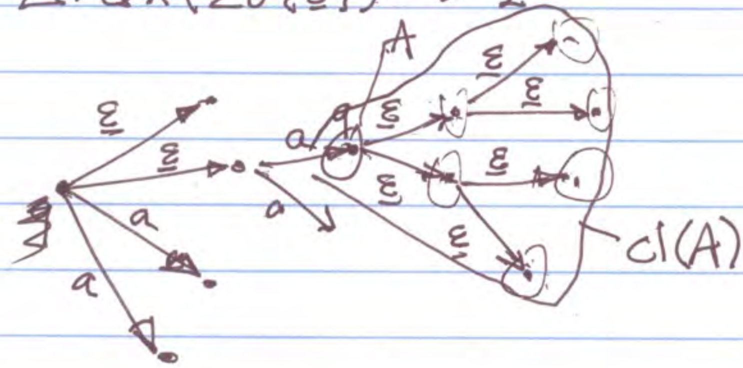
CS 4810 S22 Lecture 2/3

Automata w/ ε-transitions

New symbol ε over Σ

NFA w/ ε-transitions is defined like NFA over alphabet $\Sigma \cup \{\varepsilon\}$

$\Delta : Q \times (\Sigma \cup \{\varepsilon\}) \longrightarrow 2^Q$



ε-closure of $A \subseteq Q$: smallest set $X \subseteq Q$ such that

① $A \subseteq X$

② if $q \in X$ & $\Delta(q, \varepsilon)$ contains p, then $p \in X$.

cl(A). Extend Δ to $\hat{\Delta} : 2^Q \times \Sigma^* \longrightarrow 2^Q$

- $\hat{\Delta}(A, \varepsilon) = cl(A)$
- $\hat{\Delta}(A, xa) = \bigcup_{q \in \hat{\Delta}(A,x)} cl(\Delta(q, a))$

$x \in L(N) \overset{\Delta}{\iff} \hat{\Delta}(S, x) \cap F$.

starting state

Homomorphisms. A <u>homomorphism</u> is a map $h: \Sigma^* \longrightarrow \Gamma^*$ such that:

① $h(\varepsilon) = \varepsilon$

② $h(xy) = h(x) \cdot h(y) \qquad x, y \in \Sigma^*$

$\Sigma = \{a, b\} \quad \Gamma = \{c, d\}$

$h(a) = cc \quad h(b) = cd$

$h(abbab) = cccdcdcccd$

homomorphisms are uniquely determined by values on Σ.

$x = a_1 a_2 \cdots a_n, \ n \geq 0$

$h(x) = h(a_1) h(a_2) \cdots h(a_n)$

Regular sets are preserved under h & h^{-1}.

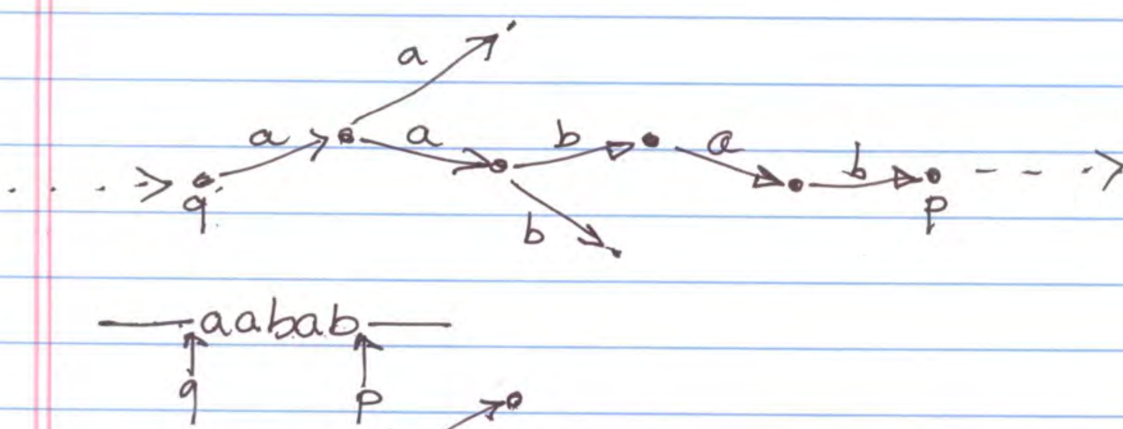
- if A is regular, & $h: \Sigma^* \to \Gamma^*$
  $A \subseteq \Sigma^*$
  is a homom., then $\{h(x) \mid x \in A\}$ is a regular subset of Γ^*.
- if $B \subseteq \Gamma^*$ is regular, so is $h^{-1}(B) = \{x \in \Sigma^* \mid h(x) \in B\}$

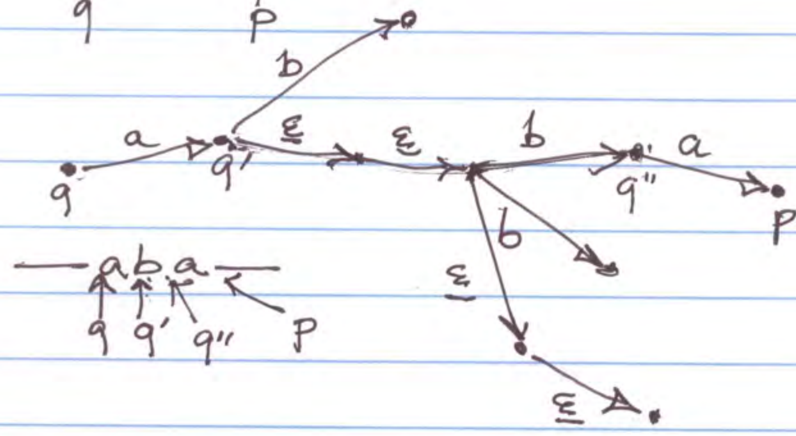
Let $N = (Q, \Sigma, \Delta, S, F)$ be an automaton with ε-trans. Let $N' = (Q, \Sigma \cup \{\underline{\varepsilon}\}, \Delta, S, F)$

Let $h: (\Sigma \cup \{\underline{\varepsilon}\})^* \to \Sigma^*$ defined by:

$h(a) = a, \ a \in \Sigma, \ h(\underline{\varepsilon}) = \varepsilon$.

Then $L(N) = \{h(x) \mid x \in L(N')\}$





Reg. sets closed under · and *.

- if $A, B \subseteq \Sigma^*$ are regular, then so is AB

  $AB = \{xy \mid x \in A \ \& \ y \in B\}$

- if $A \subseteq \Sigma^*$ is reg, so is $A^* = \bigcup_{n \geq 0} A^n$

  $= \{x_1 x_2 \cdots x_n \mid n \geq 0,\ x_i \in A,\ 1 \leq i \leq n\} \subseteq \Sigma^*$

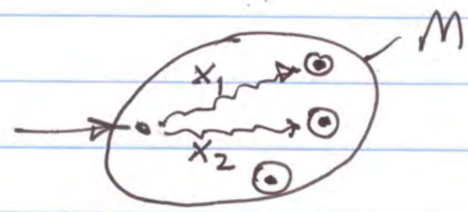
$A = L(M_1)$, $B = L(M_2)$. M_1, M_2 are NFAs.

$A \cdot B$

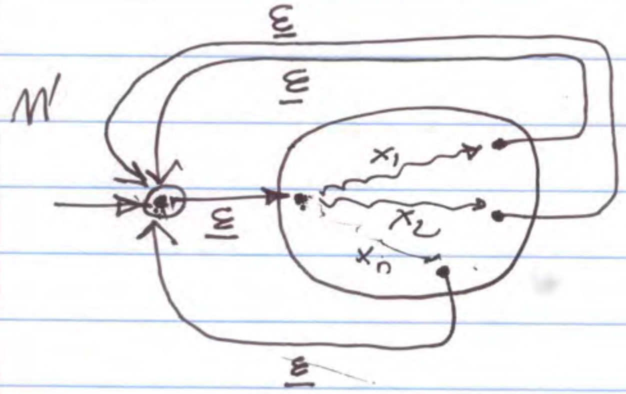
$L(M_3) = L(M_1) \cdot L(M_2)$

$z \in L(M_3) \iff \exists x, y \ \ x \in L(M_1), y \in L(M_2),$
$\ni z = xy.$

If A reg, so is A^*.



$L(M) = A$



claim: $L(M') = L(M)^*$

$z \in L(M') \iff$

$\exists x_1, \ldots, x_n,\ n \geq 0,$

$z = x_1 x_2 \cdots x_n,$

$x_i \in L(M),\ 1 \leq i \leq n.$

$\underline{x_1}\,\underline{x_2} \cdots \underline{x_n}$, $x_i \in L(M)$.

Regular expressions over Σ

- symbols a, $a \in \Sigma$, $\underline{\varepsilon}$, $\underline{\emptyset}$
- operators $+$, $\cdot$, $*$

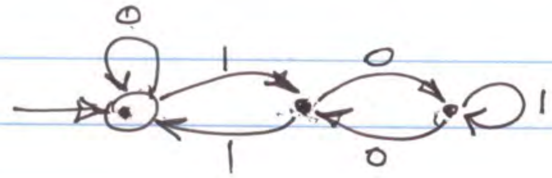
Use $\alpha, \beta, \gamma, \ldots$

- a is a reg. exp. for all $a \in \Sigma$, matched by a.
- $\underline{\varepsilon}$ is a reg. exp., matched by ε.
  ($\underline{\varepsilon}$ is boldface ε in text)
- $\underline{\emptyset}$ is a reg. exp, matched by nothing.
- if α, β are reg. exp.'s, then so is $\alpha\beta$, matched by any string xy, x matches α & y matches β.
- if α, β are reg. exp.s, so is $\alpha + \beta$, matched by any string matching either α or β.
- if α is a reg. exp, so is α^*, matched by any string $x_1 x_2 \cdots x_n$, $n \geq 0$, such that x_i matches α, $1 \leq i \leq n$.

$L(\alpha) = \{x \in \Sigma^* \mid x \text{ matches } \alpha\}$, then:

$L(a) = \{a\}$, $a \in \Sigma$; $L(\underline{\varepsilon}) = \{\varepsilon\}$; $L(\underline{\emptyset}) = \emptyset$;

$L(\alpha\beta) = L(\alpha) \cdot L(\beta)$; $L(\alpha+\beta) = L(\alpha) \cup L(\beta)$; $L(\alpha^*) = L(\alpha)^*$.



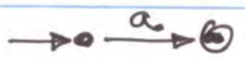
$$0^* + 0^*1(01^*0 + 10^*1)^*10^*$$

Kleene's theorem Finite automata are equivalent to reg. expressions.

① for all DFA M, $\exists \alpha$ $L(\alpha) = L(M)$

② for all α, $\exists M$ $L(M) = L(\alpha)$.

Proof of ②.

a	$\underline{\varepsilon}$	$\underline{\phi}$
$L(a) = \{a\}$	$L(\underline{\varepsilon}) = \{\varepsilon\}$	$L(\underline{\phi}) = \phi$
		

$L(\alpha + \beta) = L(\alpha) \cup L(\beta)$

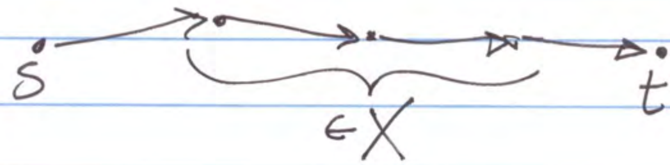
$L(\alpha\beta) = L(\alpha) \cdot L(\beta)$

$L(\alpha^*) = L(\alpha)^*$

Direction ①: Given $M = (Q, \Sigma, \delta, s, F)$, DFA, want to describe α such that $L(\alpha) = L(M)$.

Build α_{st}^X, where $s, t \in Q$, $X \subseteq Q$.

$L(\alpha_{st}^X) = \{$strings labeling paths from s to t going through intermediate states in $X\}$



α_{st}^{X}

Build by ind on $|X|$.

$$L(M) = \sum_{t \in F} \alpha_{st}^{Q}, \text{ where } s \text{ is start state.}$$