CS 486
Lectures 6/ 7: Handout on Blocked Tableau and Sequent Systems

Block tableaux rules "spelled out" (based on Smullyan 20)

We consider sets S of formulae, and isolate the one we're interested in.

left		right	
α	S, T(A ∧ B) S, T(A), T(B)	S, F(A ∧ B) S, F(A) S, F(B)	β
β	S, T(A ∨ B) S, T(A) S, T(B)	S, F(A ∨ B) S, F(A) , F(B)	α
β	S, T(A⊃B) S, F(A) S, T(B)	S, F(A⊃B) S, T(A), F(B)	α
α	S, T(∼A) S, F(A)	S, F(∼A) S, T(A)	α
*	S, T(A), F(A)		

Gentzen Systems: Multiple-conclusioned sequent rules
(Smullyan 105/106)

Hypothesis and conclusion consist of sets of formulae (Δ, Γ).

left		right	
`andL`	Δ, A ∧ B ⊢ Γ Δ, A, B ⊢ Γ	Δ ⊢ Γ, A ∧ B Δ ⊢ Γ, A Δ ⊢ Γ, B	`andR`
`orL`	Δ, A ∨ B ⊢ Γ Δ, A ⊢ Γ Δ, B ⊢ Γ	Δ ⊢ Γ, A ∨ B Δ ⊢ Γ, A, B	`orR`
`impL`	Δ, A ⇒ B ⊢ Γ Δ ⊢ Γ, A Δ, B ⊢ Γ	Δ ⊢ Γ, A ⇒ B Δ, A ⊢ Γ, B	`impR`
`notL`	Δ, ¬A ⊢ Γ Δ ⊢ Γ, A	Δ ⊢ Γ, ¬A Δ, A ⊢ Γ	`notR`
`axiom`	Δ, A ⊢ Γ, A		

We use a slightly different notation for logical connectives.
Implication is now ⇒ (instead of ⊃), negation is ¬ (instead of ∼)

Refinement Logic: Single-conclusioned sequent rules

Refinement Logic as implemented in NUPRL uses a slightly different notation for logical connectives. Instead of sets of formulae ($\varDelta$) we consider lists (H, H'). Left rules must provide the index of the formula to which the rule shall be applied.

left		right	
`andL`	$H,\ A \wedge B,\ H' \vdash G$ $\quad H,\ A,\ B,\ H' \vdash G$	$H \vdash A \wedge B$ $\quad H \vdash A$ $\quad H \vdash B$	`andR`

`orL`	$H,\ A \vee B,\ H' \vdash G$
	$\quad H,\ A,\ H' \vdash G$
	$\quad H,\ B,\ H' \vdash G$
		$H \vdash A \vee B$
		$\quad H \vdash A$
		
		$H \vdash A \vee B$
		$\quad H \vdash B$
			`orR1`
			
			`orR2`

`impL`	$H,\ A \Rightarrow B,\ H' \vdash G$ $\quad H,\ A \Rightarrow B,\ H' \vdash A$ $\quad H,\ H',\ B \vdash G$	$H \vdash A \Rightarrow B$ $\quad H,\ A \vdash B$	`impR`
`notL`	$H,\ \neg A,\ H' \vdash G$ $\quad H,\ \neg A,\ H' \vdash A$	$H \vdash \neg A$ $\quad H,\ A \vdash \mathrm{false}$	`notR`
`falseL`	$H,\ \mathrm{false},\ H' \vdash G$		
`hypothesis`	$H,\ A,\ H' \vdash A$		
Special Rules			
		$H \vdash G$ $\quad H \vdash A$ $\quad H,\ A \vdash G$	`cut` *A*
`magic`	$H \vdash A \vee \neg A$	$H \vdash G$ $\quad H,\ A \vdash G$ $\quad H,\ \neg A \vdash G$	`ClassCases` *A*
`lemma` *name*	$H \vdash L$	$H \vdash G$ $\quad H,\ L \vdash G$	`cut_lemma` *name*

In the lemma-rules L is the conclusion of a previously proven lemma with the given *name*. In `cut` and `ClassCases` A is a formula that has to be provided. The rules `ClassCases` and `cut_lemma` are composed out of simpler rules.