Problem Set 4

due Thursday, Feb 20, 2003

Reading

Please read Smullyan, Chapter XI, p. 101-108 for Tuesday, February 18

Problems

1. Let S be a set of formulas. Assume for every valuation v_i there is an $X_i \in S$ with $\mathsf{val}(X_i, v_i) = \mathsf{f}$. Show that for some n the conjunction $X_1 \wedge \ldots \wedge X_n$ is unsatisfiable.

2. Call a set S *complete* if every formula or its negation is in S.

   Show that a set is consistent and complete if and only if it is maximally consistent.

3. *"The simplest proof of the compactness theorem"*

   Let S be a consistent set and $\{p_1, p_2, \ldots\}$ be the set of all propositional variables. Construct an infinite sequence of sets B_i as follows:

   $$B_0 := \{\} \quad B_{n+1} := \begin{cases} B_n \cup \{p_{n+1}\} & \text{if } S \cup B_n \cup \{p_{n+1}\} \text{ consistent} \\ B_n \cup \{\sim p_{n+1}\} & \text{otherwise} \end{cases}$$

   Define $B^* := \bigcup B_i$. Show that there is exactly one interpretation v_0 that satisfies B^* and that S is uniformly satisfied by v_0.