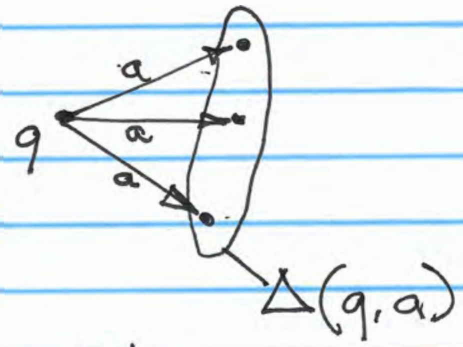
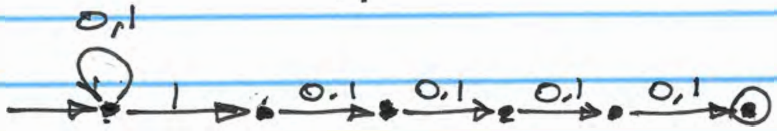
CS 4810 Lecture 2/1/2022

Nondeterminism



$P = NP$



$L(M) = \{ x \in \{0,1\}^* \mid 5^{\text{th}}$ symbol from right is $1 \}$

$100100011 \in L(M)$ $\qquad$ $11001010 \notin L(M)$

Nondeterministic finite automaton (NFA)

$N = (Q, \Sigma, \Delta, S, F)$

$Q = \{\text{states}\}$ finite set

$\Sigma =$ alphabet " "

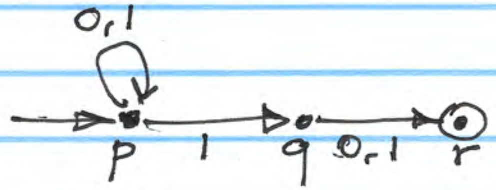
$S \subseteq Q$ start states

$F \subseteq Q$ accept states

Δ transition function

$\Delta : Q \times \Sigma \to 2^Q$

$2^Q =$ "powerset" of $Q = \{ A \mid A \subseteq Q \}$



$Q = \{p, q, r\}$
$S = \{s\}$ $\quad F = \{r\}$
$\Delta(p, 0) = \{p\} \quad \Delta(p, 1) = \{p, q\}$

		0	1
$\rightarrow$	p	$\{p\}$	$\{p, q\}$
	q	$\{r\}$	$\{r\}$
	r F	$\emptyset$	$\emptyset$

Extend $\Delta : Q \times \Sigma \rightarrow 2^Q$ to $\hat{\Delta} : 2^Q \times \Sigma^* \rightarrow 2^Q$:

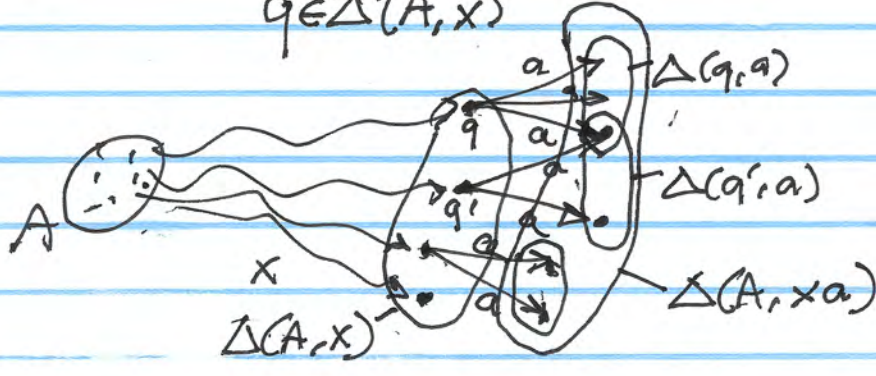
Intuitively:

$\hat{\Delta}(A, x) = \{$states could be in after scanning x starting from any state in $A \subseteq Q\}$

Define $\hat{\Delta}$ by induction on $|x|$:

$\hat{\Delta}(A, \varepsilon) \triangleq A \quad$ basis

$\hat{\Delta}(A, xa) \triangleq \bigcup_{q \in \hat{\Delta}(A,x)} \Delta(q, a) \quad$ for $x \in \Sigma^*, a \in \Sigma$



Lemma $\hat{\Delta}(A, xy) = \hat{\Delta}(\hat{\Delta}(A, x), y)$

Proof By induction on $|y|$.

Def. x is *accepted* by N if $\hat{\Delta}(S, x) \cap F \neq \emptyset$.

$L(N) = \{x \in \Sigma^* \mid x \text{ accepted by } N\}$

Given DFA $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$,

there is an NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$ such that $L(N) = L(M)$. Let

$Q_N = Q_M$

$\Delta_N(q, a) = \{\delta_M(q, a)\}$

$S_N = \{s_M\}$ $\quad F_N = F_M$

Thm Given NFA $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$ there is a DFA $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$ such that $L(M) = L(N)$.

$Q_M = 2^{Q_N}$

$\delta_M : Q_M \times \Sigma \to Q_M$

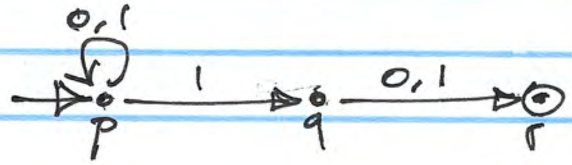
$\delta_M : 2^{Q_N} \times \Sigma \longrightarrow 2^{Q_N}$

$\delta_M(A, a) = \hat{\Delta}(A, a)$

$s_M \in Q_M$

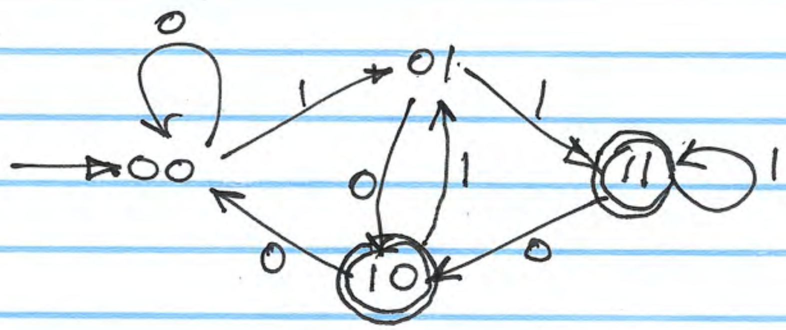
$s_M \in 2^{Q_N}$ $\quad s_M \subseteq Q_N$ $\quad s_M \triangleq S_N$ $\quad F_M \subseteq Q_M = 2^{Q_N}$

$$F_M \triangleq \{A \subseteq Q_N \mid A \cap F_N \neq \emptyset\}.$$

 $= N$

$$Q_M = \{\emptyset \quad \{p\} \quad \{q\} \quad \{r\} \quad \{p,q\} \quad \{q,r\} \quad \{p,r\} \quad \{p,q,r\}\}$$

	0	1
$\checkmark \to \{p\} \checkmark$	$\{p\}$	$\{p,q\}$
~~$\{q\}$~~	~~$\{r\}$~~	~~$\{r\}$~~
~~$\{r\}$ F~~	~~$\emptyset$~~	~~$\emptyset$~~
~~$\emptyset$~~	~~$\emptyset$~~	~~$\emptyset$~~
$\checkmark \{p,q\}$	$\{p,r\}$	$\{p,q,r\}$
$\checkmark \{p,r\}$ F	$\{p\}$	$\{p,q\}$
~~$\{q,r\}$ F~~	~~$\{r\}$~~	~~$\{r\}$~~
$\checkmark \{p,q,r\}$ F	$\{p,r\}$	$\{p,q,r\}$



Proof.

$x \in L(M) \iff$

$\hat{\delta}_M(s_M, x) \in F_M$ by def of acceptance M

$\iff \hat{\delta}_M(S_N, x) \in \{A \subseteq Q_N \mid A \cap F_N \neq \emptyset\}$

by def. of s_M & F_M

$\iff \hat{\Delta}_N(S_N, x) \in \{A \subseteq Q_N \mid A \cap F_N \neq \emptyset\}$

by Lemma (to follow!)

$\iff \hat{\Delta}_N(S_N, x) \cap F_N \neq \emptyset$

$\iff x \in L(N)$ by def of acceptance for N.

$\Rightarrow L(M) = L(N)$.

Lemma $\hat{\delta}_M(A, x) = \hat{\Delta}_N(A, x)$, $A \subseteq Q_N$, $x \in \Sigma^*$

Proof. By ind. on $|x|$.

$\hat{\delta}_M(A, \varepsilon) = A = \hat{\Delta}_N(A, \varepsilon)$ ← Basis

$$
\begin{aligned}
\hat{\delta}_M(A, xa) &= \hat{\delta}_M(\hat{\delta}_M(A, x), a) && \text{def of } \hat{\delta}_M \\
&= \hat{\delta}_M(\hat{\Delta}_N(A, x), a) && \text{by ind. hyp.} \\
&= \Delta_N(\hat{\Delta}_N(A, x), a) \\
&= \hat{\Delta}_N(A, xa). && \text{def of } \hat{\Delta}_N
\end{aligned}
$$

Lemma

$$\hat{\delta}_M(A, a) = \bigcup_{q \in A} \Delta_N(q, a) = \hat{\Delta}_N(A, a)$$