CS486: Spring 1999

Problem Set 4:

Due Thursday, February 25, 1999

Reading:

Please read Smullyan, Chapter 4, pp. 43–52 for Tuesday Feb 23.

Exercises from Smullyan:

Please submit each homework problem on a separate piece of paper.

1. Prove the formulae in Smullyan, p24, Exercise 6,7,8 using Refinement Logic
2. Show that the rules `ClassCases` and `cut_lemma` are *redundant* in Refinement Logic: Show how to transform a proof that uses these rules into one that does not use them.
3. Show how to implement the rule `impR`*

   $\Delta \vdash A \Rightarrow B \ \vee \ \Gamma^*$ `impR`*
   $\quad \Delta,\ A \ \vdash B \ \vee \ \Gamma^*$

   Hint: use the `magic` *or* `ClassCases` *rule*

Additional Exercises (not required)

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4. Can the formulae in Smullyan, p24, Exercise 5 be shown without the `magic` rule?
5. Prove that the rule `orAssocL` can be derived in Refinement Logic

   $\Delta \vdash$ `A ∨ (B ∨ C)` by `orAssocL` $\quad$ $\Delta \vdash$ `(A ∨ B) ∨ C` by `orAssocR`
   $\quad \Delta \vdash$ `(A ∨ B) ∨ C` $\quad\quad\quad\quad$ $\Delta \vdash$ `A ∨ (B ∨ C)`

   $\Delta \vdash$ `A ∨ B` by `orCommut` $\quad$ $\Delta \vdash$ `(A ∨ B) ∨ C` by `orCommutL`
   $\quad \Delta \vdash$ `B ∨ A` $\quad\quad\quad\quad$ $\Delta \vdash$ `(B ∨ A) ∨ C`

6. For $\Gamma = G_1, G_2 ..., G_n$ define Γ_i^* to be the formula $G_1 \vee (G_2 \vee (...(G_{i-1} \vee (Gi{+}1 \vee ...G_n))))$ Describe a tactic that "implements" the derived rules

   $\Delta \vdash \Gamma^*$ `SwapOut` i $\quad$ $\Delta \vdash G_i \vee \ \Gamma_i^*$ `SwapIn` i
   $\quad \Delta \vdash G_i \vee \ \Gamma_i^*$ $\quad\quad\quad\quad$ $\Delta \vdash \Gamma^*$

   Try to use the `THEN, THENL, ORELSE, Repeat` notation where possible
