CS486: Spring 1999

Problem Set 9:

Due Thursday, April 29, 1999

Exercises

Please submit each homework problem on a separate piece of paper.

1. Using the available constants and functions formally define the following concepts

   (a) `increasing(`l`)`, saying that l is an increasing list

   (b) `index(`l`,`i`)`, computing the i-th element of l (for $1 \leq i \leq |l|$)

   (c) `multiplicity(`x`,`l`)`, the number of occurrences of x in l (0 to $|l|$ possible)

   With these concepts state the theorem that any list can be sorted into an increasing order.

   What would be the computational meaning of a proof of that theorem?

2. Consider this proposition over $\mathbb{N}$:

$$(\forall x, y)(x < y^2 \Rightarrow \exists\, r(r^2 \leq x \wedge x < (r+1)^2)$$

   (a) Prove this (by induction on y).

   (b) Derive a program for computing the integer root of x from this proof (Hint: consider the right starting value of y.)