Microfacet models for reflection and refraction

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Cornell University CS 6630 Fall 2015

(based on presentation for
Walter, Marschner, Li, and Torrance EGSR ’07)

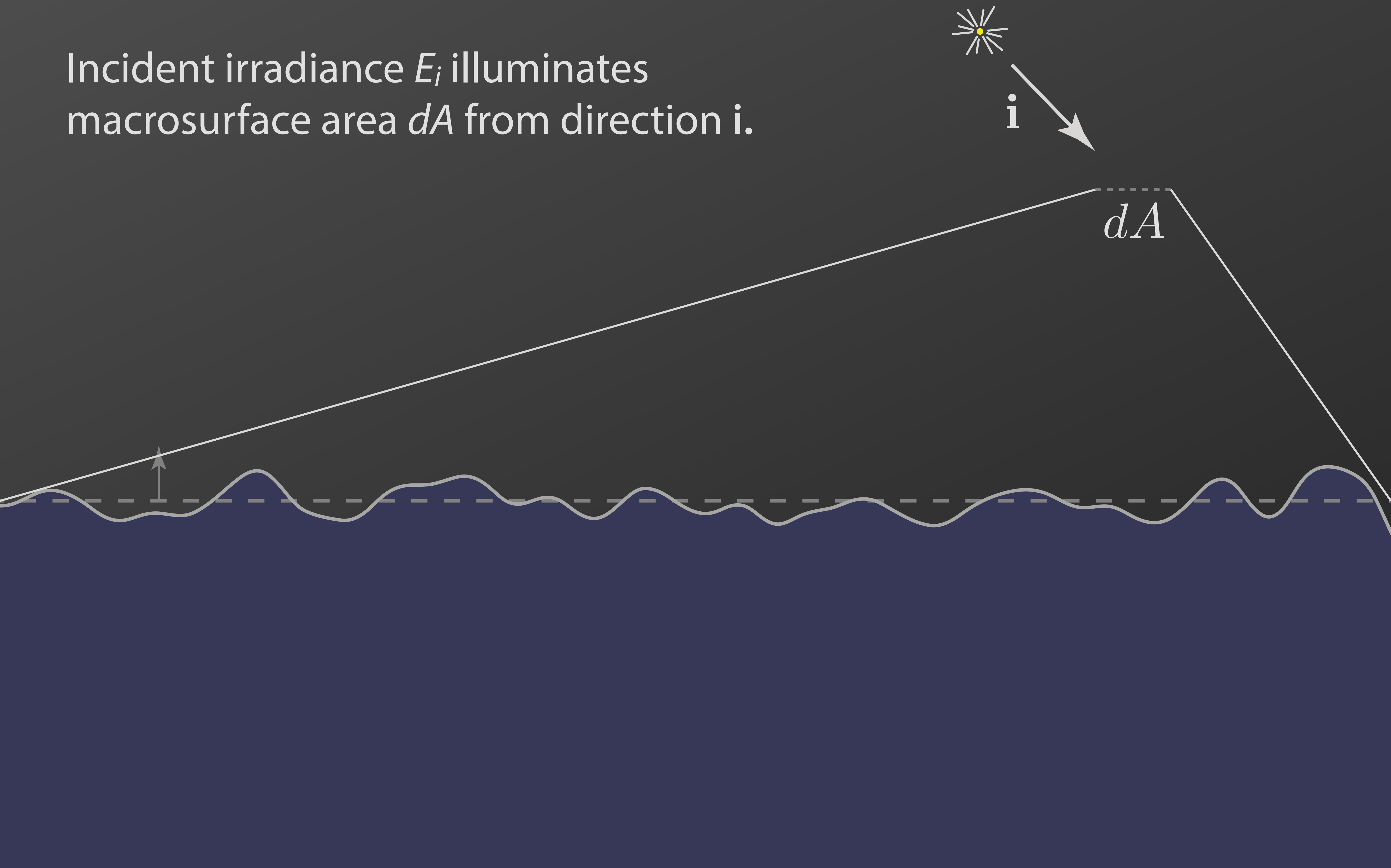
Microfacet scattering models

Rough dielectric surface

- smooth at wavelength scale
- rough at microscale
- flat at macroscale

Microfacet scattering models

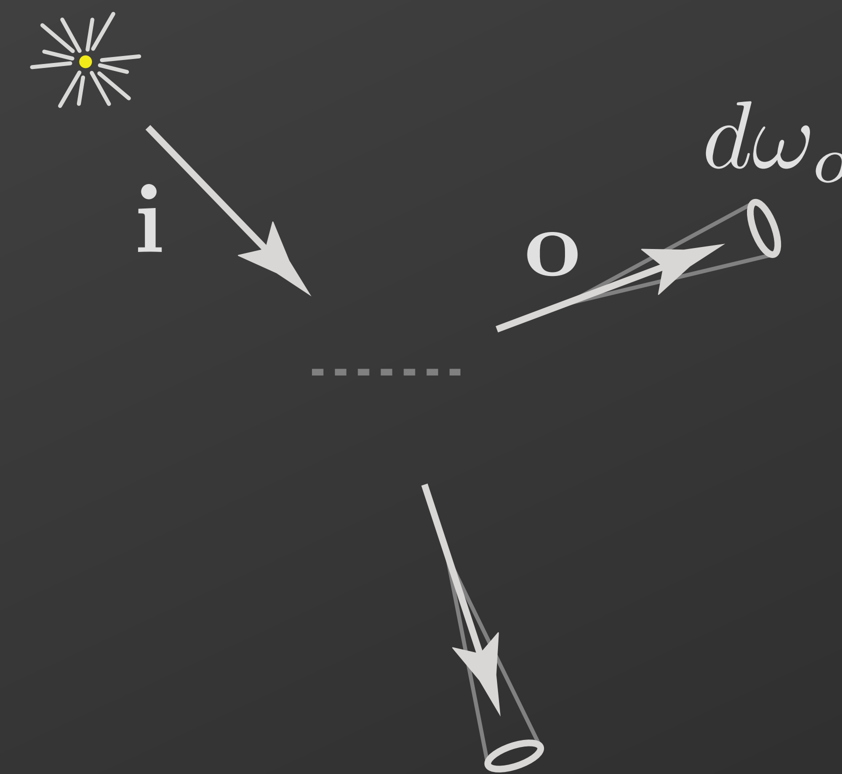
Incident irradiance E_i illuminates macrosurface area dA from direction **i**.

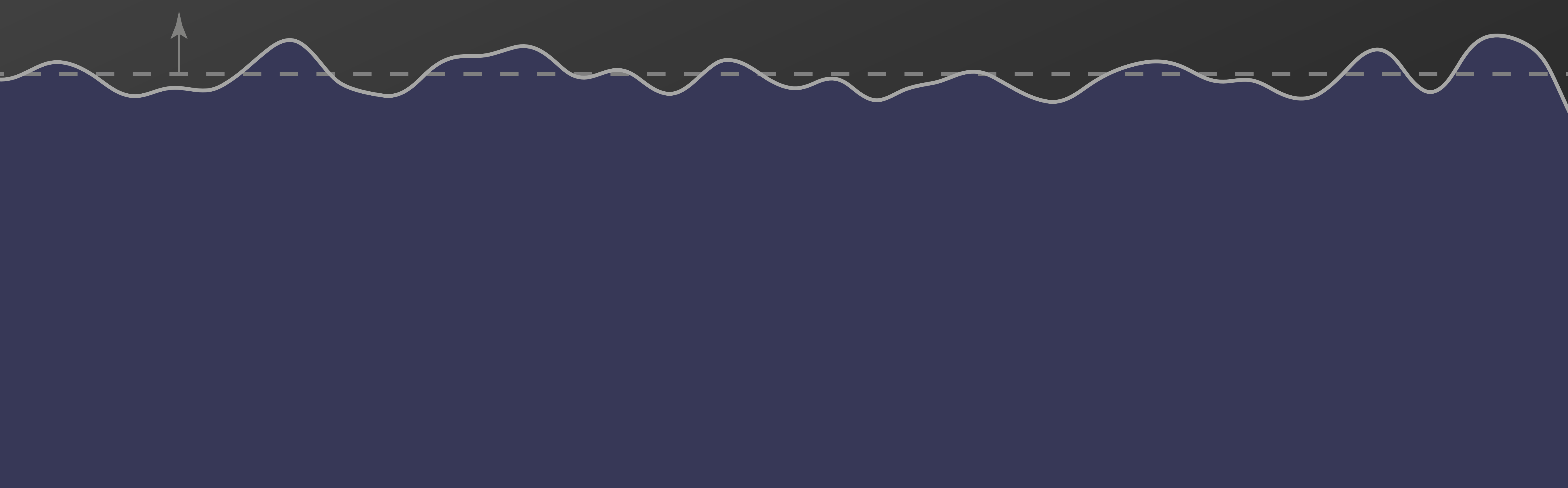


Microfacet scattering models

Incident irradiance E_i illuminates macrosurface area dA from direction **i**.

Scattered radiance L_r or L_t measured in direction **o** in solid angle $d\omega_o$.





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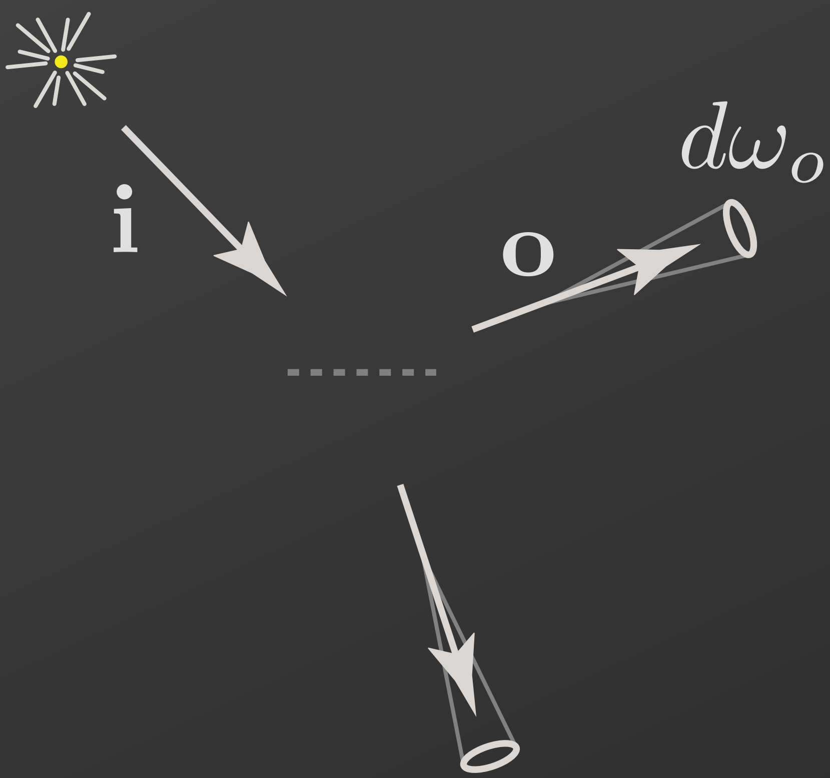
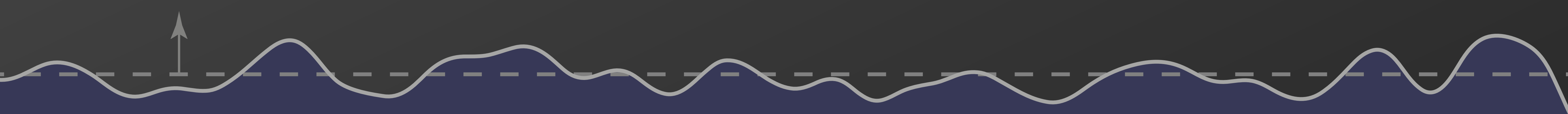




$$f_s(\mathbf{i}, \mathbf{o}) = \frac{L_{r,t}}{E_i}$$

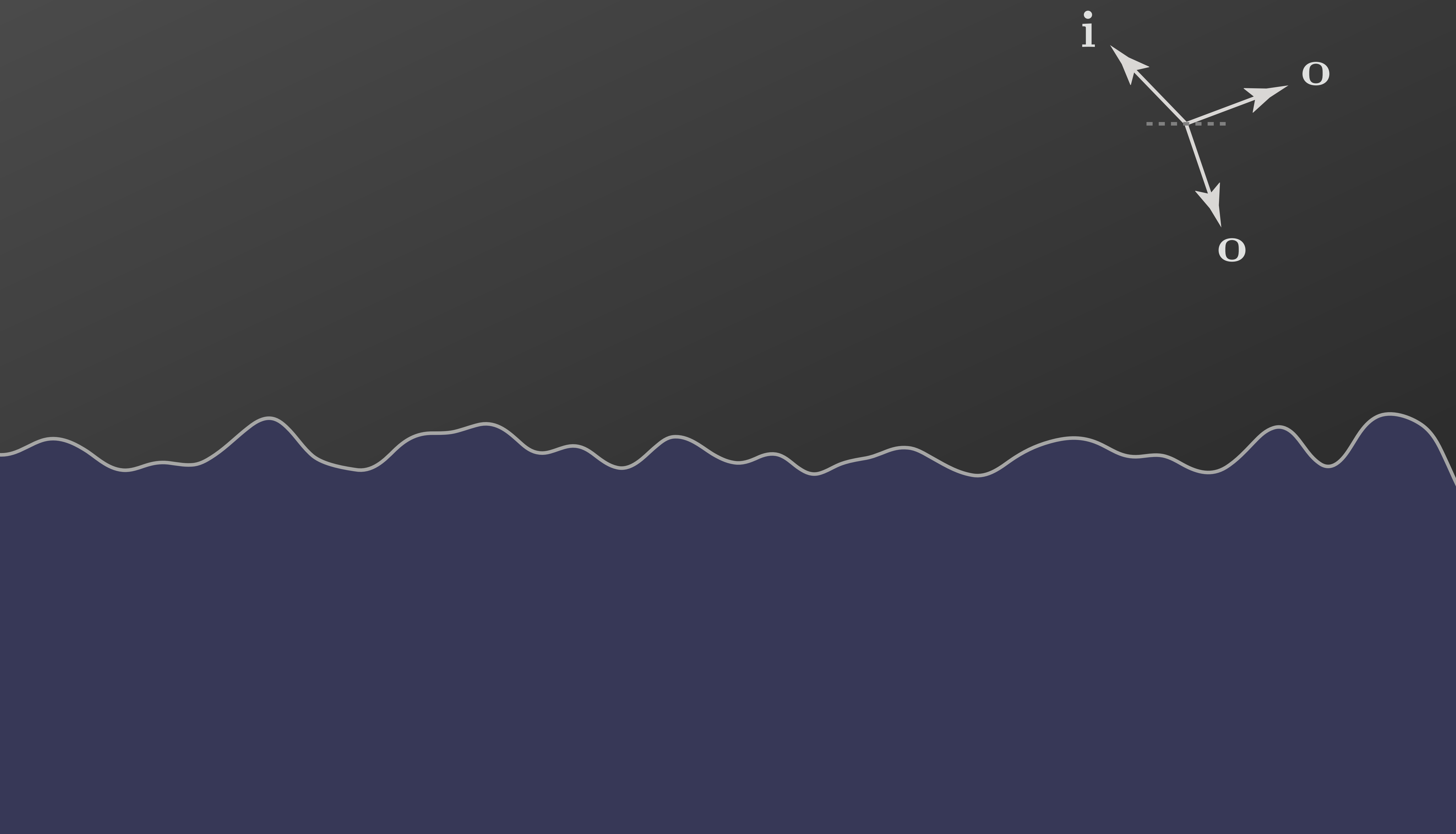
Bidirectional **S**cattering **D**istribution **F**unction

“half-vector” function
$\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution
$D(\mathbf{m})$

shadowing–masking
$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation
$\rho(\mathbf{i}, \mathbf{o})$



“half-vector” function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

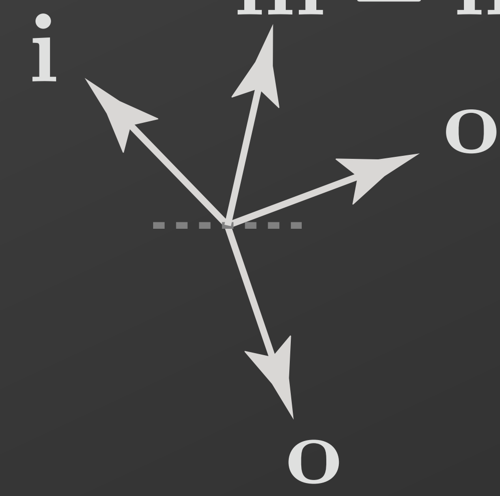
normal distribution $D(\mathbf{m})$

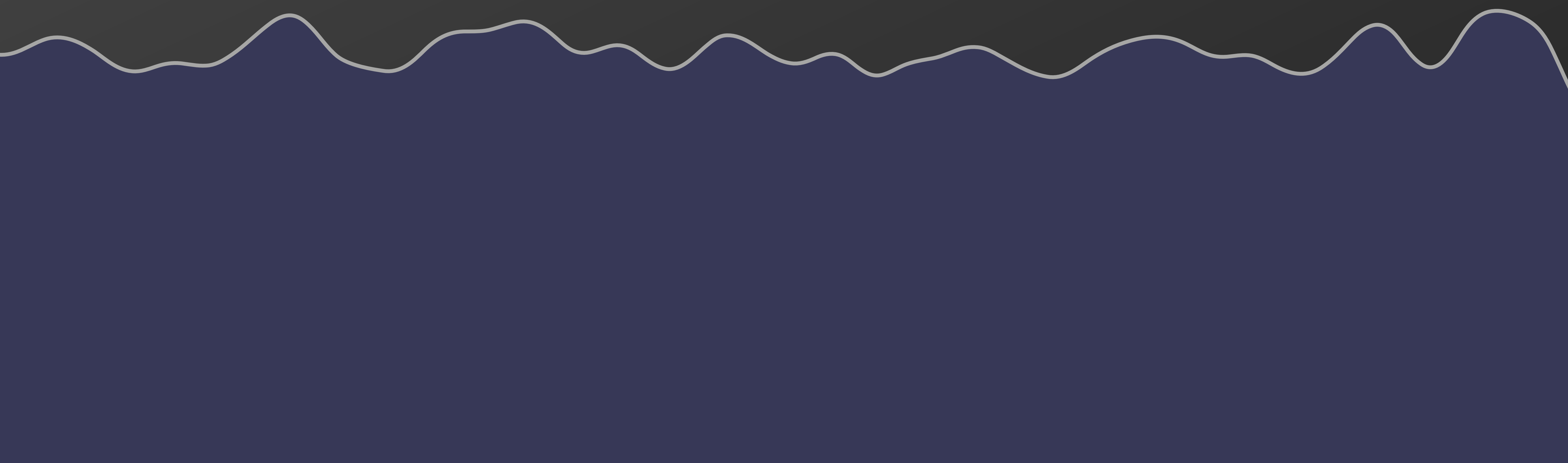
shadowing–masking $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation $\rho(\mathbf{i}, \mathbf{o})$

Gives the one microsurface normal **m** that will scatter light from **i** to **o**.

$$\mathbf{m} = \mathbf{h}(\mathbf{i}, \mathbf{o})$$





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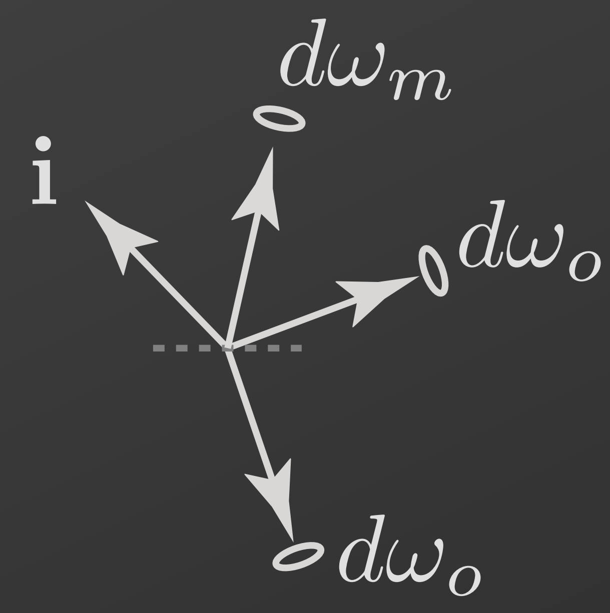
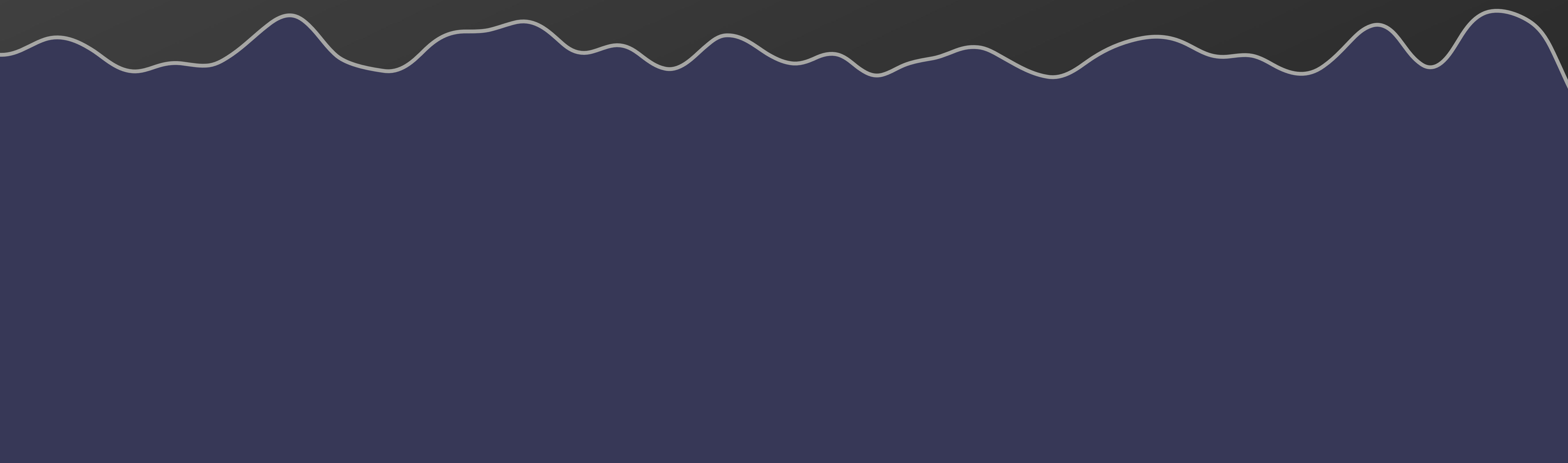


The size of the set of relevant normals $d\omega_m$ relative to the receiving solid angle $d\omega_o$ is determined by **h**.



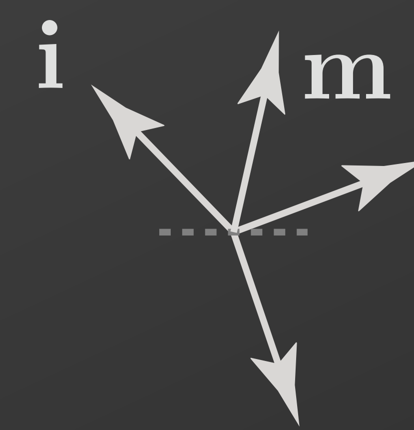
"half-vector" function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

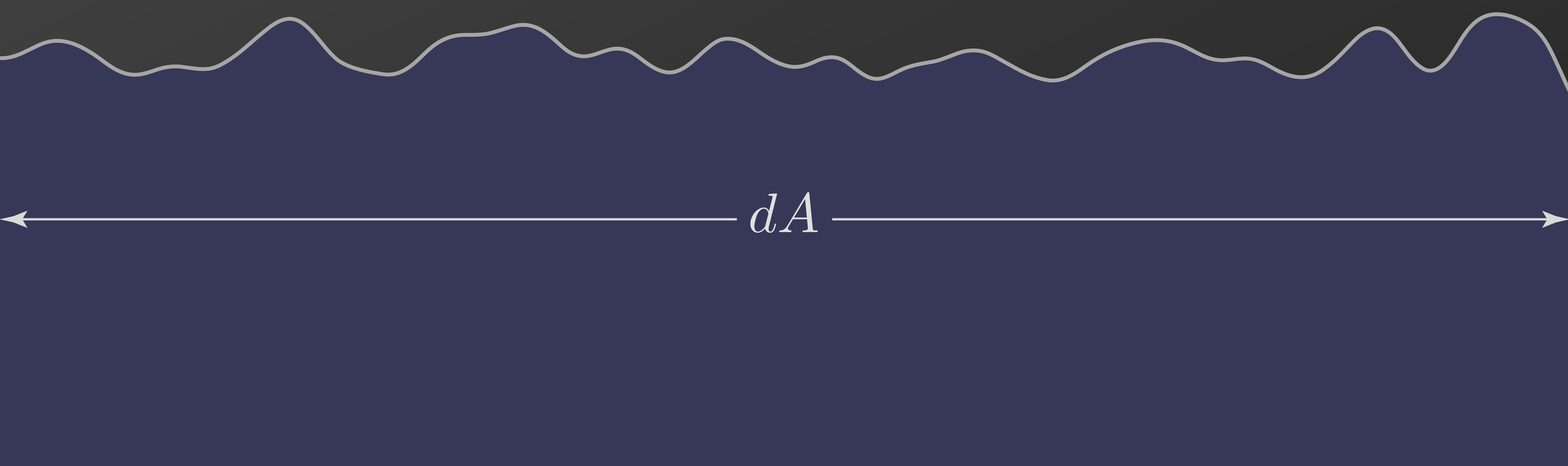
normal distribution $D(\mathbf{m})$

shadowing–masking $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation $\rho(\mathbf{i}, \mathbf{o})$

Measures density of microsurface area with respect to microsurface normal.





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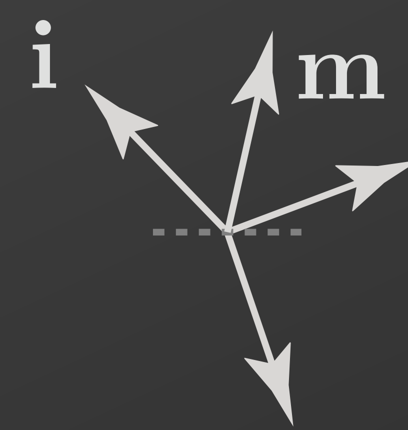

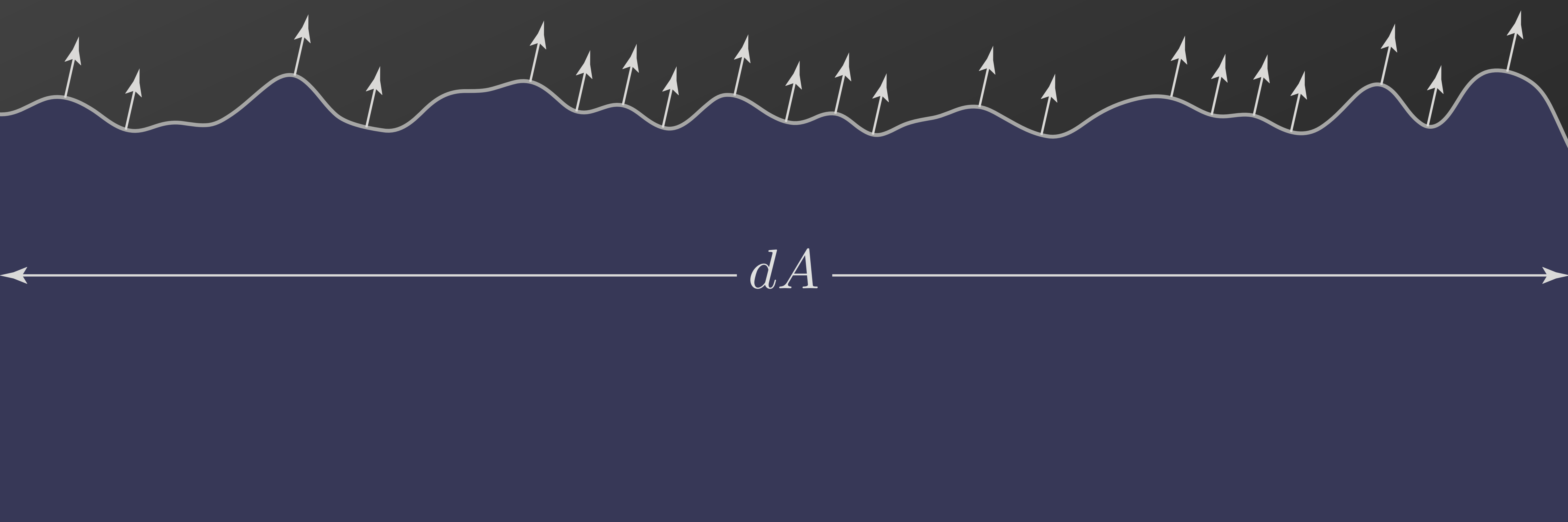

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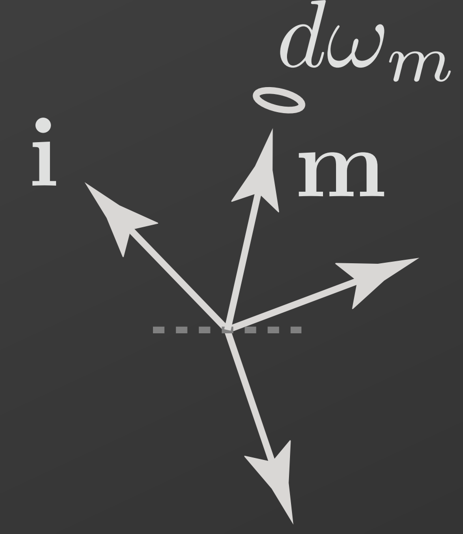
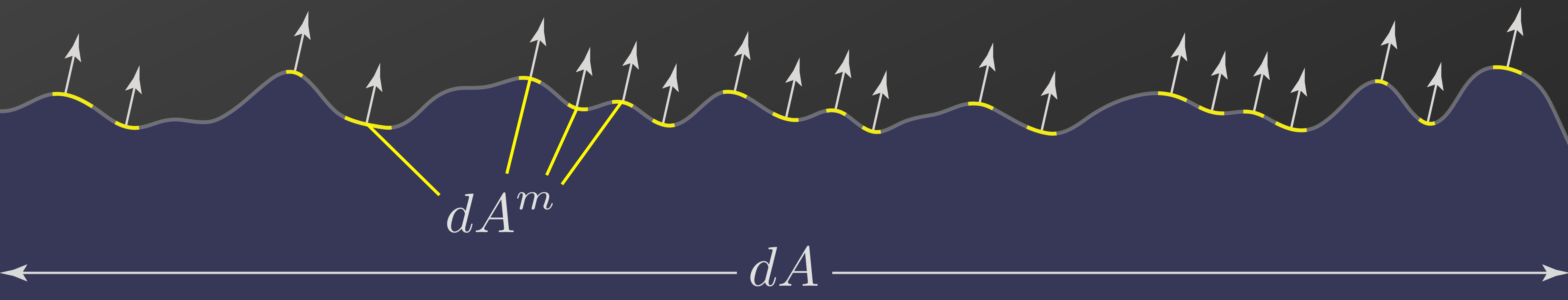


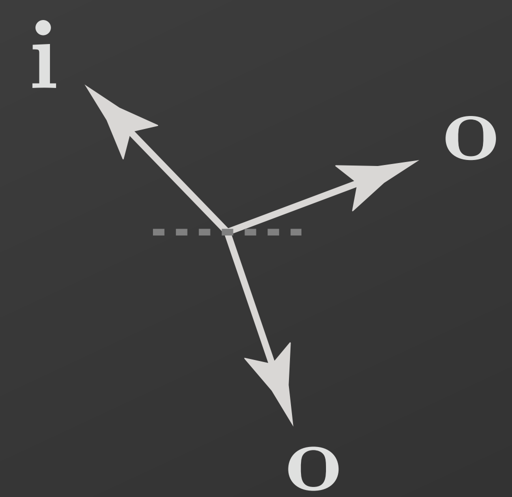
The ratio of relevant microsurface area dA^m to macrosurface area dA is $D(\mathbf{m})d\omega_m$.

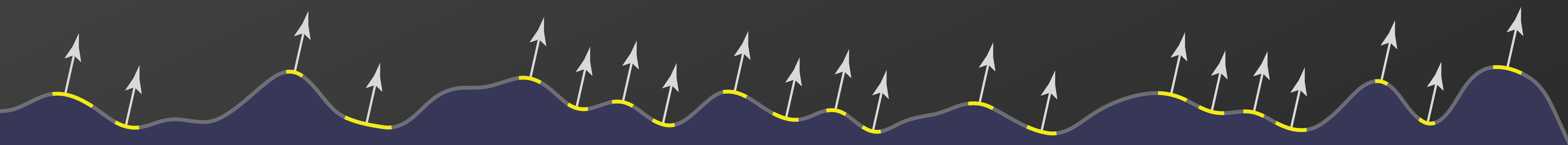


$$dA^m = D(\mathbf{m})\, d\omega_m\, dA$$

“half-vector” function	normal distribution	shadowing–masking	attenuation
$\mathbf{h}(\mathbf{i}, \mathbf{o})$	$D(\mathbf{m})$	$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$	$\rho(\mathbf{i}, \mathbf{o})$

Measures the fraction of points with microsurface normal **m** that are visible in directions **i** and **o**.





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“half-vector” function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

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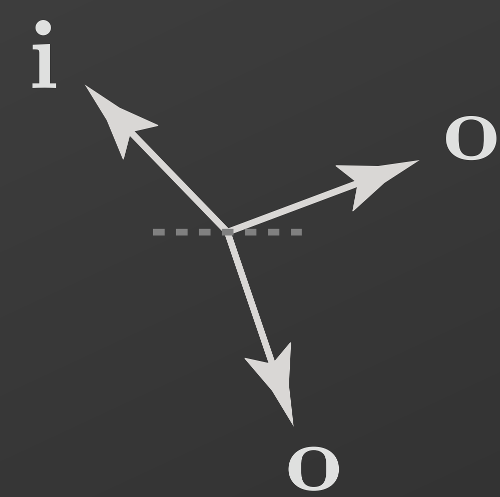

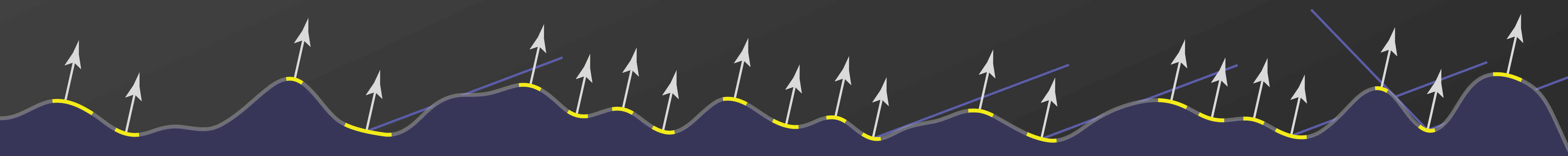

$$dA^m = D(\mathbf{m})\, d\omega_m\, dA$$

“half-vector” function $\mathbf{h}(\mathbf{i}, \mathbf{o})$ | normal distribution $D(\mathbf{m})$ | **shadowing–masking $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$** | attenuation $\rho(\mathbf{i}, \mathbf{o})$

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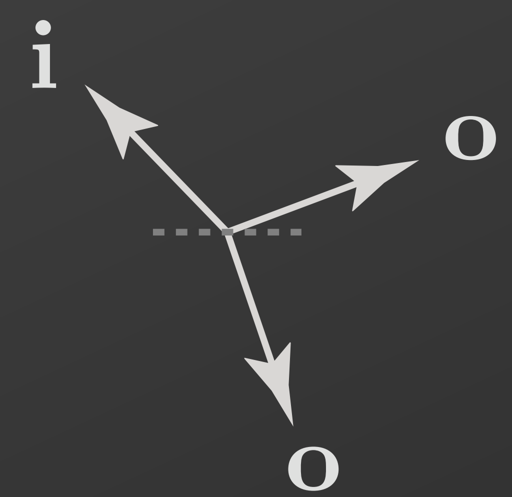

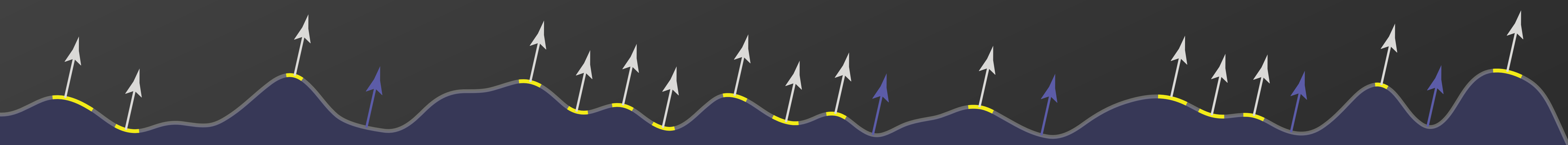

$$dA^m = D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, d\omega_m\, dA$$

“half-vector” function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution $D(\mathbf{m})$

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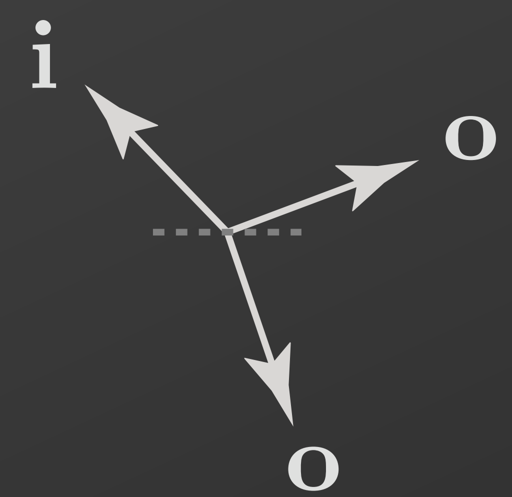
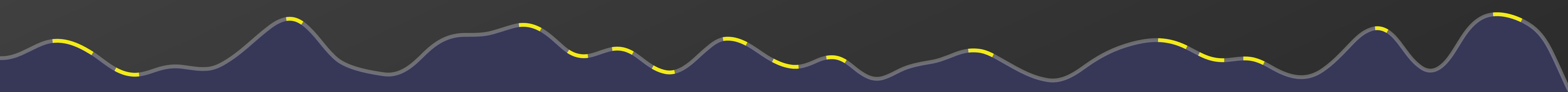


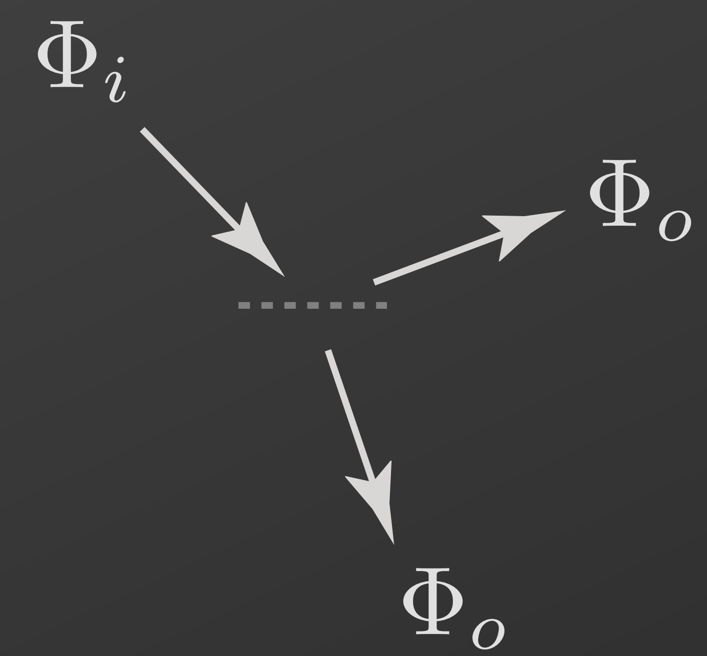
We now know the size of the scattering area, which determines how much light reflects.

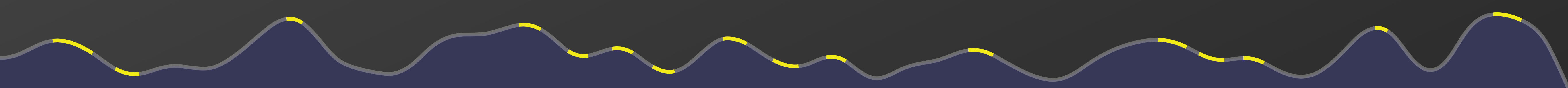


$$dA^m = D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, d\omega_m\, dA$$

"half-vector" function	normal distribution	shadowing–masking	attenuation
$\mathbf{h}(\mathbf{i}, \mathbf{o})$	$D(\mathbf{m})$	$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$	$\rho(\mathbf{i}, \mathbf{o})$

Gives the fraction of the power incident on the scattering area dA^m that is scattered.





$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) \, dA^m dE_i$$

$$dA^m = D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, d\omega_m \, dA$$

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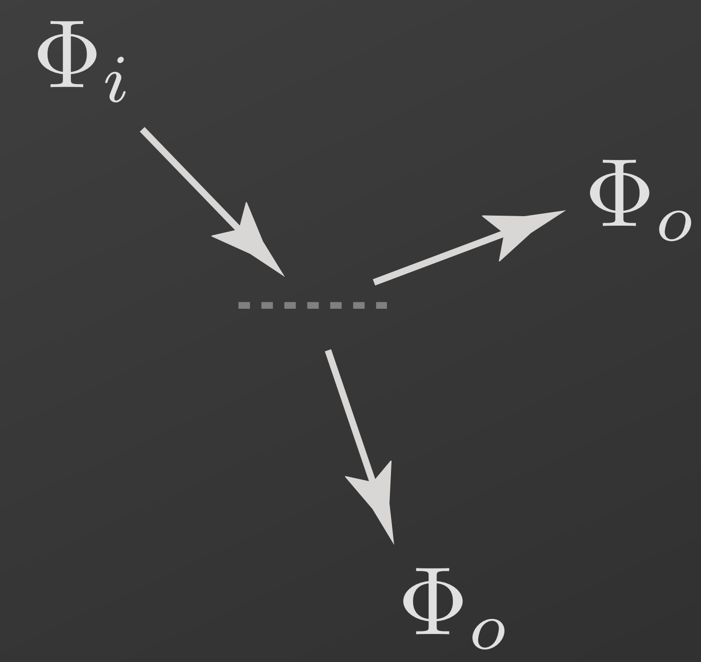
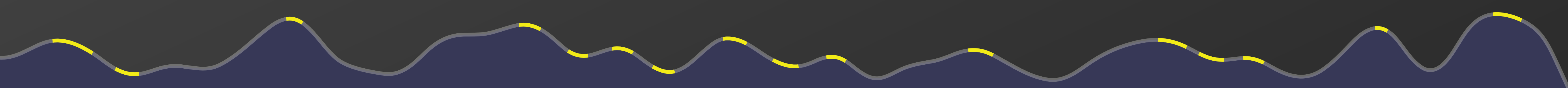


This scattered power is related to the incident irradiance by the attenuation and the scattering area, projected in the incident direction.



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$$dA^m = D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, d\omega_m \, dA$$

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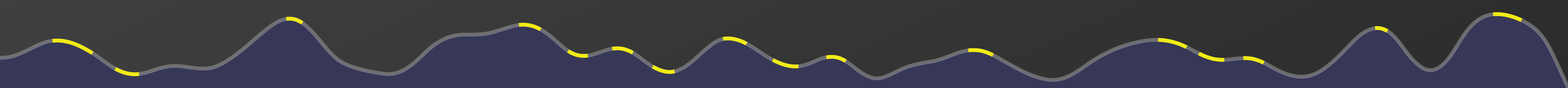
normal distribution $D(\mathbf{m})$

shadowing–masking $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation $\rho(\mathbf{i}, \mathbf{o})$

The BSDF is the ratio of scattered radiance to incident irradiance:

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{dL_o}{dE_i} = \frac{d\Phi_o^m / (dA \, |\mathbf{o} \cdot \mathbf{n}| \, d\omega_o)}{dE_i}$$



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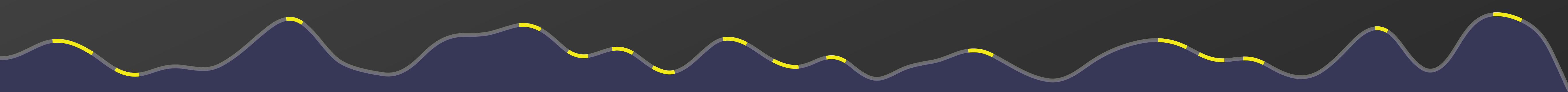

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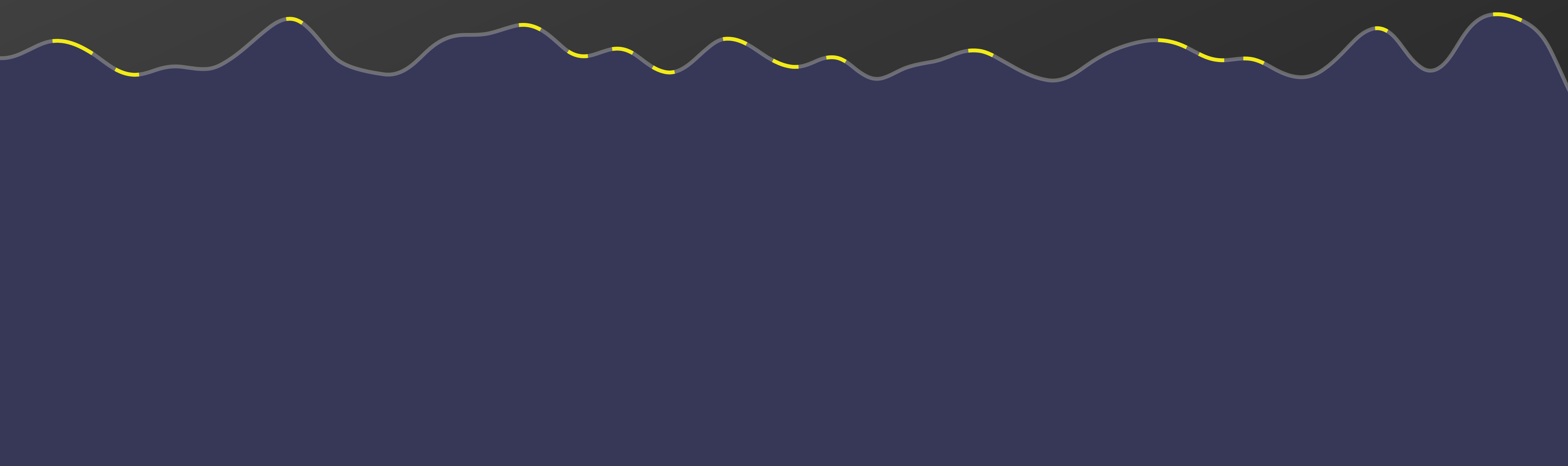

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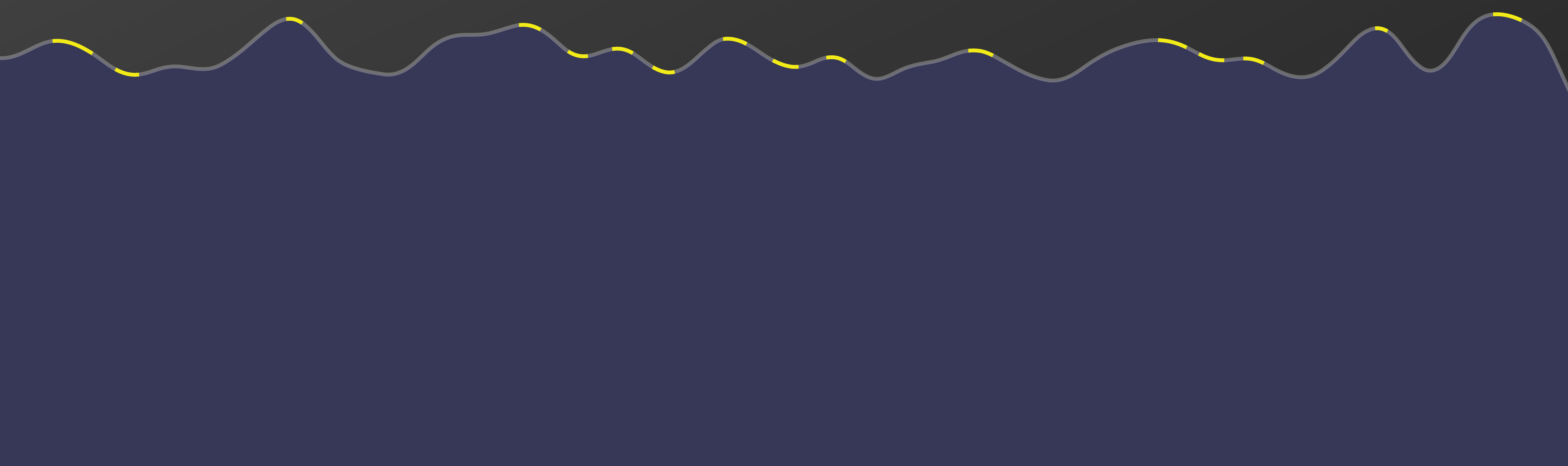
Fresnel reflection

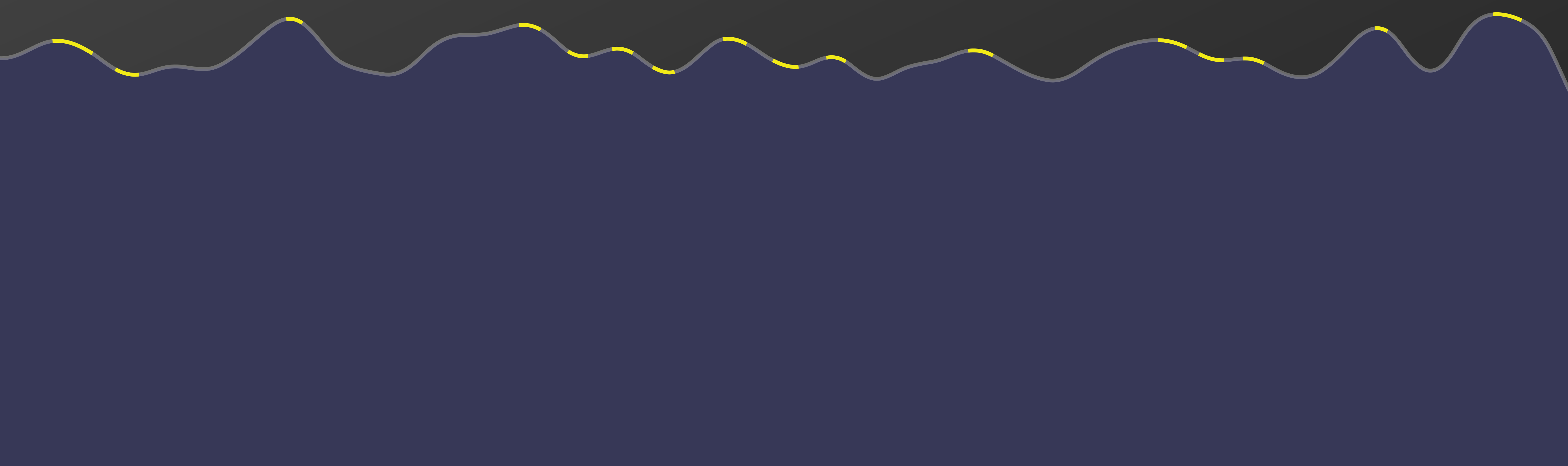
$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



"half-vector" function	normal distribution	shadowing–masking	attenuation
$\mathbf{h}(\mathbf{i}, \mathbf{o})$	$D(\mathbf{m})$	$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$	$\rho(\mathbf{i}, \mathbf{o})$

Fresnel reflection | surface roughness

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



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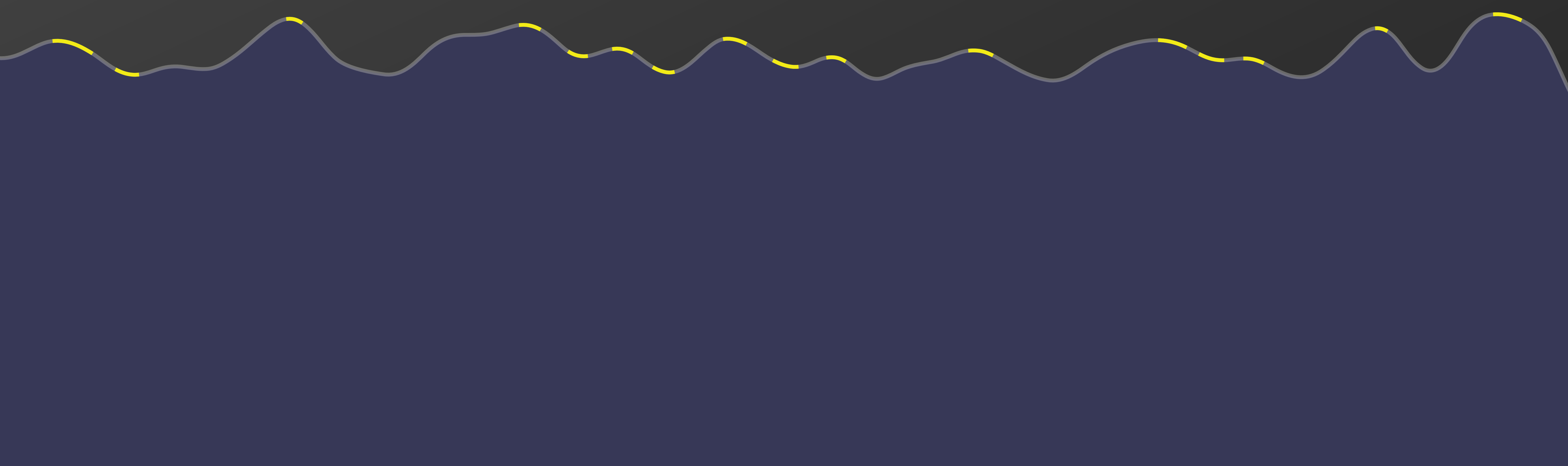
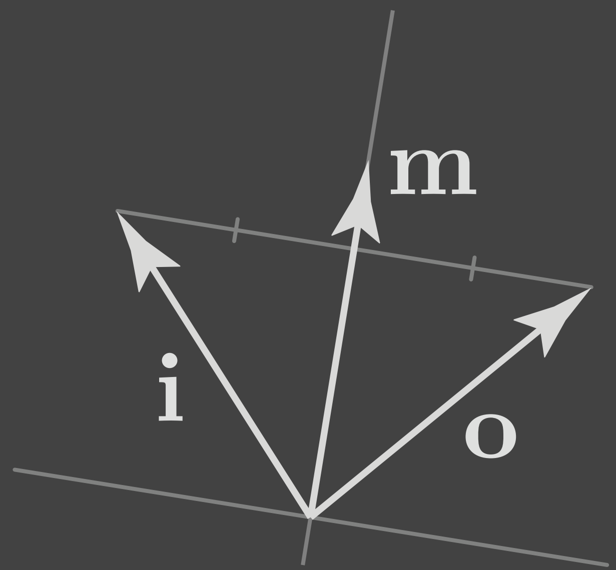
determined by geometry



Construction of half-vector

reflection



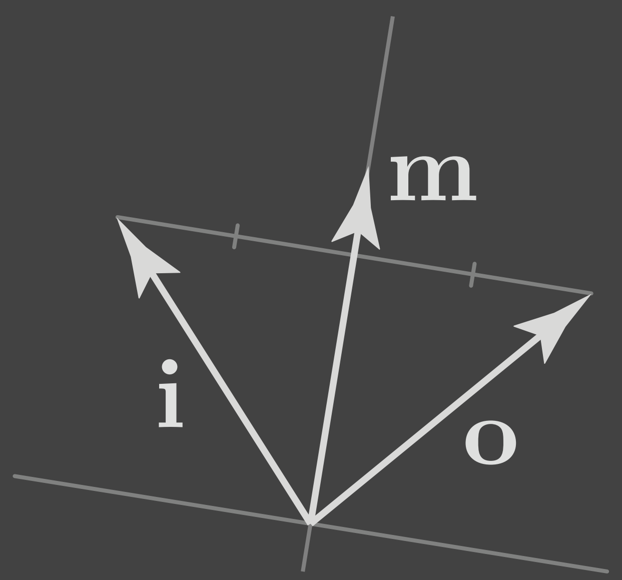
$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

refraction

Construction of half-vector

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



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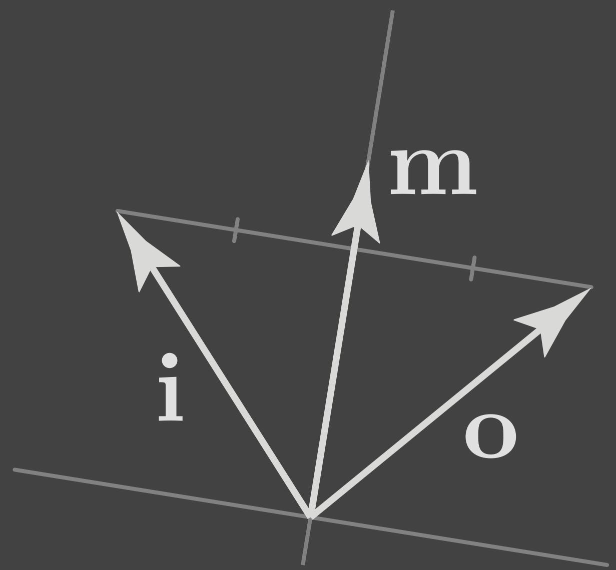

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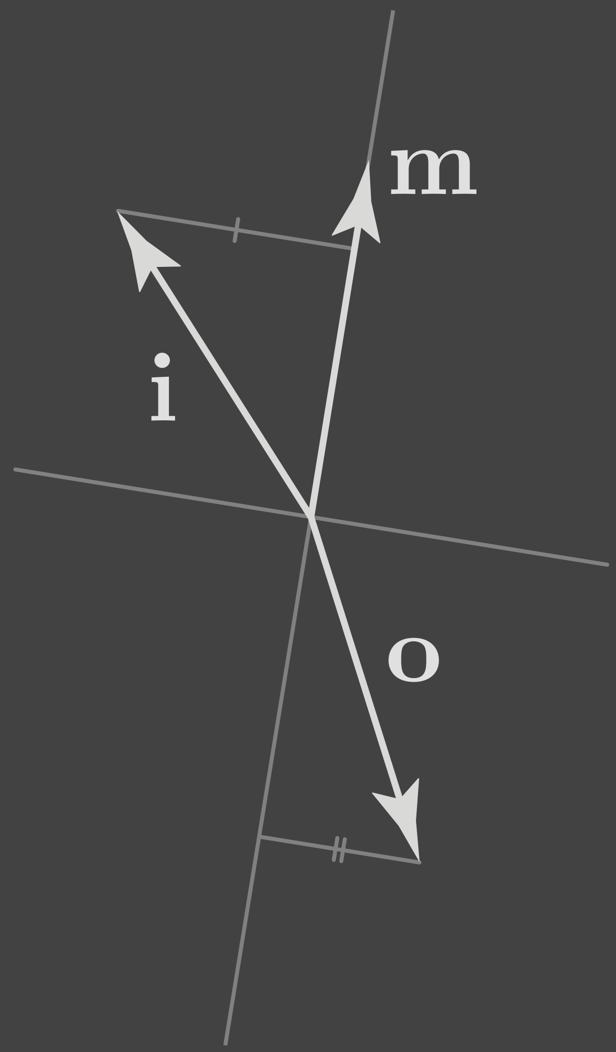

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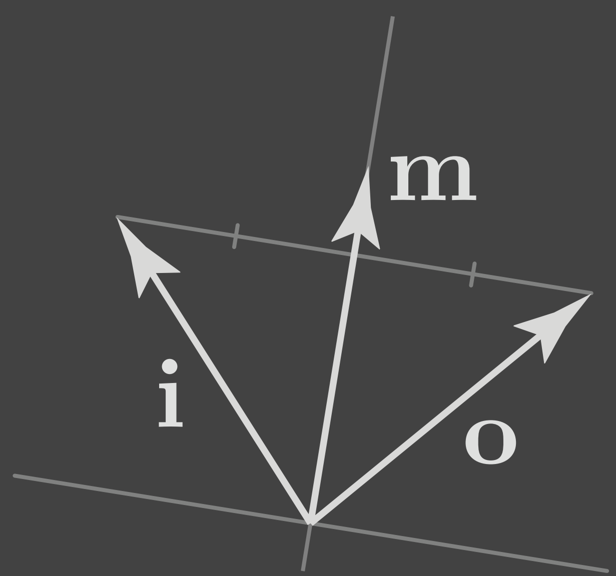

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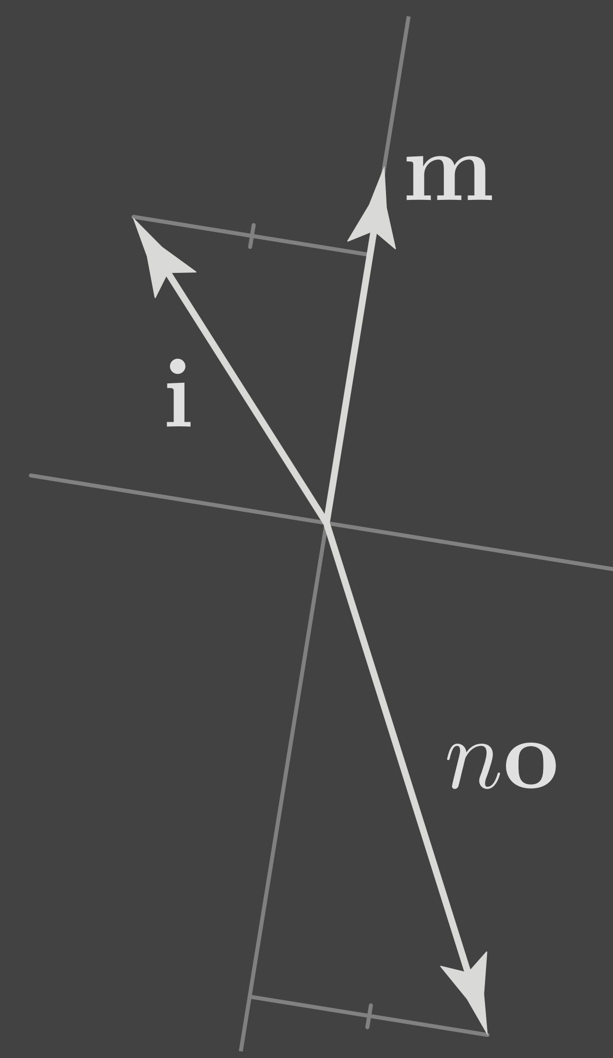

$\mathbf{i} + n\mathbf{o}$ parallel to $\mathbf{m}$

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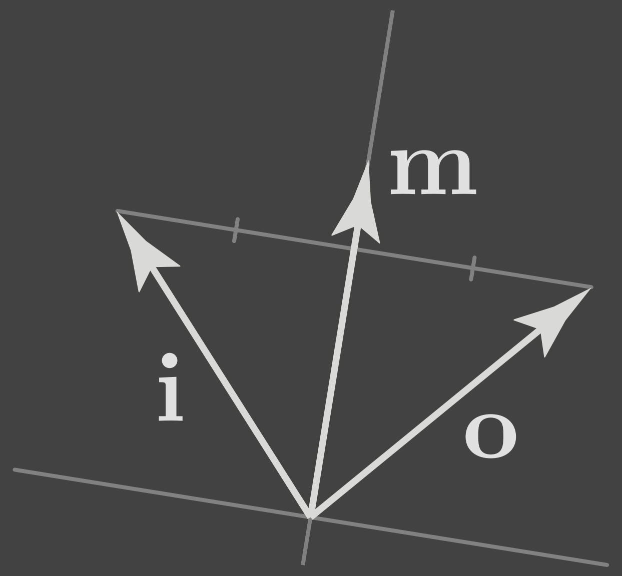
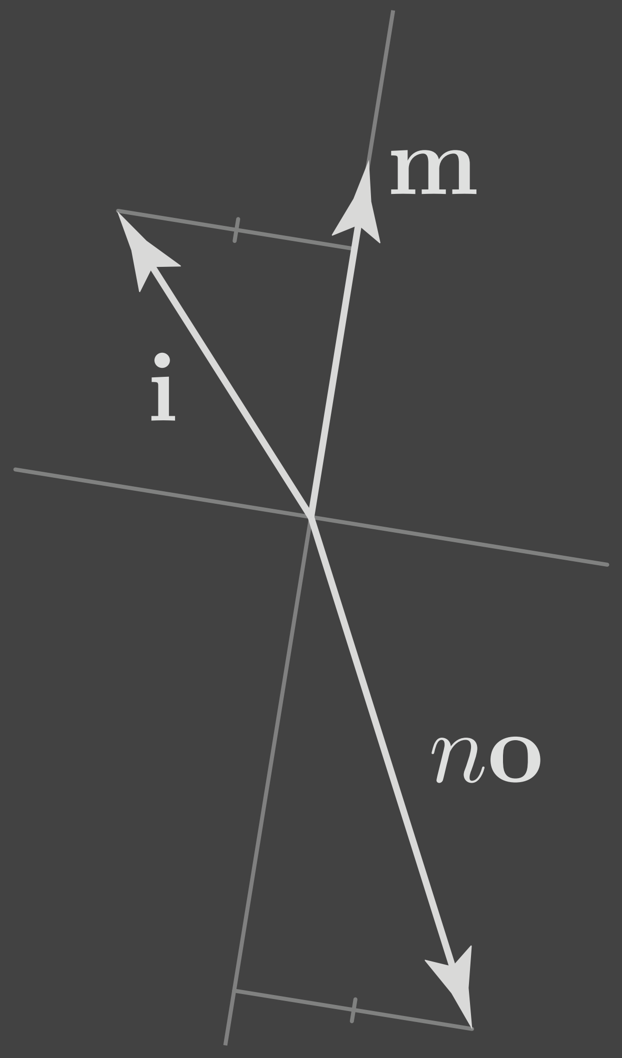


$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

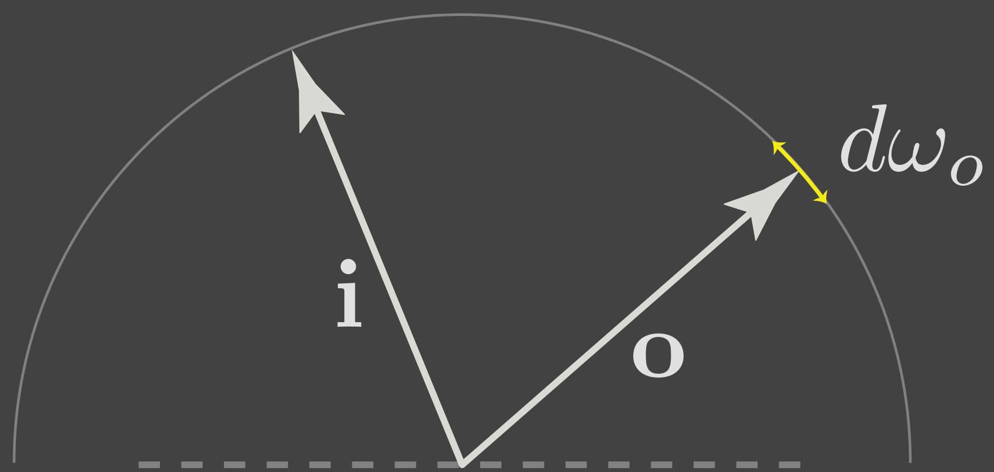


$\mathbf{i} + n\mathbf{o}$ parallel to $\mathbf{m}$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



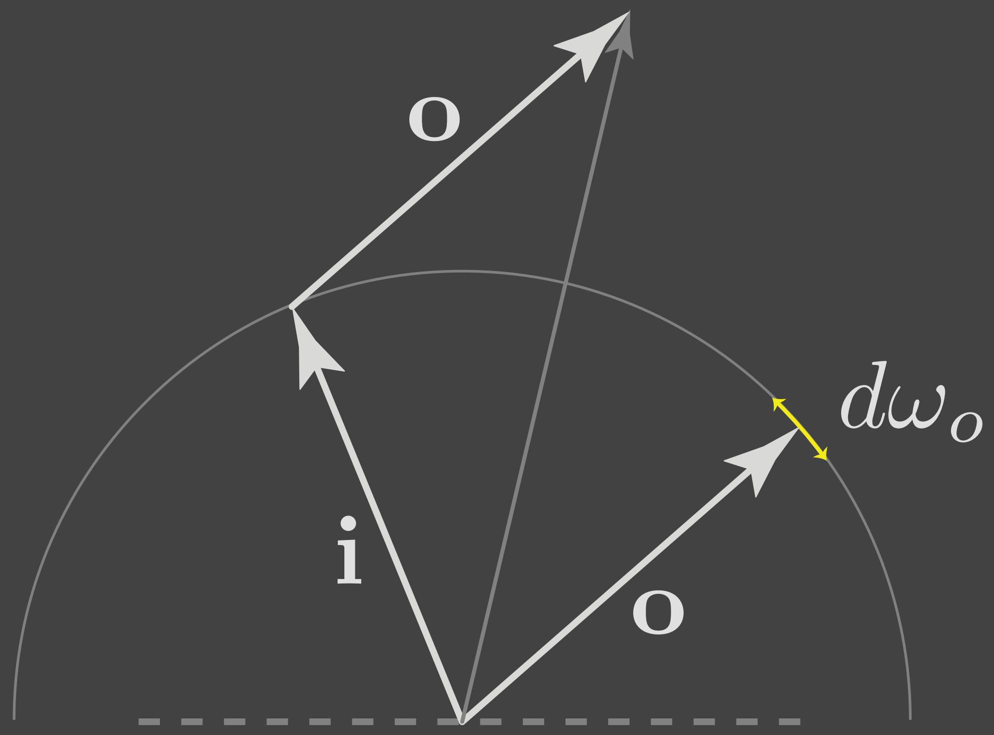
refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \mathrm{normalize}(\mathbf{i} + \mathbf{o})$$



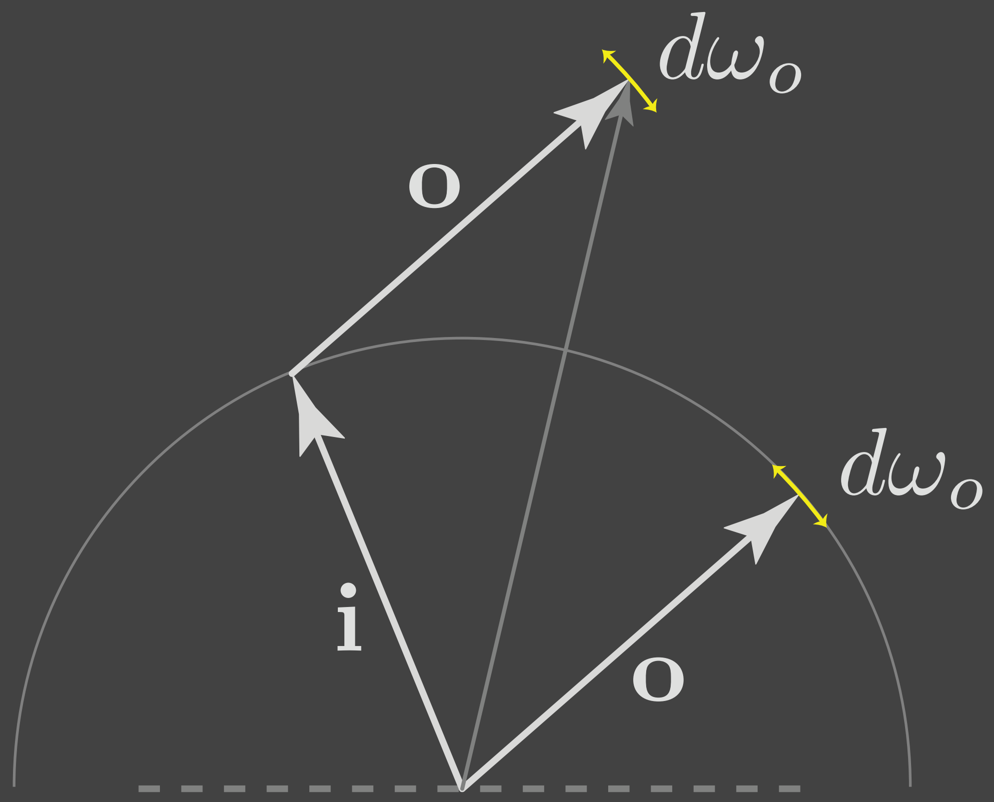
refraction

$$\mathbf{h}_t = -\mathrm{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \mathrm{normalize}(\mathbf{i} + \mathbf{o})$$



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Construction of half-vector solid angle

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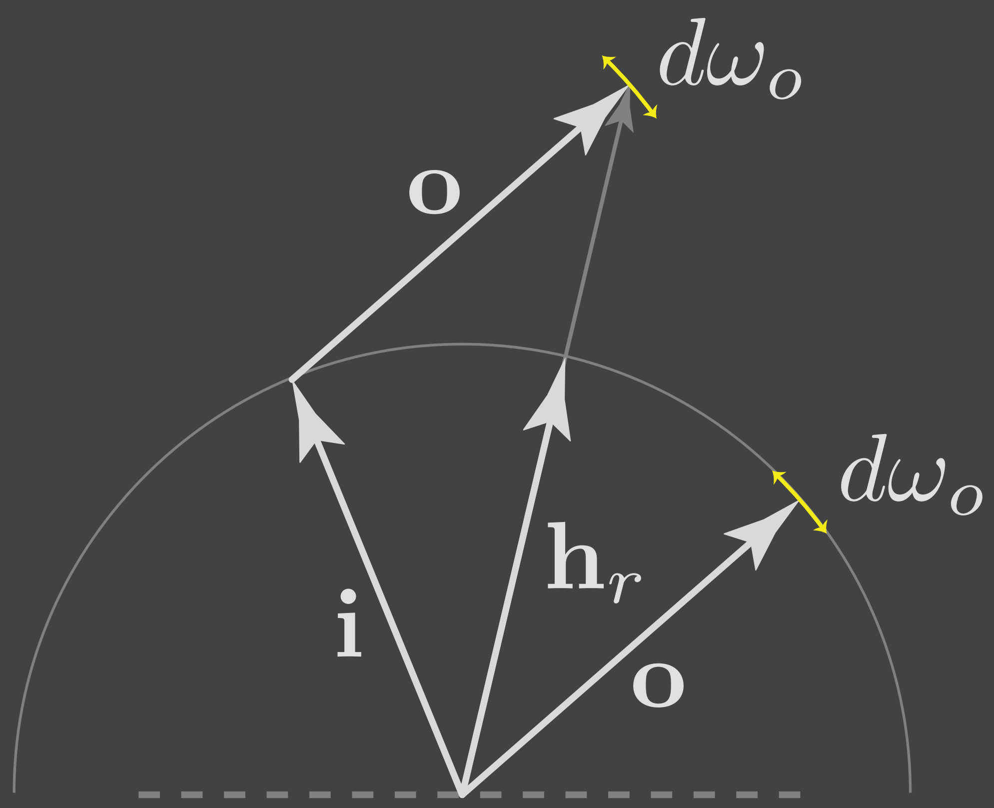

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Construction of half-vector solid angle

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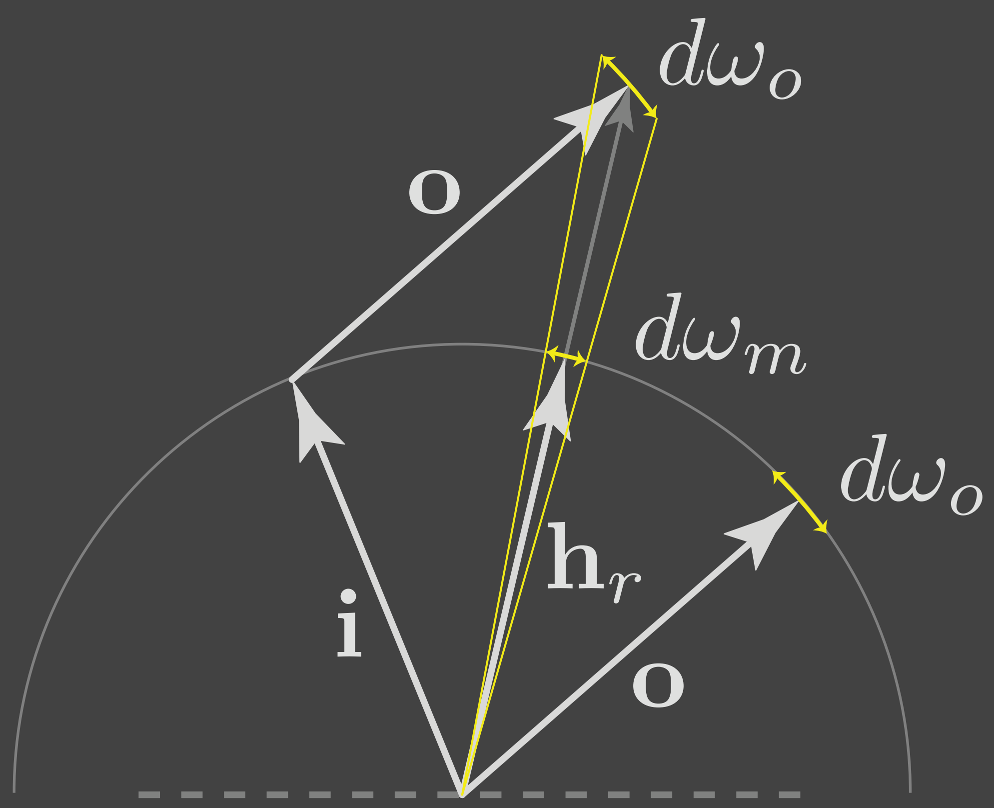

$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

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Construction of half-vector solid angle

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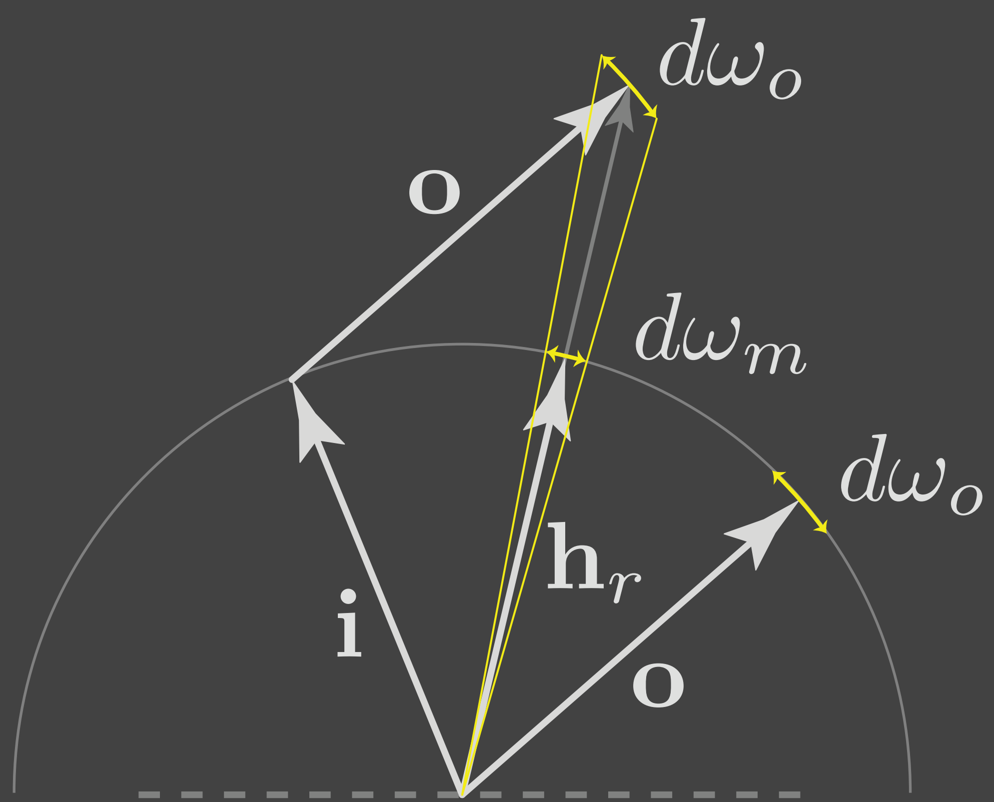

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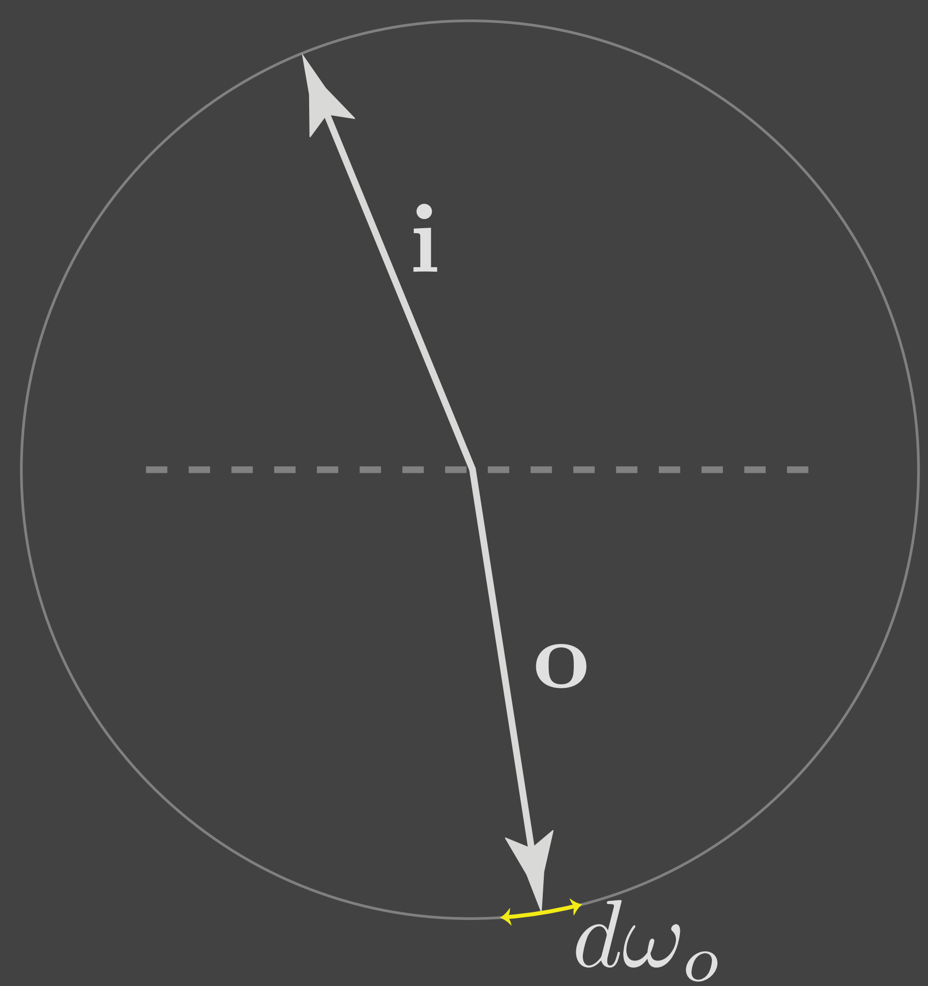

Construction of half-vector solid angle

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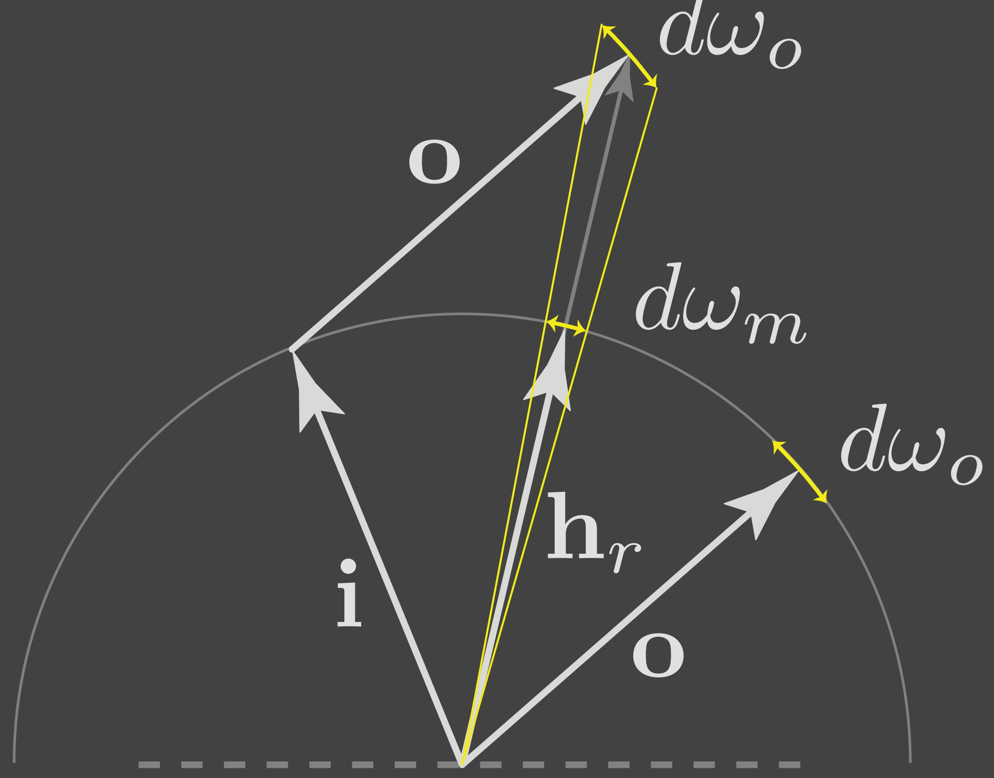

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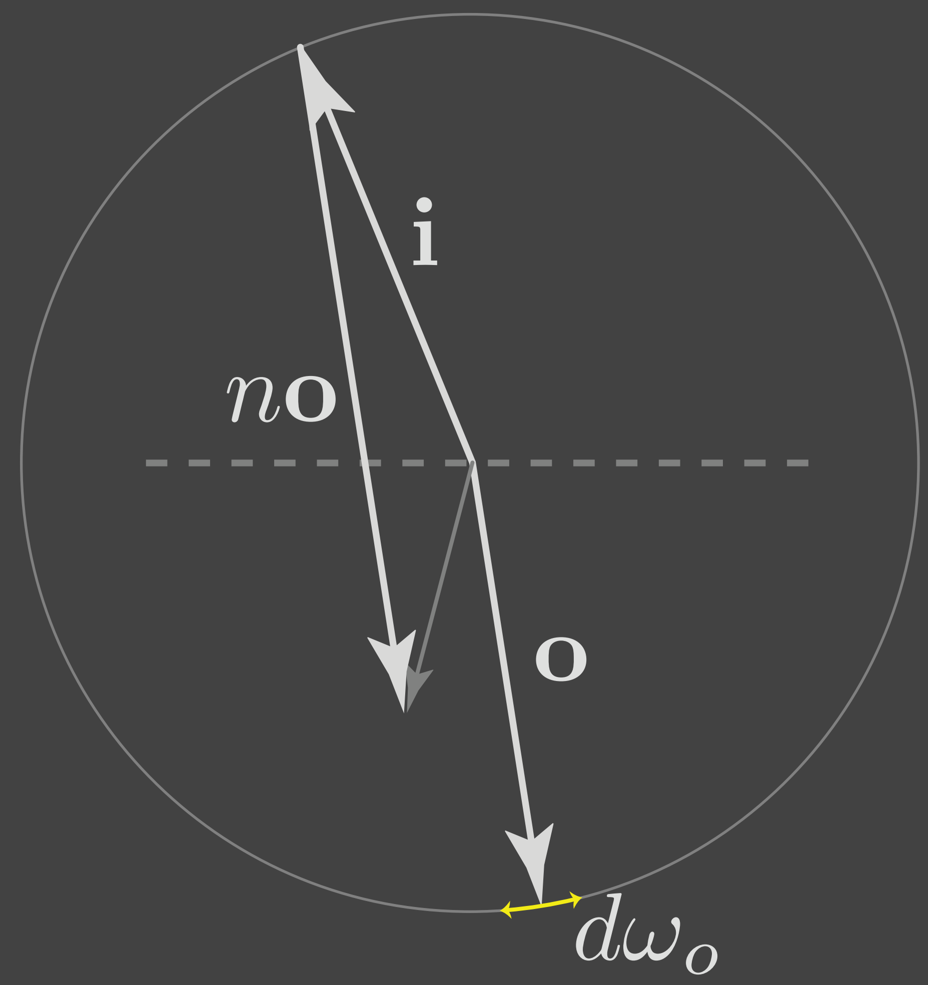
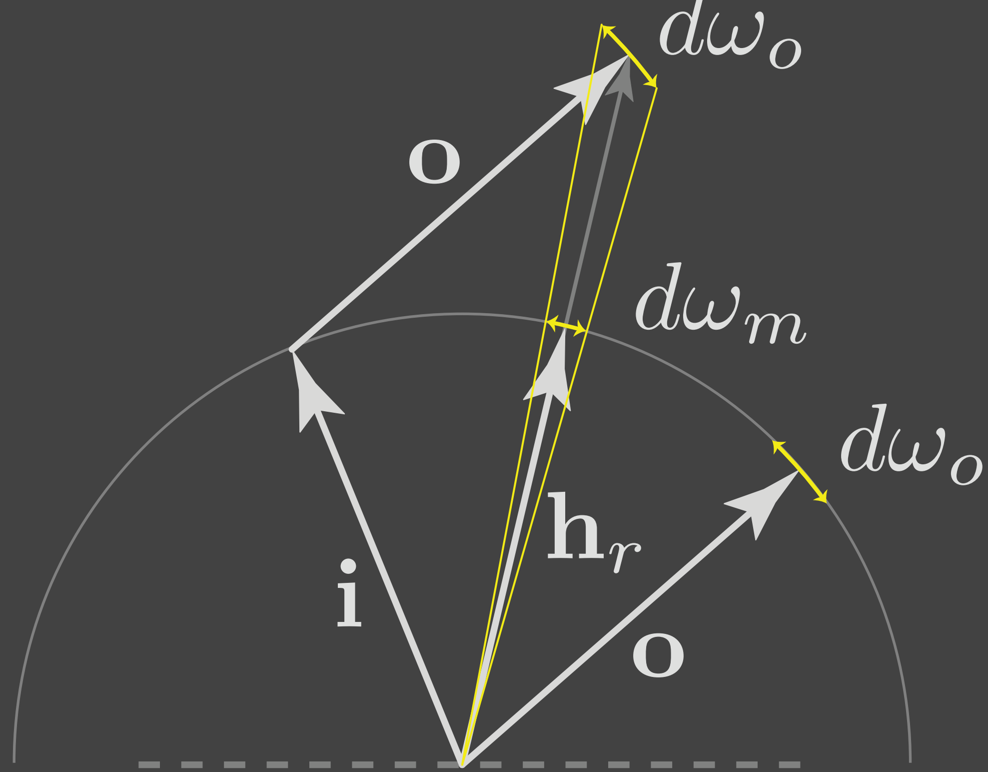


Construction of half-vector solid angle

reflection

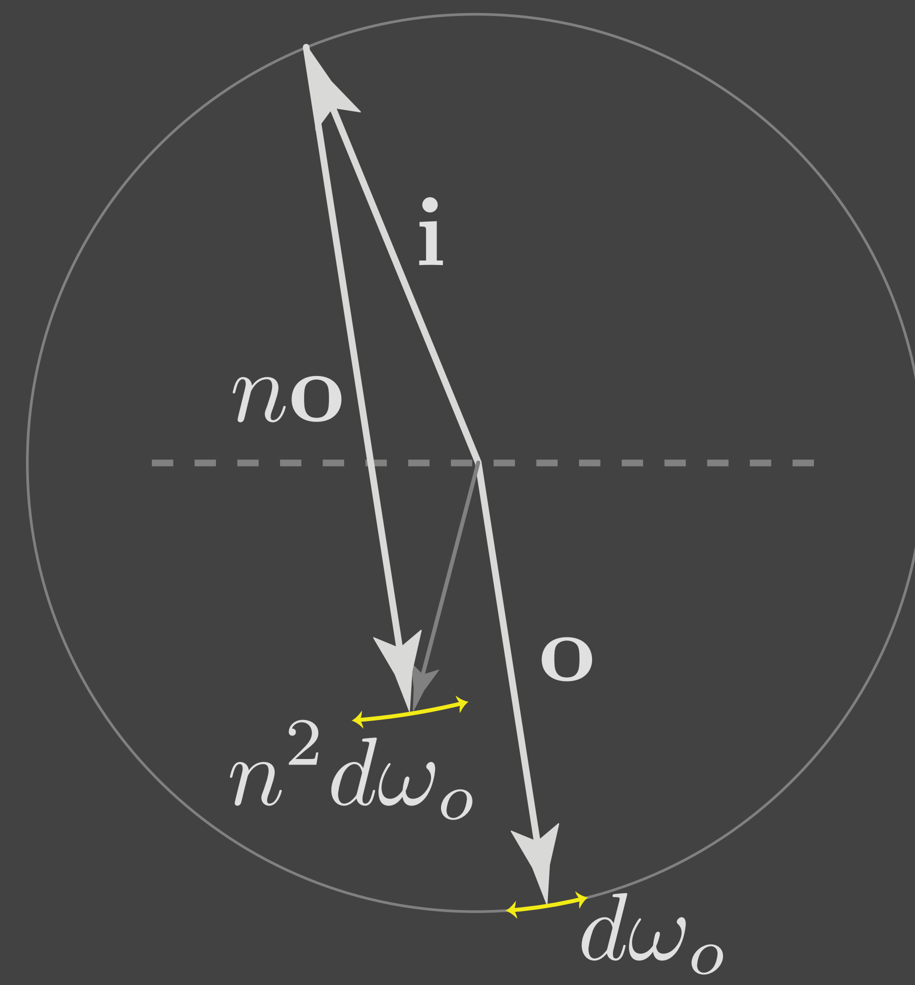
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2}\, d\omega_o$$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

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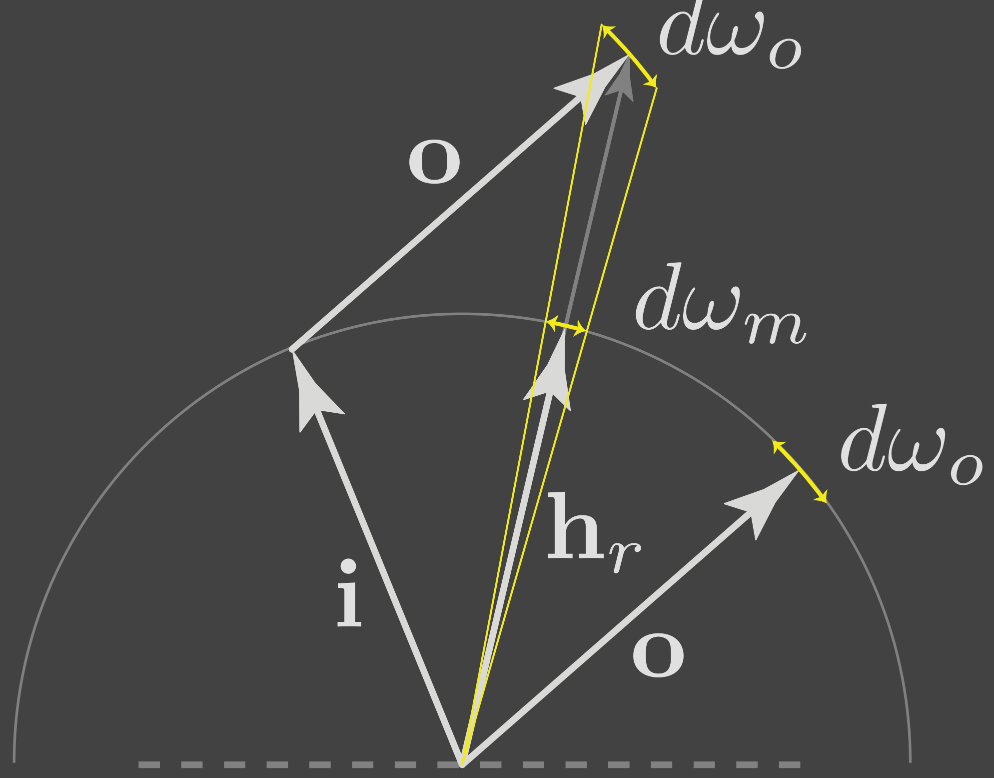

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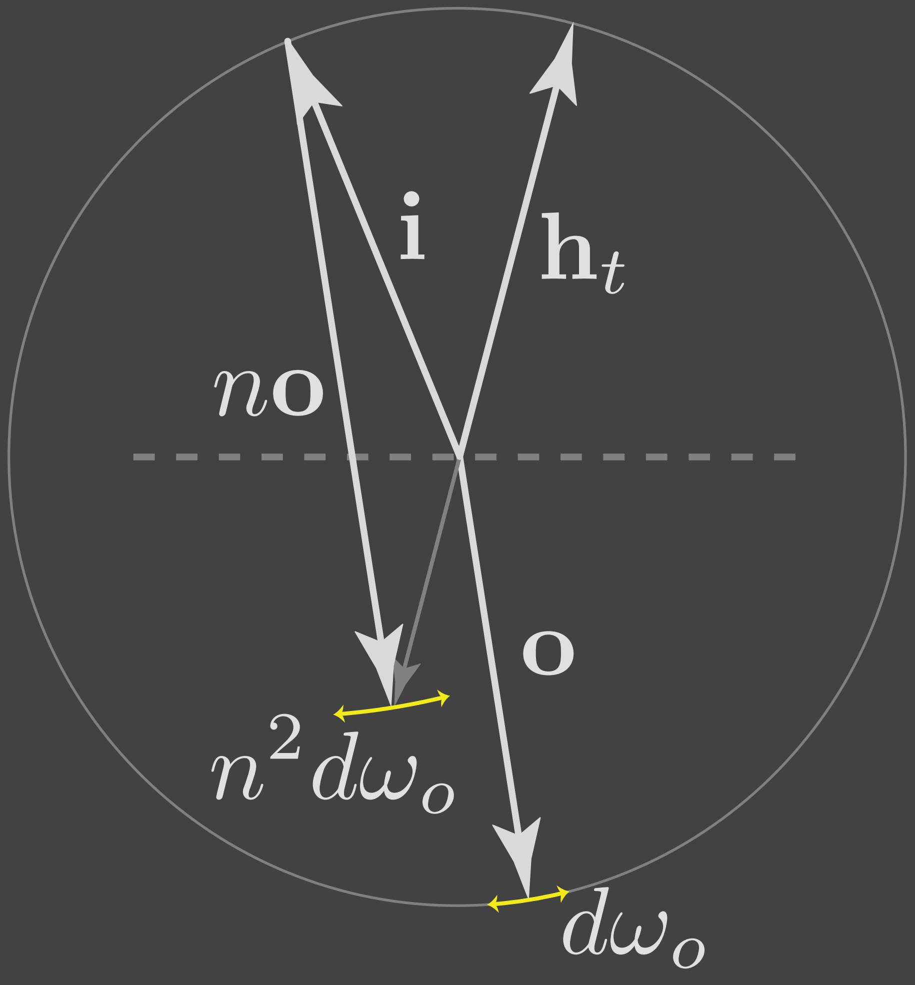

Construction of half-vector solid angle

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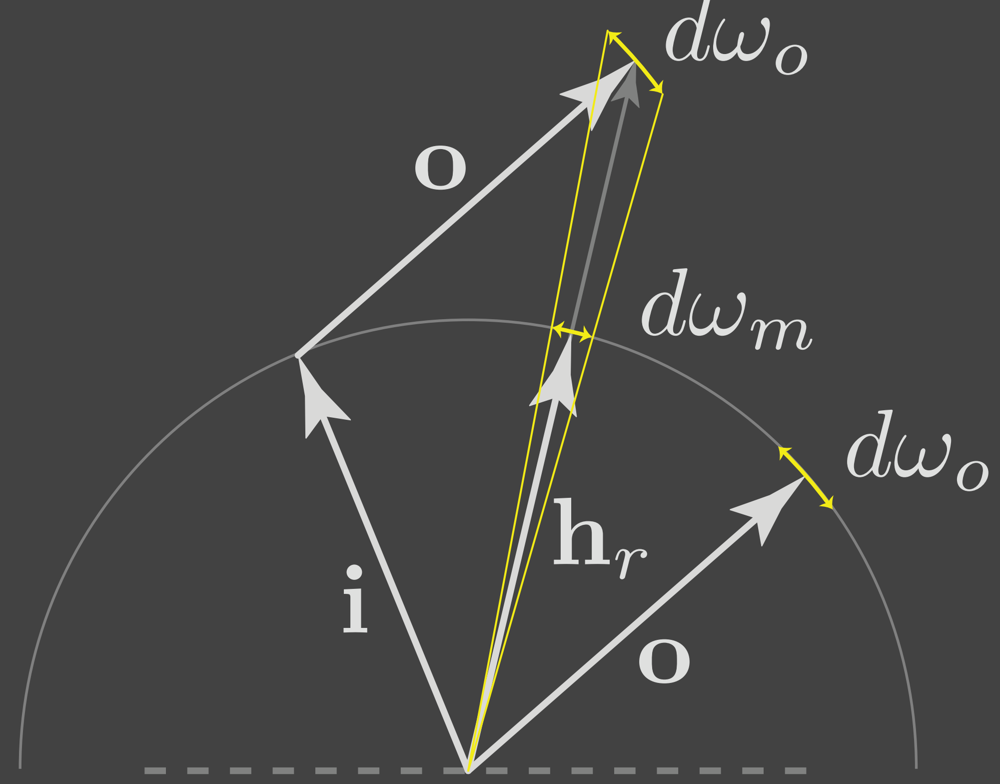
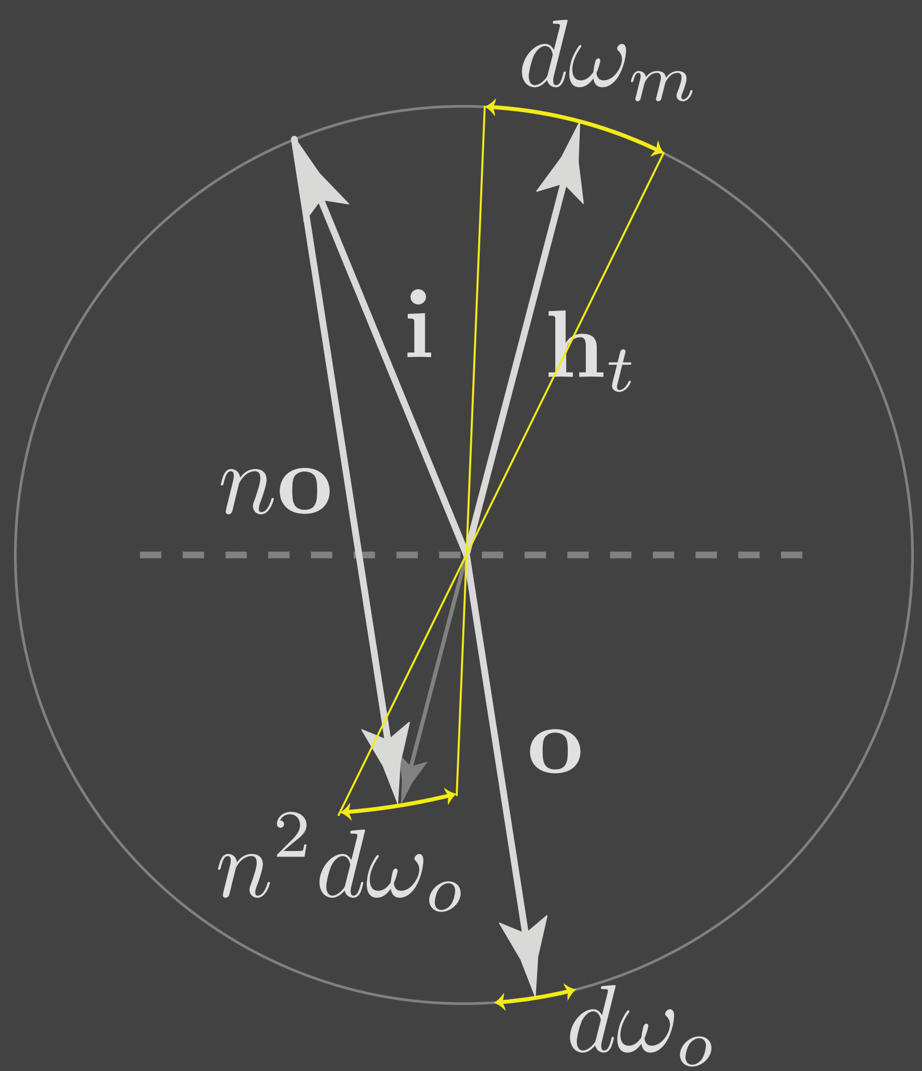


$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_t|}{\|\mathbf{i} + n\mathbf{o}\|^2} \, n^2 d\omega_o$$

Result: scattering functions

reflection

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|}\, \rho(\mathbf{i}, \mathbf{o})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, \frac{d\omega_m}{d\omega_o}$$

transmission

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|}\, \rho(\mathbf{i}, \mathbf{o})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, \frac{d\omega_m}{d\omega_o}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,F(\mathbf{i},\mathbf{m})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{|\mathbf{o}\cdot\mathbf{m}|}{\|\mathbf{i}+\mathbf{o}\|^2}$$

transmission

$$f_s(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,\rho(\mathbf{i},\mathbf{o})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{d\omega_m}{d\omega_o}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,F(\mathbf{i},\mathbf{m})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{|\mathbf{o}\cdot\mathbf{m}|}{\|\mathbf{i}+\mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,(1-F(\mathbf{i},\mathbf{m}))\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{n^2|\mathbf{o}\cdot\mathbf{m}|}{\|\mathbf{i}+n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|\,|\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|} \, \frac{F(\mathbf{i}, \mathbf{m})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|} \, (1 - F(\mathbf{i}, \mathbf{m}))\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|\, |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|\, |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m}))\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{1}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|} \, \frac{F(\mathbf{i}, \mathbf{m})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|\,|\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\,|\mathbf{o} \cdot \mathbf{n}|} \, \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m}))\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{F(\mathbf{i}, \mathbf{m})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4|\mathbf{i}\cdot\mathbf{n}|\, |\mathbf{o}\cdot\mathbf{n}|}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|\, |\mathbf{o}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\, |\mathbf{o}\cdot\mathbf{n}|}\, \frac{n^2(1 - F(\mathbf{i}, \mathbf{m}))\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Fresnel reflectance

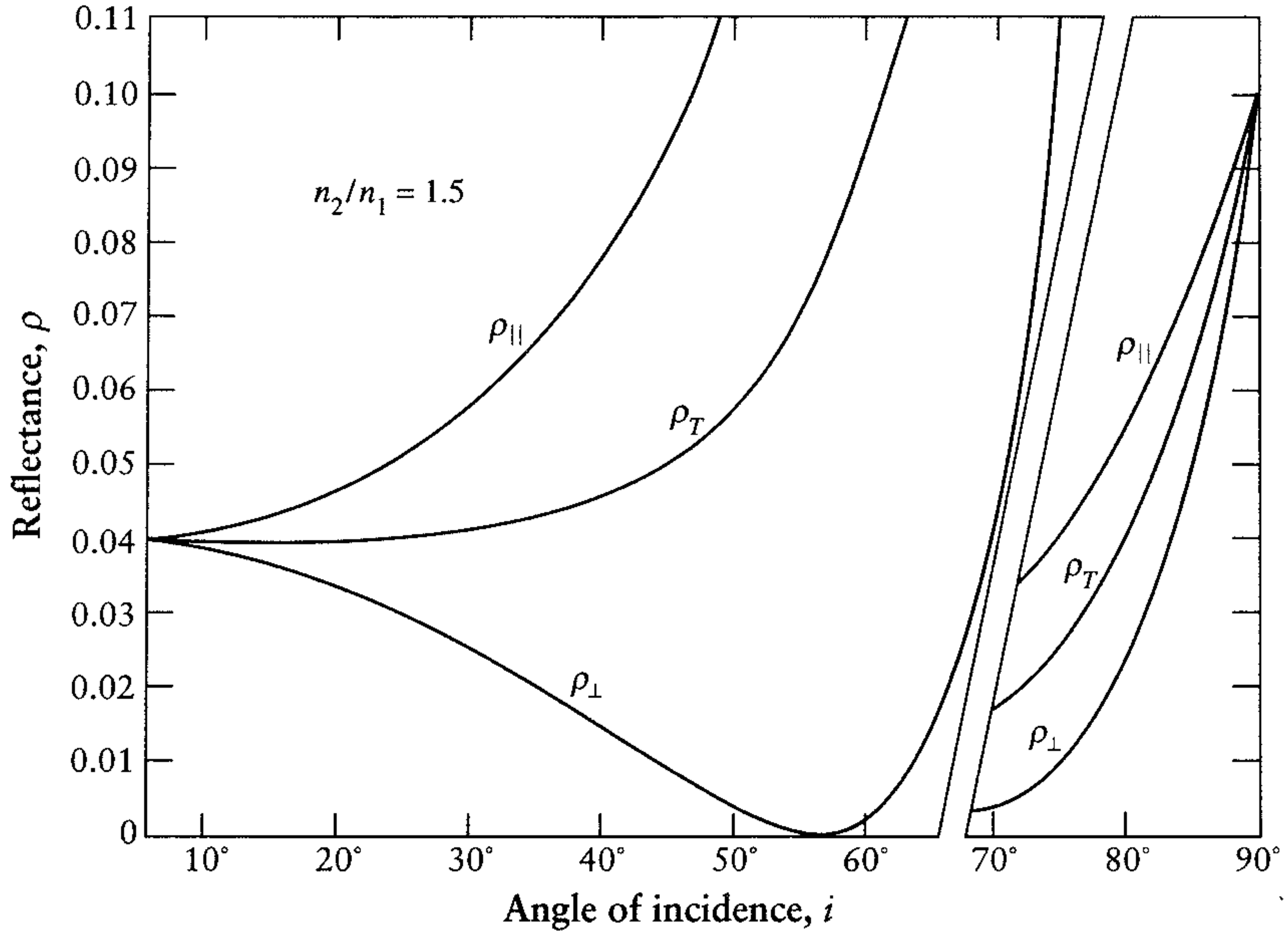


FIGURE 15.8

The Fresnel reflectance for an air-glass boundary with index of refraction 1.5. We show the two polarized components and the term for unpolarized light. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.2, p. 400.

Glassner, *Principles of Digital Image Synthesis*

Fresnel reflectance

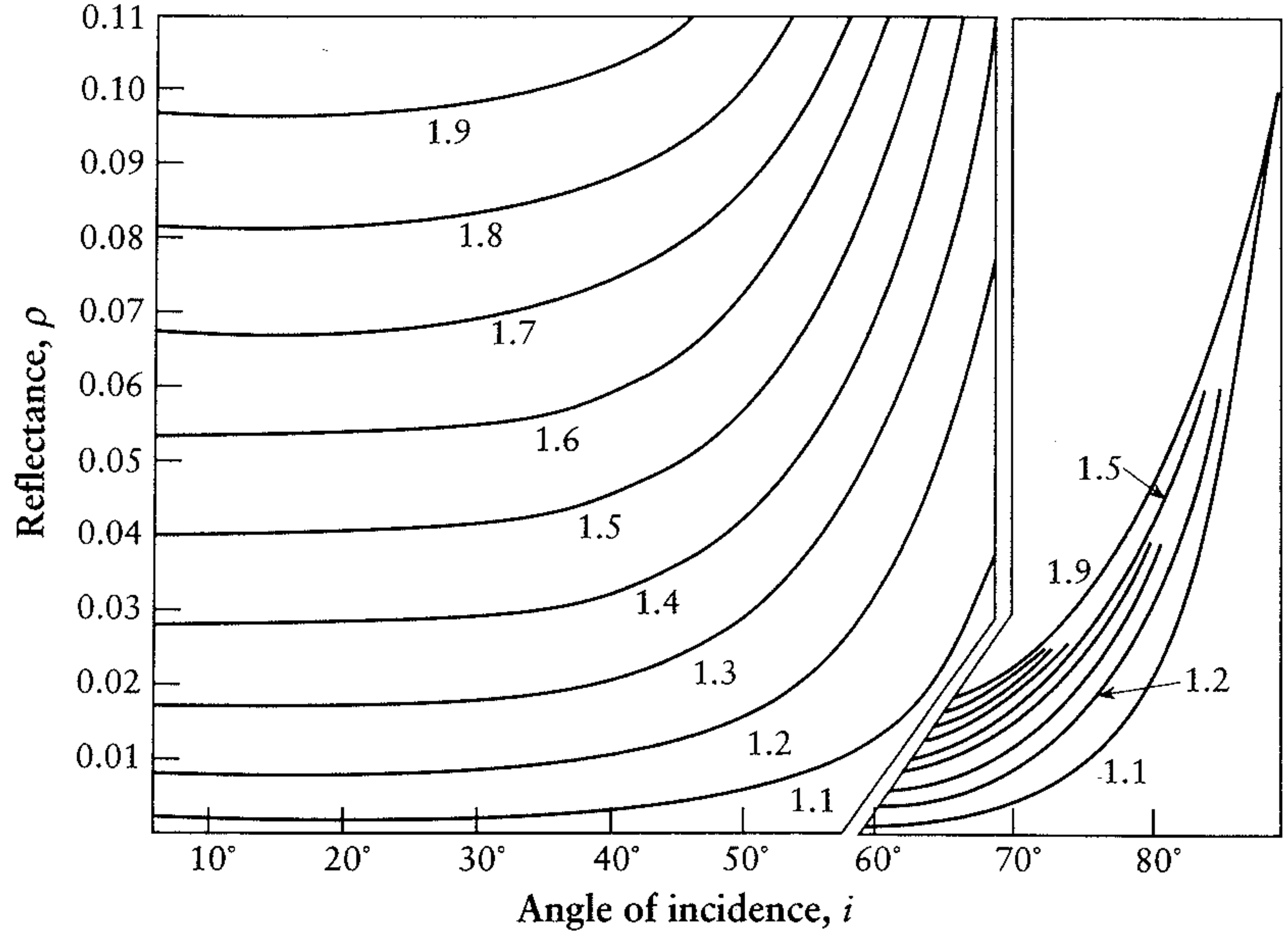


FIGURE 15.9

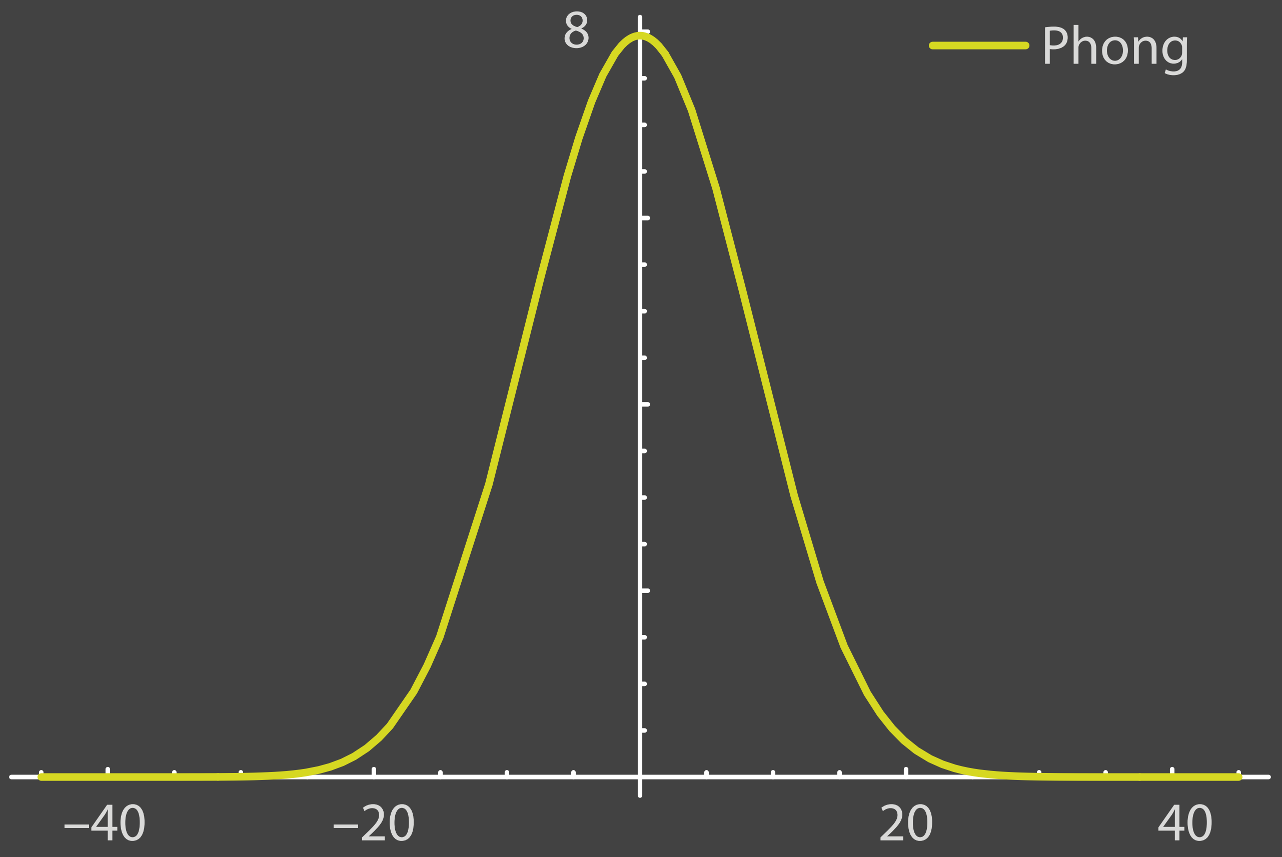
The Fresnel reflection for unpolarized light for different indices of refraction. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.3, p. 401.

Glassner, *Principles of Digital Image Synthesis*

Normal distributions

Choice of distribution is determined by surface

- Phong, Beckman are popular choices
- "GGX" distribution is another option
- [Smith 67] gives a way to produce smooth Gs



$D(\theta_m)$

Normal distributions

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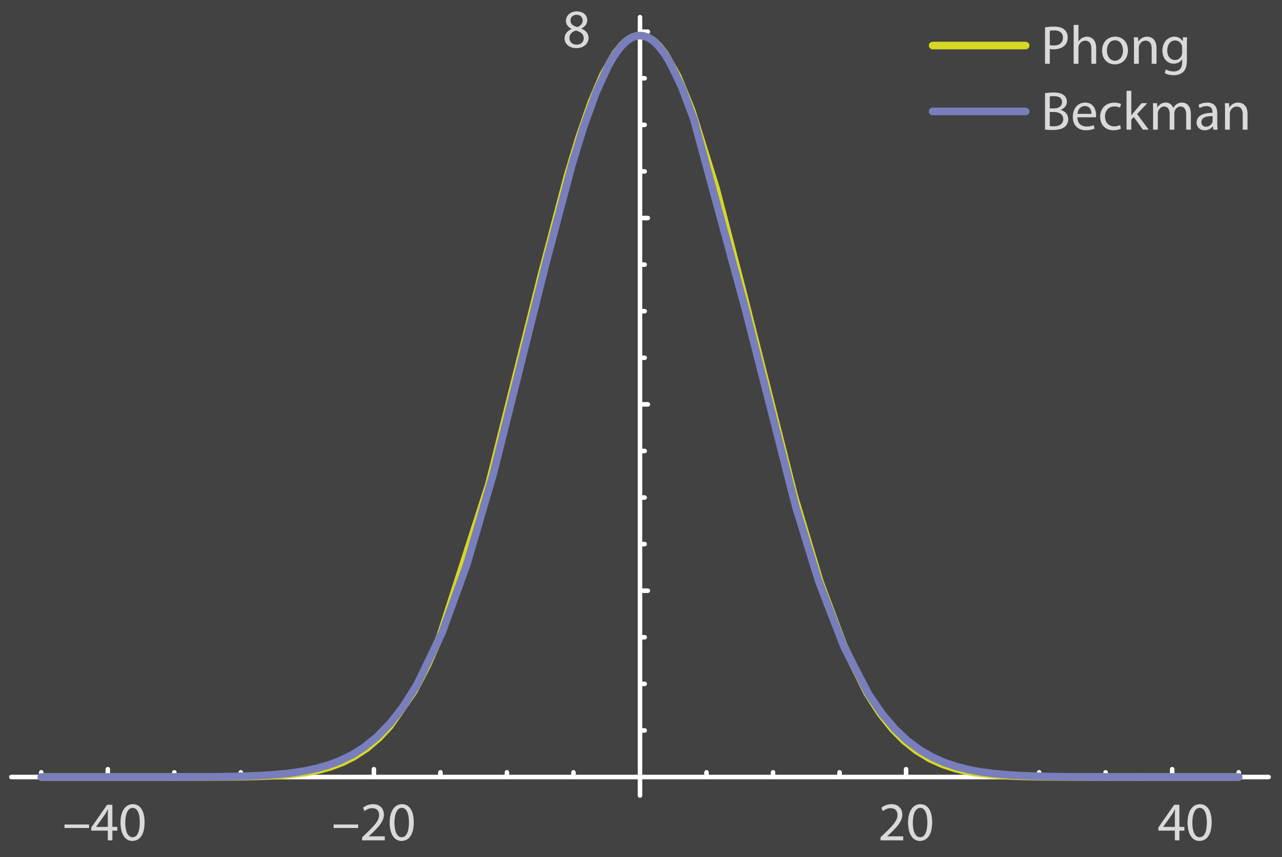

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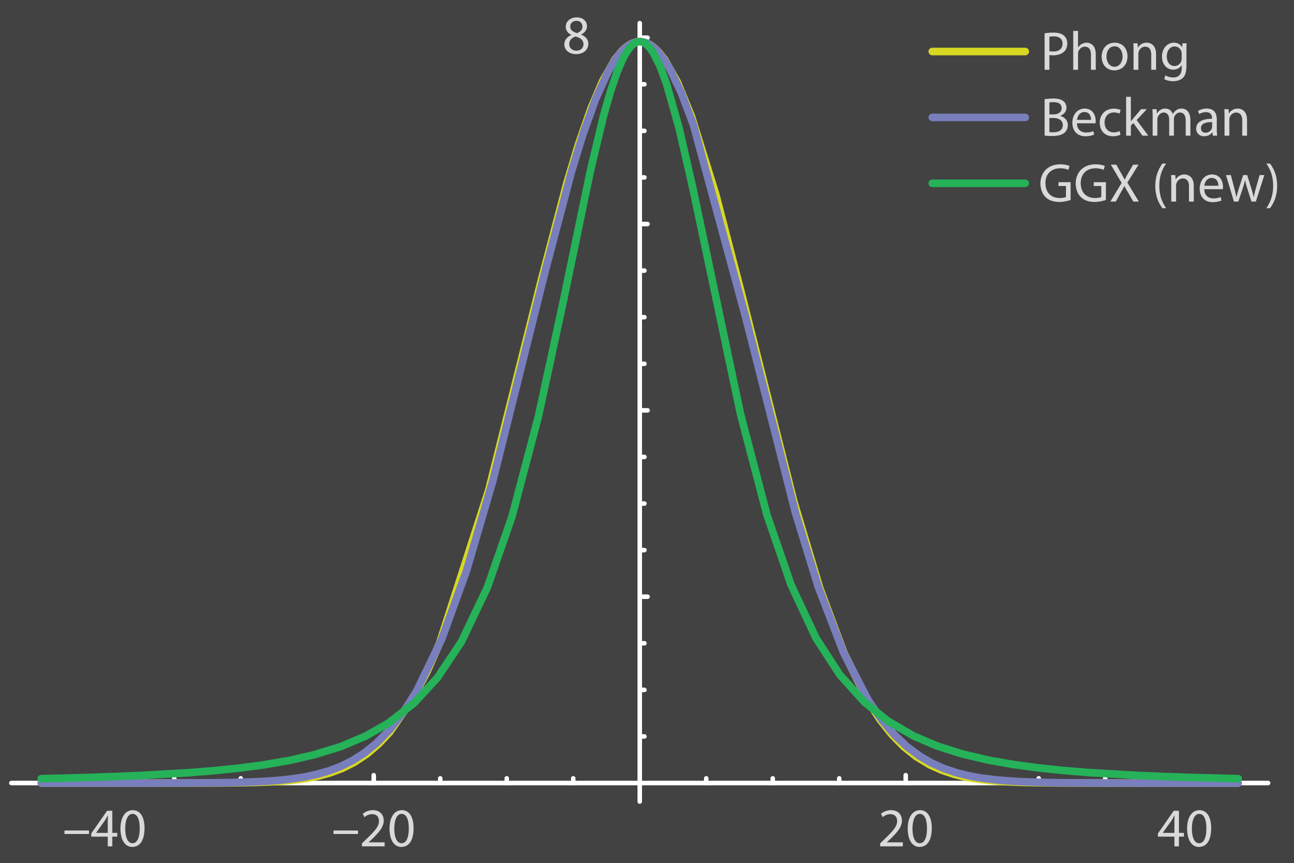

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