CS6630 Homework 2

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We saw in class that a region of space $R \subset \mathbb{R}^3$, emitting light uniformly and isotropically with emission coefficient ϵ, acts as an isotropic source of intensity ϵV, where V is the volume of R. This is true regardless of the shape of the volume.

However, things are a bit different if the emission is from the interior of a piece of dielectric medium.

1. Show that a cylindrical dielectric emitter ($\eta = 1.33$) whose thickness is negligible compared to its radius (i.e. disc-shaped), ignoring the light that is internally reflected at the surface, has a point-source intensity of

$$I(\omega) = T(\eta, \mu)\epsilon V \frac{\mu}{\eta^2 \mu'}.$$

   Where μ and μ' are cosines (i.e. dot products of unit vectors), and T is the Fresnel transmission factor. Infer the definitions of μ and μ'. Compute numerically the fraction of emitted power that is missing.

2. Argue that if we account for light that is internally reflected at the disc's surfaces, but disregard light emitting from the edges of the disc, the intensity is simply

$$I(\omega) = \epsilon V \frac{\mu}{\eta^2 \mu'}.$$

3. What fraction of the total emitted power $4\pi\epsilon V$ is accounted for in the integral of the distribution in (2)? Figure this out directly, but also confirm it numerically.[1] Where does the missing power go?

Forget about the disc and instead consider an infinitely wide layer of emitting and absorbing dielectric medium, with emission ϵ, absorption σ_a, and thickness d_0.

4. Show that the radiance emitting from this layer is

$$L_e(\omega) = T(\eta, \mu)(1 - \alpha)\frac{\epsilon}{\eta^2 \sigma_a} + T(\eta, \mu)^2 \alpha L_b$$

[1] This will also help confirm your numerical answer above.

Where L_b is the radiance entering from the other side (the "back") and $\alpha = \exp(-\sigma_a d_0/\mu')$.

5. What is the radiance emitted by a very thick layer?

Now replace the self-emitting medium with a more ordinary scattering medium of scattering coefficient σ_s and phase function f_p, still with absorption coefficient σ_a. Suppose it is illuminated by parallel light coming from a direction ω_i, on the same side of the layer as the viewer, that would produce irradiance E_i on a surface facing the source. Ignore internal reflection.

6. Show that the irradiance available for scattering at depth d is

$$E_i'(d) = T(\eta, \mu_i)\frac{\mu_i}{\mu_i'} \exp(-\sigma_t d/\mu_i')E_i.$$

   Also define μ_i and μ_i'. (This formula gives scalar irradiance, or equivalently in this context, irradiance for a surface perpendicular to the illumination direction.)

7. Show that the radiance due to single scattering exiting the layer is

$$L_s(\omega_s) = \frac{T(\eta,\mu_i)T(\eta,\mu_s)}{\eta^2}\frac{\sigma_s}{\sigma_t} f_p(\omega_i', \omega_s')\frac{\mu_i}{\mu_i' + \mu_s'} \left(1 - \exp\left(-\frac{\mu_i' + \mu_s'}{\mu_i'\mu_s'}\sigma_t d_0\right)\right) E_i.$$

   Provide definitions for the ωs and μs.

8. What is the radiance scattered by a very thick layer?

9. Work out the radiance when the source is on the opposite side of the layer from the viewer.