CS6110/6116 Problem Set 6 Due Friday May 4, 2012

CS6116 students should do the * problems. We use ideas from the handout on integer square root using standard and "efficient induction" as presented by Christoph Kreitz (Lect 35, 36).

1. Extend the pure λ-calculus evaluator to the applied λ-calculus with pair, spread, inl, inr, decide, 0, s, fix, and the induction form ind(n; b; u, i.h) (like R_0 in Gödel's T). Use a "lazy" (call by name) evaluation strategy.

* 2.(a) Write an "efficient" loop based program in IMP to compute integer square root using the idea behind Kreitz's recursive program. You can add $n \div 4$ as an IMP primitive, giving integer division as in $9 \div 4 = 2$.

(b) Add an assertion as a loop invariant that "documents" the while loop.

(c) Show how to transform Kreitz's program to tail recursion using closures and continuations.

3. (a) Explain the meaning of $((\mathbb{N} \to \mathbb{N}) \to (\mathbb{N} \to \mathbb{N})) \to (\mathbb{N} \to \mathbb{N})$ as a type, and explain <u>equality</u> on this type.

(b) Give an element of this type.

(c) Explain the meaning of this type if each $\mathbb{N}$ is replaced by $\overline{\mathbb{N}}$.

(d) Give an example of a member of the type with $\overline{\mathbb{N}}$ that is not in the type with $\mathbb{N}$.

4. Write two interesting and fair final exam questions.

* Note: Two more questions will be added on Monday April 30.

CS6110/6116

Problem Set 6 continued

5.(a) Prove the principle of complete induction (or course-of-values induction) from standard induction

$$* \quad \forall x(\forall y(y<x \Rightarrow A(y)) \Rightarrow A(x)) \Rightarrow \forall x. A(x)$$

Hint consider proving $\forall y(y<x).A(y)$ by induction on x.

(b) Give a realizer for $*$.

6.(a) Using complete induction, prove the efficient induction principle $(P(0) \;\&\; \forall x.(P(x \div 4) \Rightarrow P(x))) \Rightarrow \forall x\, P(x)$

(b) Give a realizer for efficient induction.

Note, I broke one problem into two, so there will be one more problem posted on Monday April 30. Here it is.

7. Consider these recursive types $rec(T.\, F(T))$ for various examples of F.

(a) $F(T) = \mathbb{N} + T \times T$ is F monotone? Explain.

(b) $F(T) = \mathbb{N} + (T \to \mathbb{N})$ is F monotone? Explain.

(c) $F(T) = \mathbb{N} + (\mathbb{N} \to T)$ show F is monotone.

(d) $F(T) = \mathbb{N} + T$ is F monotone? Explain.