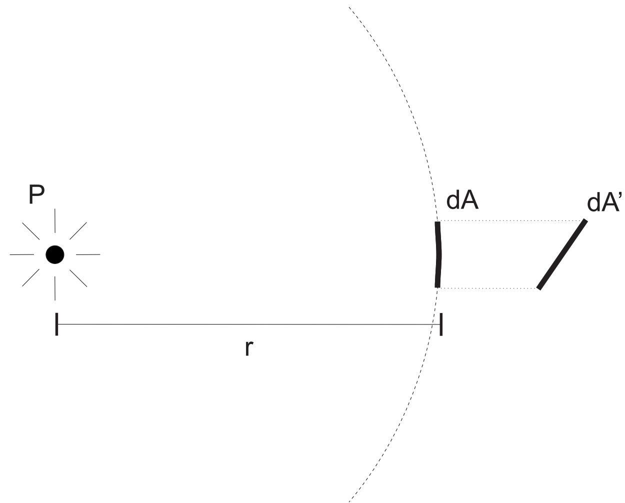

Figure 1: A point source with power, P, illuminates two small surfaces, dA and dA', at a distance of r. Note: dA' is meant to superimpose dA, but is drawn to the side for clarity.

2.3.1 Example: Irradiance from a point source

The small patch, dA, in Figure 1 can be considered as a fraction of the surface area of a sphere with radius r centered on the point source. The surface area of this sphere is:

$$Area = 4\pi r^2$$

The flux per unit area through this sphere is is uniform and therefore is:

$$E = \frac{P}{4\pi r^2}$$

The area dA must experience the same density of flux, so the total flux through dA is its area times this flux density.

$$\begin{aligned} E &= \frac{d\Phi}{dA} = \frac{P}{4\pi r^2} \\ d\Phi &= \frac{P\ dA}{4\pi r^2} \end{aligned}$$

The patch dA' is not perpendicular to the source. However, clearly from the diagram, the same amount of total flux passes through dA' as dA. The area of dA' can be related to the area of dA (assume an angle of θ between them):

$$dA' = \frac{dA}{cos\ \theta}$$

Finally, the flux density through dA' can be calculated by dividing the total flux through dA' by its area.

$$E = \frac{P\ cos\ \theta}{4\pi r^2}$$

2.4 Solid Angle Distribution of Flux (I in $\frac{W}{sr}$)

The solid angle distribution of flux is called *intensity*, I. It measures the flux per unit solid angle, Ω.

$$I = \frac{d\Phi}{d\Omega}$$

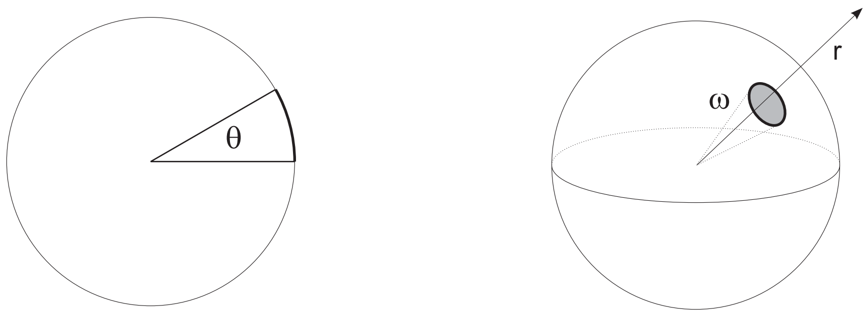

Figure 2: Illustrates the analogy between angles and solid angles.

2.4.1 Aside: Solid Angle

Solid angles in 3D are analogous to simple angles in 2D. Figure 2 attempts to demonstrate this analogy. In 2D, an angle can be measured in radians. A radian is the arc length of a unit circle subtended by an angle, θ. The entire circumference has a radian measure of 2π. Similarly in 3D, a solid angle can be measured in steradians. A steradian is the surface area of the unit sphere subtended by the solid angle, ω. The entire surface area is said to have a steradian measure of 4π.

Solid angles are used to describe sets of directions. Consider the ray, r, drawn in the unit sphere in Figure 2, clearly r intersects the sphere at a particular point, and this point is uniquely determined by the direction of r. There is a direct mapping from directions to points on the unit sphere, and the set of points on the unit sphere subtended by the solid angle ω is also a set of directions subtended by the solid angle.

Refer once again to Figure 1. It would be useful to able to calculate the solid angle of directions subtended by the area, dA. In the example from the previous section, the area dA was described as a fraction of the area of the sphere which intersects it.

$$\text{fractional area} = \frac{dA}{4\pi r^2}$$

Clearly this must also be the fraction of the total solid angle at point, P, subtended by dA.

$$d\Omega = 4\pi \cdot \frac{dA}{4\pi r^2} = \frac{dA}{r^2}$$

Additionally, generalization to non-perpendicular surfaces proceeds exactly as in the previous example and yields the same additional cosine term.

$$d\Omega = \frac{\cos\theta \, dA}{r^2}$$

2.5 Radiance (L in $\frac{W}{m^2 \cdot sr}$)

Radiance is the flux density per unit area per unit solid angle[1]. It represents the amount of flux through a neighborhood in position-direction (ray) space. Thus radiance is a second derivative of flux.

$$L = \frac{d^2\Phi}{dAd\Omega}$$

These derivatives are unordered, so:

$$L = \frac{d^2\Phi}{dAd\Omega} = \frac{d^2\Phi}{d\Omega dA}$$

Radiance can also be expressed in terms of the area and solid angle densities of the previous sections. The earlier formulas for irradiance and intensity, $E = \frac{d\Phi}{dA}$ and $I = \frac{d\Phi}{d\Omega}$, can be substituted into the second derivitive above.

$$L = \frac{d}{d\Omega}\left(\frac{d\Phi}{dA}\right) = \frac{dE}{d\Omega}$$

$$L = \frac{d}{dA}\left(\frac{d\Phi}{d\Omega}\right) = \frac{dI}{d\Omega}$$

One of the most useful properties of radiance is that it is conserved along straight line paths. This property is valuable because it simplifies many rendering computations. However, the proof of this is left to the next lecture.

[1] Note that the area is stipulated to be perpendicular to the direction of the solid angle. For generally oriented surfaces, a factor of $cos\ \theta$ needs to be introduced: $L = \frac{d^2\Phi}{cos\ \theta\ dAd\Omega}$, to account for the projection. This will be covered in future lectures.