

CS 482 Summer 2005
Homework Assignment #6

Out: July 28

Due: August 3

Question 1

To help pay for your European travels, you've decided to help the Cornell Fitness Centre schedule their student employees at the Helen Newman gym for the next semester. At any given time they want some students working the front desk to hand out towels and check IDs, some other students in the cardio room to ensure fair time usage of the treadmills, some students in the weight room to help out the scrawny guy who decided today was the day he would bench 300lbs., etc. They have divided the weekly working hours into a certain number of shifts, and when students apply for the job they are asked to specify which shifts they are available to work.

In particular, there are n shifts $S = \{s_1, s_2, \dots, s_n\}$, where s_i represents a time interval, such as Monday 3-5pm. Each shift s_i must be covered by a minimum number m_i of people. Furthermore, there are m students $T = \{t_1, t_2, \dots, t_m\}$ available to work. Student t_i has listed a subset of shifts $S_i \subseteq S$ that she is able to work, and a maximum number p_i of shifts she is willing to work each week. You may assume that for each student t_i , the shifts in S_i are not conflicting, i.e. for any two s_j and s_k in S_i , their times do not overlap. Given these parameters, the fitness centre is asking you to find a weekly schedule satisfying these restrictions.

- (a) Come up with an algorithm that, given S, m_i, T, S_i , and p_i , determines whether it is possible to satisfy all the shift requirements, and if so, returns the assignments of students to shifts.
- (b) Now suppose that there are c types of jobs $J = \{j_1, j_2, \dots, j_c\}$ done at Helen Newman, and suppose that each student is best qualified for one of these c types. For example, Bob may be very proficient at folding towels, whereas Charlie is a natural at checking IDs. Then we will assume that each student only applies for positions in his or her specialty area. Furthermore, each shift had better hire students of more than one of these specialties, lest they end up with only towel folders. Given these new parameters, can you still find an algorithm to find a schedule satisfying all shift requirements?

Question 2

Suppose you are the artistic director of a film production studio, and you want to decide which movies to make next year. At your disposal is a list of potential scripts, and for each script you have a list of actors who have agreed to star in the film, should you choose to make it. Each film has an expected revenue, and each actor demands a certain salary for starring in your film. You would like to select a subset the films to produce that will maximize your total profit (film revenue minus actors' salaries).

More formally, you have a set $F = \{f_1, \dots, f_n\}$ of film scripts and a set $A = \{a_1, \dots, a_m\}$ of actors. If you produce film f_i you have promised to hire actors $A_i \subseteq A$ to star in it, so there is a set A_i associated with each f_i . Unfortunately, actors do not work for free; you have to pay actor a_j a salary of s_j for participating in any number of your films (i.e. you just pay the actor once, and he will act in as many films as you require him for). On the bright side, most movies you make tend to have ticket and endorsement revenue: you expect film f_i to have a revenue of r_i .

For example, suppose you have 4 actors and their demanded salaries: Natalie Portman at \$2M, Keanu Reeves at \$4M, Brad Pitt at \$7M, and the Olsen Twins, who may as well work for free, at \$0.7M for the two of them combined. You have three scripts: the Olsens and Keanu have agreed to star in *You're Invited to Mary Kate and Ashley's Zion Rave Party*, should you make it, with \$0.1M estimated revenue. Portman, Keanu and Pitt have agreed to star in *With Names Like These, Who Cares What It's Called*, with \$4.5M revenue. Lastly, Portman and Pitt have agreed to star in *Mr an Mrs Sith*, with \$9.1M revenue. The profit of making just *Names* is $4.5 - (2 + 7 + 4) = -8.5$ M, whereas the profit of making just *Sith* is $9.1 - (2 + 7) = 0.1$ M. The profit of making both though, is 0.6M, since Keanu and Brad are only getting paid the one time for starring in two films. (So, maybe this isn't realistic.)

(a) Give an algorithm to select a subset of films to make that maximizes your profit.

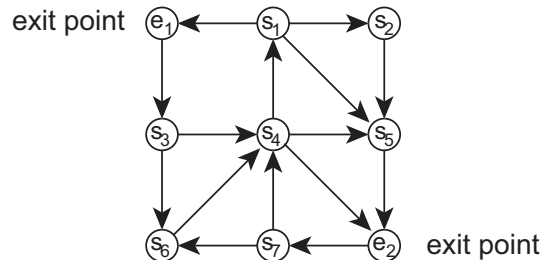
(b) Now suppose that you are given the go-ahead to make all the films in the example. Provided that you can only pay an actor from the revenues of movies he actually stars in, the head honcho wants to know if you can cover all the actors' the salaries. Use an argument based on a minimum-capacity cut to show why you cannot break even. Also, provide an explanation for this fact that would be understandable to your boss, who has not taken a course on algorithms. (So, for example, this explanation should not involve the words *flow*, *cut*, or *graph* in the sense we use them in class.)

Question 3

It's a special year at Hogwarts School of Witchcraft and Wizardry, because this year they will be hosting the TriWizard Tournament, in which young wizards from three schools compete in a few extraordinary challenges.

In order not to give anything away (and to avoid inevitable copyright lawsuits), we will describe one of the challenges with slight modifications. Suppose there is a maze of hedge-lined uni-directional pathways, and we are going to magically beam m students down in this maze, and ask them to make their way out to one of the p exit points, with one snag: there is a magic spell cast at the intersection of any two paths in the maze, and after one student crosses the intersection, an invisible barrier appears and blocks any later student from using that intersection. It is your duty, as challenge organizer, to determine a fair way to place down the students so that it's possible for everyone to make their way out without getting stuck inside the maze.

In particular, suppose you have a maze with n intersection points s_1, \dots, s_n , directed pathways between them, and p exits points, and you place each of the $m \leq p$ students down in the maze, student i at intersection point s_{k_i} . Can you determine whether there are intersection-disjoint paths for each of the m wizards from their starting points s_{k_i} to distinct exit points? Don't forget the paths are directed.



- State two different intersection points in the example to which you can beam two wizards so that there is no way for them to exit on intersection-disjoint paths.
- State two different intersection points in the example *excluding exit points* to which you can beam two wizards so that both wizards can get out along intersection-disjoint paths.
- Given arbitrary input, state and prove an algorithm that determines whether intersection-disjoint paths exist, and if so, returns one such set of paths.