

CS 482 Summer 2005  
**Homework Assignment #9**

Out: August 15

Due: August 18

### Question 1

Suppose you are given a graph  $G$ . For free. And for some reason, you decide you're going to color all the nodes. Sadly, years of Cornell tuition have you at the point where all you can only afford 3 colors: red, blue, and green. You have no reason to think that  $G$  is 3-colorable, but you're in something of a rebellious mood, so you say "what the heck, I'm gonna color it anyway". However, to appease your instructors, you decide that you'll try to make the coloring as good as possible. In other words, you're going to try to minimize the number of *bad edges*, i.e. edges for which both end points have been assigned the same color.

- (a) Prove that it is NP-Complete to get any constant factor approximation for the problem of finding the minimum number of bad edges required in any coloring. In other words, if we define  $Opt(G)$  to be the minimum number of bad edges in any coloring of  $G$ , prove that for any  $\rho \geq 1$  it is NP-Complete to find a coloring of  $G$  with at most  $\rho Opt(G)$  bad edges.
- (b) It turns out things aren't quite as grim if we want to maximize the number of *good edges*, namely the edges that aren't bad. This is clearly the same goal, just a different measure of approximation. Suppose we define  $Opt'(G)$  to be the maximum number of good edges in any coloring of  $G$ . Give a randomized algorithm that returns a coloring with  $\frac{2}{3}Opt'(G)$  good edges in expectation.