CS 486
Spring 1999
Quiz 8

April 15, 1999

1. Give a formula in Presburger Arithmetic that represents the predicate $x \geq y$.

   $(\exists z) x = y + z$

2. Can division $x \div y$ be represented in Presburger Arithmetic?

   NO, otherwise we could represent multiplication as follows

   $x * y = z \Leftrightarrow (x=0 \wedge z=0) \vee (z \div y = x \wedge (z-1) \div y = (x-1))$

3. Check wether the following sentences are valid or not valid in the theory $\mathcal{Q}$.

$(\forall x)(x + S\,0 = S\,x)$	☒ valid	☐ not valid
$(\forall x)(x * S\,0 = x)$	☐ valid	☒ not valid
$(\forall x)(\exists y)(S\,y = x)$	☐ valid	☒ not valid
$(\forall x,y)(x + y = y + x)$	☐ valid	☒ not valid
$(\forall x,y)(x + y \neq y + x)$	☐ valid	☒ not valid

4. Can an inconsistent extension of $\mathcal{Q}$ be decidable?

   YES, a theory in which everything is true
   (we're not talking about finite axiomatization here)

5. If we extend $\mathcal{Q}$ by the induction scheme from Presburger Arithmetic, will $(\forall x)(S\,x \neq x)$ become valid?

   YES, because we have an extension of Presburger Arithmetic in this case and I proved the law explicitly.