

Problem Set 6

Due Date: Thurs, March 6

Reading

skim Chapter IV, p. 43-51 for Tuesday, March 4

Problems

1. A proof rule is *reversible* if its main goal is equivalent to the conjunction of all its subgoals.

- (a) Show that all rules of Gentzen's multi-conclusioned sequent calculus are reversible.
- (b) Show that the refinement logic rule **impL** is irreversible

2. For the completeness proof of refinement logic we have introduced the following rules.

$$\frac{H \vdash A \vee (B \vee C)}{H \vdash (A \vee B) \vee C} \text{ by } \mathbf{orAssocL}$$

$$\frac{H \vdash (A \supset B) \vee G^*}{H, A \vdash B \vee G^*} \text{ by } \mathbf{impR^*}$$

Prove that both rules can be *derived* in refinement logic, i.e. give a proof fragment that simulates the rules.

3. Prove that the **magic** rule is equivalent to the following rule

$$\frac{H \vdash G \text{ by } \mathbf{contradiction}}{H, \sim G \vdash \mathbf{f}}$$

Show how to derive **contradiction** in refinement logic and how to derive **magic** in a refinement logic with **contradiction** (and without **magic**).

*Note that using the **cut** rule is not necessary but simplifies the derivation.*