Problem Set 11

Due Date: Thurs, May 1, 2003

Problems

1. Show that all functions that are representable in the theory $\mathcal{Q}$ are computable.

2. Let $B_1(x)$ and $B_2(x)$ be formulas in the language of $\mathcal{Q}$ with x as sole free variable. Show how to construct formulas G_1 and G_2 such that

   $\models_Q G_1 \Leftrightarrow B_1(\ulcorner G_2 \urcorner)$ and $\models_Q G_2 \Leftrightarrow B_2(\ulcorner G_1 \urcorner)$.

3. Let *Prov* be a provability predicate for the theory $\mathcal{Q}$ and X and Y be formulas in the language of $\mathcal{Q}$. Assume $\models_Q Prov(\ulcorner X \urcorner) \supset Y$ and $\models_Q Prov(\ulcorner Y \urcorner) \supset X$

   Show that both X and Y are theorems in $\mathcal{Q}$.

4. A pair of natural numbers a and a is called a *stamps pair* if for all $n \geq a+b$ there are natural numbers i and j such that $n = i*a + j*b$.

   (a) Prove in λ-PRL that 3 and 5 is a stamps pair.

   (b) Using λ-PRL notation, describe the algorithm implicitly contained in your proof.

   (c) **Extra credit.** *What other pairs of natural numbers are stamps pairs?*

5. Prove in λ-PRL that every non-empty list of integers has a maximal element.

   `∀l:list. (∼(l=[]) ⊃ ∃i:int.(i∈l ∧ ∀j:int.(j∈l ⊃ j≤i)))`

   Give a λ-PRL definition of the predicate $i \in l$, prove the above statement in λ-PRL's refinement logic, and describe the extracted algorithm in λ-PRL notation.