CS 482 Summer 2005

Homework Assignment #4

Out: July 18 Due: July 22

Question 1

Suppose you are working at a beer factory, and you want some beer. Various sized bottles of beer come out of the beer-making machine every minute. Each bottle i has a volume v_i. You know the schedule of the bottle volumes, namely $v_1, v_2, \ldots, v_n$.

Normally you'd just take all n bottles, but you figure someone would catch on. So instead, you decide to ensure that in any consecutive sequence of 3 bottles, there is at least one bottle which you do not take. In other words, for any i, if you consider bottles i, $i+1$, and $i+2$, at least one of these will be left behind. Your goal is to calculate the maximum total volume of beer you can possibly take without violating this constraint.

For example, if we have 5 bottles, with volumes of 6,7,20,8,9, then the optimal solution would be to take bottles 2, 3, and 5, for a total volume of 36.

a) One of your friends has the following plausible idea: Repeatedly select the largest volume bottle that doesn't get you 3 in a row. Give an example where this algorithm fails to find the optimal solution.

b) Give an algorithm to solve the problem. Prove that your algorithm is correct and give a brief analysis of its running time.

Question 2

Rather than play the stock market, you've decided to do the unconventional and invest in the natural resource of the great north: timber. To facilitate this investment, you've recently purchased a large plot of land from which you will produce wood. The idea is as follows: you will use this land to grow trees. At some future date you will chop down these trees for timber, which you will sell at the current market price. You will then replant your trees and repeat the process.

As this is an investment venture, you want to maximize your profit. To this end,

you have industry estimates of the price of timber for each of the next n years, as well as environmental estimates of the growth rate of your trees. For each year you want to decide whether to chop down your trees and replant, or keep the trees for another year so as to benefit from a higher price in a later year.

To be concrete, for each of the next n years you know the price of timber, $p_1, p_2, \ldots, p_n$, and the forest growth rate, $g_1, g_2, \ldots, g_n$ (where g_i represents the fraction of growth from year $i-1$ to year i). If you decide to chop down trees in year i then you will immediately plant 100 trees in the same year. The trees will then regrow according to estimated growth rates g_{i+1}. Given that you plan on having this plot of land for the next n years, what is your maximum profit?

For example, suppose the price of timber in the next three years is $\$10, \$20$, and $\$15$ while the growth rates are $\frac{3}{2}, \frac{4}{3}$ and 3. If we cut down in years 2 and 3 the profit is $20(100 \cdot \frac{3}{2} \cdot \frac{4}{3}) + 15(100 \cdot 3) = 8500$, whereas if we just cut down in year 3 the profit is $15(100 \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot 3) = 9000$.

Question 3

Cadbury is planning its advertising strategy for the next year for its n major products. Since the n products are quite different (well, to chocolate connoisseurs) each advertising effort will focus on a single product. In units of millions of dollars, a total of B is available for advertising next year, where the advertising expenditure for each product must be an integer greater than or equal to 1. The sales of each product is directly proportional to its advertising expenditure, but it is different for each product. Given $s_{i,b}$, the estimated sales of product i if given b million dollars in advertising expenditures, determine how much to spend on each product in order to maximize total sales.

For example, suppose there are 3 products with $s_{i,b}$ as given in the following table.

Advertising	Products		
Expenditure	Creme Eggs	Dairy Milk	Fruit & Nut
1	7	4	6
2	10	8	9
3	14	11	13
4	17	14	15

If you only have 6 million dollars in budget, then one way to maximize total sales would be to spend 3 million on the Creme Eggs campaign, 2 million on Dairy Milk's, and 1 million on Fruit & Nut's to obtain 28 million in sales.