CS486: Spring 1999

Problem Set 8:

Due Thursday, April 15, 1999

There will be other problems handed out on Thursday, April 8.

Exercises

Please submit each homework problem on a separate piece of paper.

1. Using the group axioms from lecture (Group-L) (see below) prove these properties informally from the axioms.

   (a) $1 \cdot x = x$ (left identity)

   (b) $x \cdot y = x \supset y = 1$ (identity is unique)

   (c) $y \cdot x = 1 \supset y = x^{-1}$ (inverse is unique)

   (d) $x = y \supset x^{-1} = y^{-1}$ (inverse is functional)

   (e) $(x^{-1})^{-1} = x$

Group Theory Axioms

E1	$x = y \supset (x = z \supset y = z)$
E2	$x = y \supset x \cdot z = y \cdot z$
E3	$x = y \supset z \cdot x = z \cdot y$
G1	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$
G2	$a \cdot 1 = a$
G3	$a \cdot a^{-1} = 1$

2. Prove the associativity of addition

$$(\forall x, y, z)(x + (y + z) = (x + y) + z)$$

in Presburger Arithmetic.

3. Show how integer division $x \div y$ and a test for prime numbers t_p (with $t_p(x) = 1$ if x is prime and 0 otherwise) can be represented in Peano Arithmetic.

   It is not necessary to give a detailed proof of the "correctness" of the corresponding predicates.

4. Show that the following laws are not valid in $\mathcal{Q}$

   (a) $(\forall x, y)\ (x + y = y + x)$
   (b) $(\forall x, y, z)\ (x + (y + z) = (x + y) + z)$
   (c) $(\forall x)\ (0 + x = x)$
   (d) $(\forall x, y)\ (x * y = y * x)$
   (e) $(\forall x)\ (0 * x = 0)$

   Find an interpretation of $\mathcal{Q}$ that violates all these five laws at once.