# Problem Set 7

Due Date: Thurs, March 13, 2003

## Reading

Please read the handout on second order propositional logic (on either the 2001 or 2003 CS486 web site); Smullyan, Chapter IV, p. 43-52; and Suppes' *Introduction to Logic*, pages 43-54.

## Problems

1. Give Refinement logic rules for $P^2$.

2. Prove or disprove these $P^2$ formulas:

    (a) $\forall p \forall q ((p \supset q) \supset ((p \supset \bot) \supset (q \supset \bot)))$

    (b) $\forall p \exists q ((p \supset q) \supset ((p \supset \bot) \supset (q \supset \bot)))$

    (c) $(\forall p(\sim p)) \supset \neg \exists p.p$

    (d) $\forall p \exists q ((p \vee q) \supset p)$

    (e) $\forall p \exists q ((p \vee q) \wedge \sim (p \wedge q))$

3. Reduce these $P^2$ formulas to $P^0$ formulas.

    (a) $\forall p(p) \supset \bot$

    (b) $\forall p \forall q ((\sim p \vee q) \supset (p \supset q))$

    (c) $\forall p \forall q ((p \supset p \vee q) \wedge (p \wedge q \supset p))$

4. Solve exercise 4, page 50 of Smullyan.

5. Solve exercise 3, page 52 of Smullyan.

6. Solve exercise 2, page 55 of Suppes' *Introduction to Logic*.

7. There is a simple proof for cut elimination in $P^2$. State the theorem and outline a proof. Details are not necessary.

**Optional Problem:**

Develop the idea that Refinement Logic is a calculus of "problems." Try to define the idea of a problem $P$ and a solution for it, say $s$. Use a semantics based on the idea that $s$ *solves* $P$.