Problem Set 5

Due Date: Thurs, Feb 27

Reading

Please study Smullyan, Chapter XI, p. 101-108, and skim Chapter IV, p. 43-51 for Thurs, Feb 27.

Problems

1. Give a top-down Gentzen proof of formulas (2), (4), (6), and (8) on page 24.

2. Recall the lecture presentation of Smullyan's definition of a tree. A tree is a 4-tuple $< s, a, p, f >$ where S is a set of nodes, $a \in S$, p maps $\{x : S | x \neq a\}$ into S; it computes the *predecessor* of a node, the function f maps S to $\mathbb{N}^+ = \{1, 2, 3, \ldots\}$. The two axioms are:

   Ax 1. For all x in S, $f(x) = 1$ iff $x = a$.

   Ax 2. For all x in S, $f(x) = f(p(x)) + 1$.

   Define $L(i) = \{x : S | f(x) = i\}$.

   Prove carefully that $L(i+1) = \{x : S | p(x) \in L(i)\}$ and describe the result graphically.

3. Recall that *Refinement Logic* is a single conclusion (top down) Gentzen system in which the rule $\frac{H \vdash P \vee Q}{H \vdash P, Q}$ is replaced by $\frac{H \vdash P \vee Q}{H \vdash P}$ or $\frac{H \vdash P \vee Q}{H \vdash Q}$ and the rule $\frac{H \vdash G}{H, X \vee \sim X \vdash G}$ for any formula x.

   Prove the following formulas in Refinement Logic:

   (a) $((P \supset Q) \supset P) \supset P$

   (b) $(P \supset Q) \supset \sim Q \supset \sim P$

   (c) $(\sim Q \supset \sim P) \supset (P \supset Q)$

   (d) $\sim (P \vee Q) \supset \sim P \vee \sim Q$

4. Write down the rules for a Gentzen system based on the *Sheffer stroke* and one based on *joint denial* (see p. 14 of Smullyan and p.30).

5. Produce Tableau rules and Refinement rules for a logic with the constants t, f (Smullyan, p. 13), but without $\sim$. Define $\sim P$ as $P \supset f$ and show how to replace any deduction using the $\sim$ rules by one using $P \supset f$ instead.