CS6630 Homework 1

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Consider a point source at the origin, radiating isotropically with power Φ. We saw already that the irradiance of this source at a location $\mathbf{r}$ and normal $\mathbf{n}$ is

$$E(\mathbf{r}, \mathbf{n}) = \begin{cases} \frac{\Phi}{4\pi} \frac{\cos\theta}{\|\mathbf{r}\|^2} & \text{when } \mathbf{r} \cdot \mathbf{n} \leq 0 \\ 0 & \text{when } \mathbf{r} \cdot \mathbf{n} \geq 0. \end{cases}$$

1. What is the net irradiance $\bar{E}(\mathbf{r}, \mathbf{n})$?
2. Derive from the definition that the scalar irradiance at $\mathbf{r}$ is

   $$\phi(\mathbf{r}) = \frac{\Phi}{4\pi} \frac{1}{\|\mathbf{r}\|^2}$$

   That is, it is the same as the irradiance on a surface facing the source.
3. Derive from the definition that the vector irradiance at $\mathbf{r}$ is

   $$\vec{E}(\mathbf{r}) = \frac{\Phi}{4\pi\|\mathbf{r}\|^3}\mathbf{r}$$

4. Sketch or plot a map of the vector irradiance field over a plane that passes through the source.

Now replace the point source with a spherical emitting surface of radius R centered at the origin, with constant radiance L and total power Φ.

5. Give an expression for L in terms of Φ and R.
6. By integrating the incident light at $\mathbf{r}$, conclude that the irradiance $E(\mathbf{r}, \mathbf{n})$ is the same as the point source when the normal points directly at the origin.
7. Sketch the Nusselt analog of the incident hemisphere for the computation of irradiance when the normal does and does not point directly at the origin.
8. Show that the net irradiance $\bar{E}(\mathbf{r}, \mathbf{n})$, and therefore the vector irradiance also, is in fact the same as the point source for all $\mathbf{n}$, for all $\mathbf{r}$ outside the sphere.

9. For what $\mathbf{r}$ and $\mathbf{n}$ does the net irradiance differ materially from the irradiance? Here "materially" means that flipping normal vectors doesn't suffice to make them the same.

10. By integrating the incident light at $\mathbf{r}$, conclude that the scalar irradiance is

$$\phi(\mathbf{r}) = 2\pi\, L \left(1 - \frac{\sqrt{\|\mathbf{r}\|^2 - R^2}}{\|\mathbf{r}\|} \right).$$

    This does not match the point source, but show that it does agree in the limit of $\|\mathbf{r}\| \gg R$.

Finally, consider a spherical particle of radius R that scatters light isotropically, with albedo α. That is, a α times the total power that lands on its surface is scattered uniformly in all directions.

11. Assuming that the particle is small compared to variations in the radiance field, the scattered intensity is proportional to the fluence at the particle's position. What is the constant of proportionality?