Problem Set 1:

Due Thursday, February 4, 1999

Reading: Please read Smullyan, pp. 1 –26.

Exercises:

1. On page 7, Smullyan gives the *abstract syntax* for propositional formulas; call the type *Formula.*

   Please write a data type definition for *Formula* in either Java or ML or Dylan, and convince us that it implements the type *Formula.*

2. Give truth table definitions for your understanding of the following English connectives:

   (a) P only if Q

   (b) P if and only if Q

   (c) P on condition Q

3. Define an *Assignment* as a (computable) mapping from propositional variables (p. 6) to Booleans $\mathbb{B}$. That is, a member of the type $Var \rightarrow \mathbb{B}$.

   Write a recursive function, *val*, from *Formula*, and *Assignment*, to $\mathbb{B}$ that defines the truth value of the formula according to the page 11 definition.

4. A truth table for a formula can be considered as a proof attempt. If the formula has value true for each row of the table, then the table is a *proof* that the formula is a *tautology.*

   Using the definition of *consistency* and *completeness* on p. 25, show the truth-table proofs are consistent and complete.

5. Solve the following exercises on p. 24.

   (3) $((p \supset q) \wedge (p \supset r)) \supset (p \supset (q \wedge r))$

   (6) $\sim (p \vee q) \supset (\sim p \ \wedge \sim q)$

and see if these are tautologies using tableau

- $(p \supset (p \supset q)) \supset p$
- $((p \supset q) \vee (p \supset r)) \supset (p \supset (q \vee r))$.