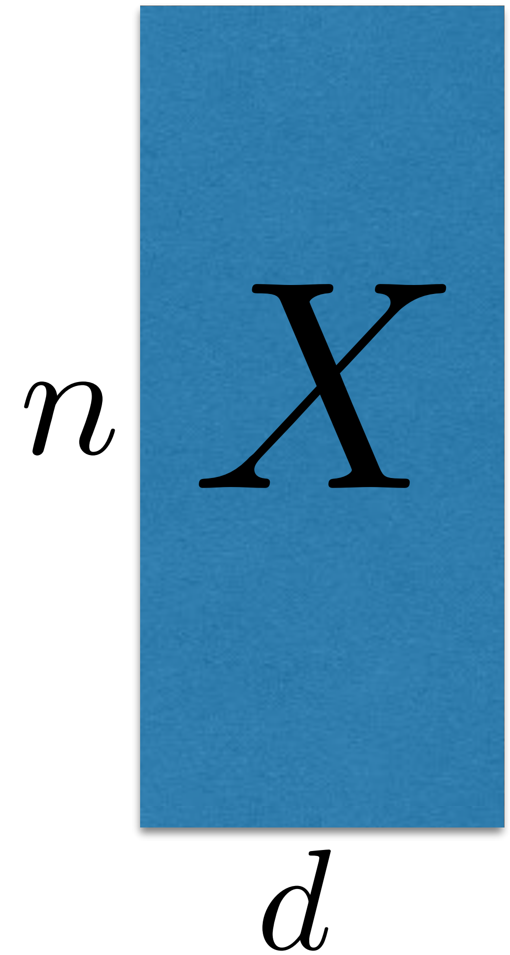
Machine Learning for Data Science (CS4786)
Lecture 11

Random Projections & Canonical Correlation Analysis

Course Webpage :
http://www.cs.cornell.edu/Courses/cs4786/2017fa/

The Tall, THE FAT AND THE UGLY



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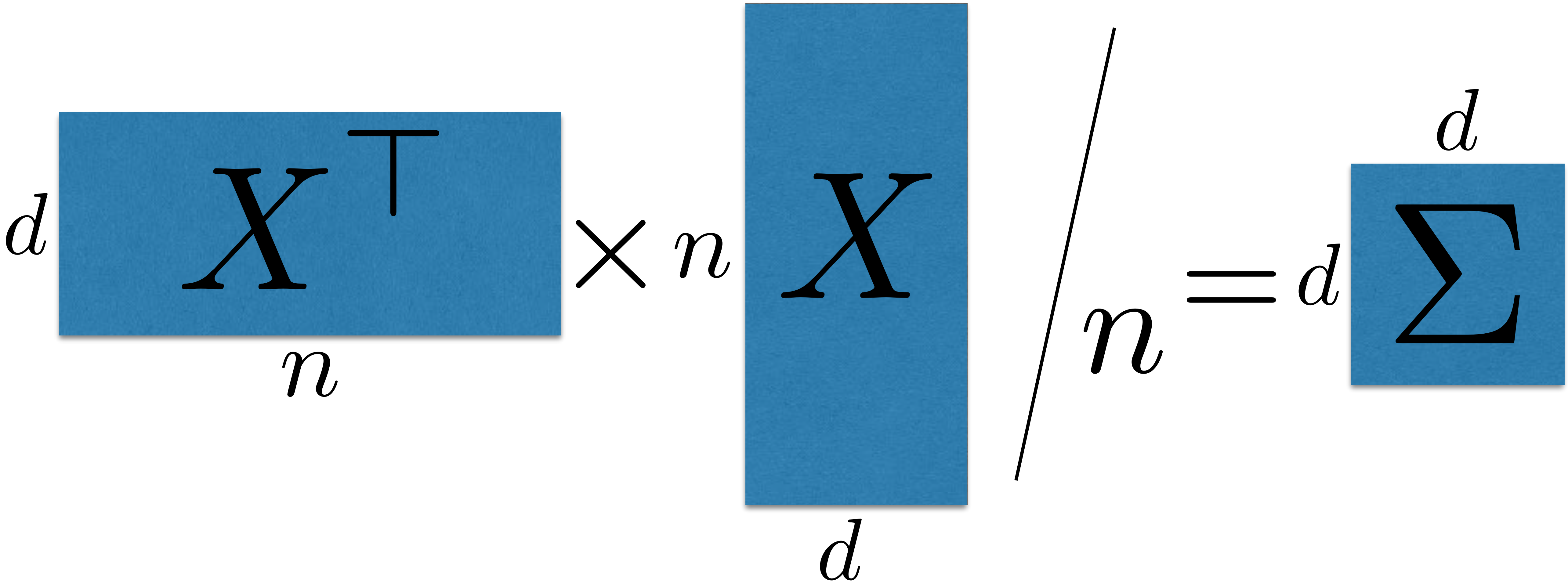

The Tall, the Fat and the Ugly

$$X^\top \times X \,/\, n = \Sigma$$

$$W = \mathrm{Eigs}(\Sigma, K)$$

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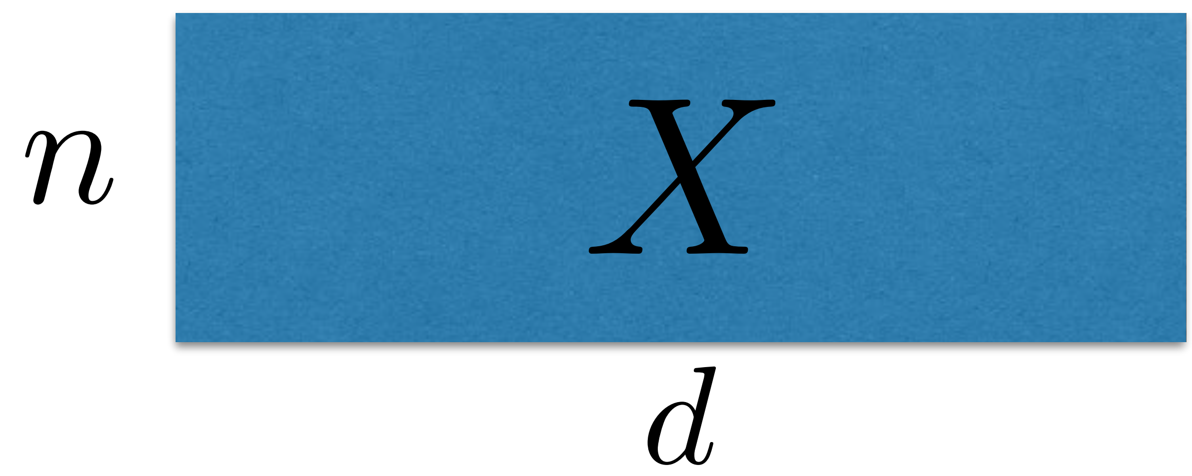

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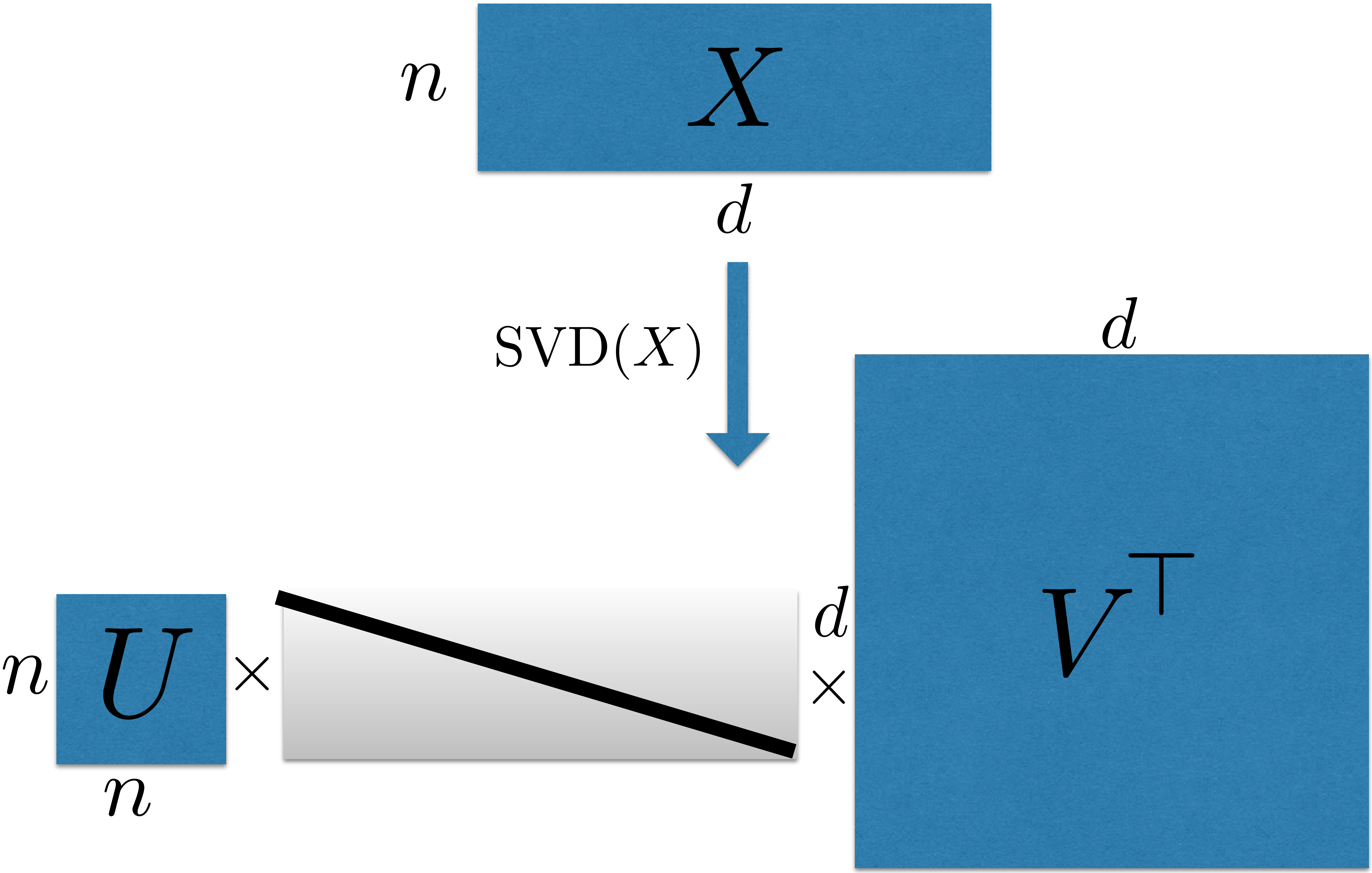

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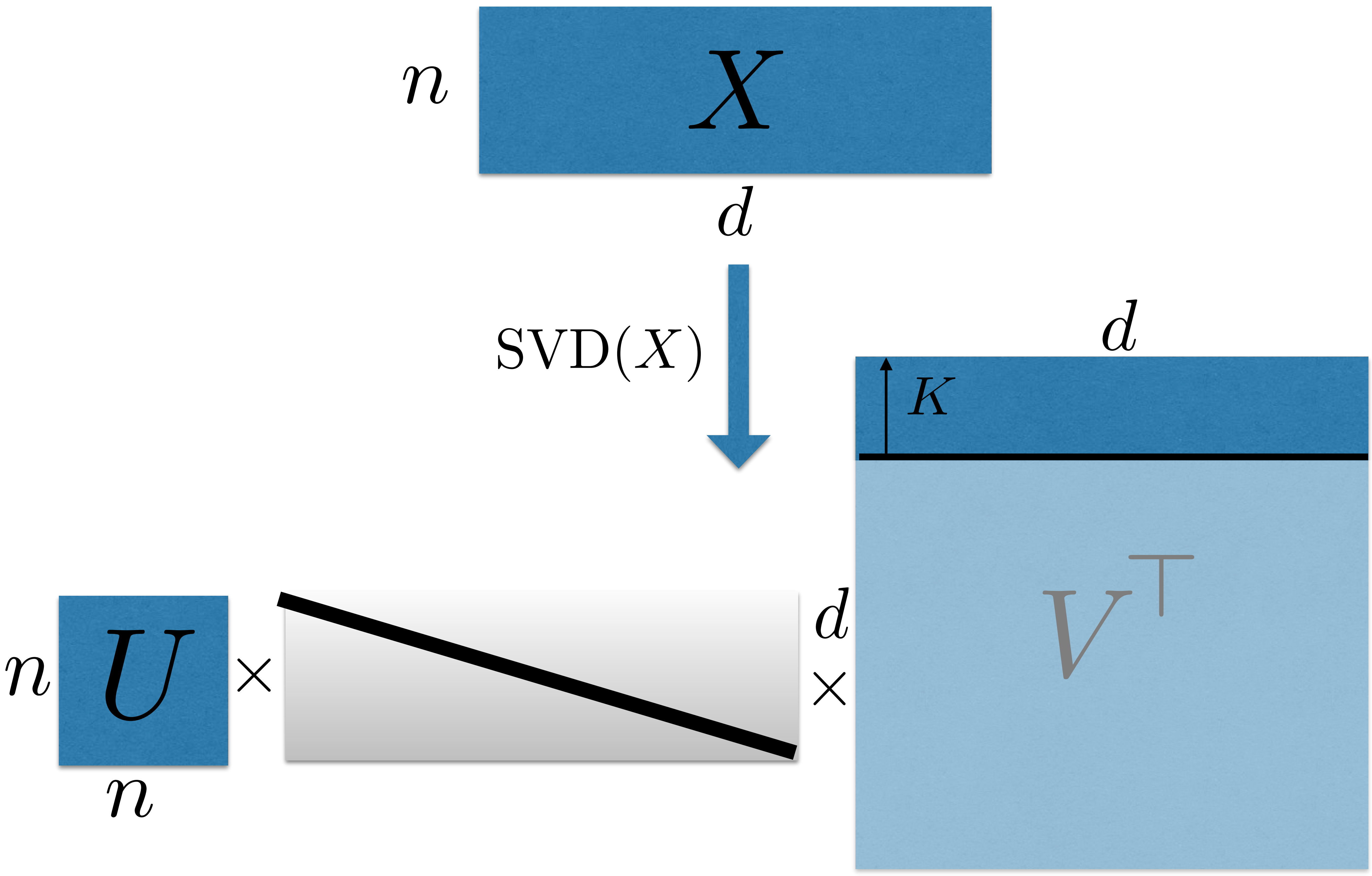

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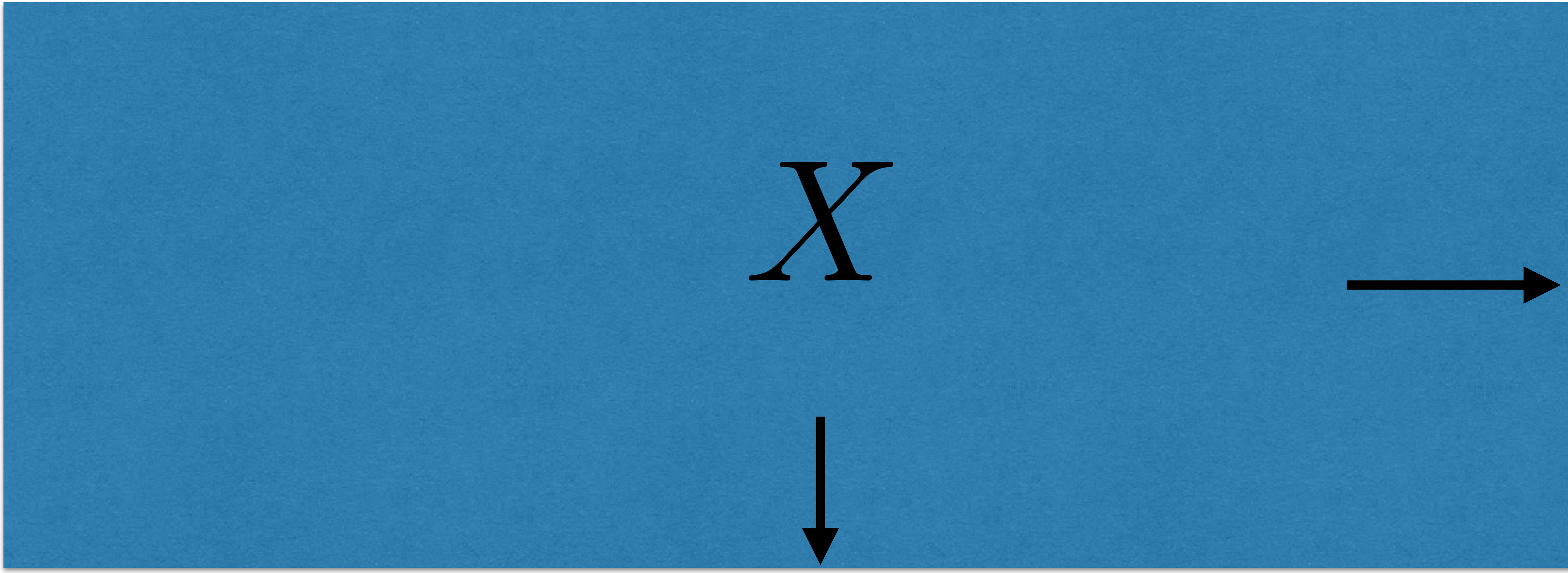

- d and n so large we can't even store in memory
- Only have time to be linear in $\text{size}(X) = n \times d$

I there any hope?

Pick a Random W

Pick a Random W

$$Y = X \times \left.\underbrace{\begin{bmatrix} +1 & \dots & -1 \\ -1 & \dots & +1 \\ +1 & \dots & -1 \\ & \vdots & \\ +1 & \dots & -1 \end{bmatrix}}_{K} d \right/ \sqrt{K}$$

Why should Random Projections even work?!

Random Projection

- What does “it works” even mean?

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  Distances between all pairs of data-points in low dim. projection is roughly the same as their distances in the high dim. space.

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  Distances between all pairs of data-points in low dim. projection is roughly the same as their distances in the high dim. space.

  That is, when K is "large enough", with "high probability", for all pairs of data points $i, j \in \{1, \ldots, n\}$,

$$(1-\epsilon)\left\|\mathbf{y}_i - \mathbf{y}_j\right\|_2 \leq \left\|\mathbf{x}_i - \mathbf{x}_j\right\|_2 \leq (1+\epsilon)\left\|\mathbf{y}_i - \mathbf{y}_j\right\|_2$$

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$$= \sum_{i=1}^{d} (W[i,1] \cdot \tilde{\mathbf{x}}[i])^2 + 2 \sum_{i'>i} (W[i,1] \cdot \tilde{\mathbf{x}}[i]) \left(W[i',1] \cdot \tilde{\mathbf{x}}[i']\right)$$

WHY SHOULD RANDOM PROJECTIONS EVEN WORK?!

Say $K = 1$. Consider any vector $\tilde{\mathbf{x}} \in \mathbb{R}^d$ and let $\tilde{\mathbf{y}} = \tilde{\mathbf{x}}\, W$. Note that

$$\begin{aligned}
\tilde{\mathbf{y}}^2 &= \left(\sum_{i=1}^{d} W[i,1] \cdot \tilde{\mathbf{x}}[i]\right)^2 \\
&= \sum_{i=1}^{d} (W[i,1] \cdot \tilde{\mathbf{x}}[i])^2 + 2 \sum_{i'>i} (W[i,1] \cdot \tilde{\mathbf{x}}[i]) \left(W[i',1] \cdot \tilde{\mathbf{x}}[i']\right) \\
&= \sum_{i=1}^{d} W^2[i,1]\tilde{\mathbf{x}}^2[i] + \sum_{i'>i} \left(W[i,1] \cdot W[i',1]\right) \cdot \left(\tilde{\mathbf{x}}[i] \cdot \tilde{\mathbf{x}}[i']\right)
\end{aligned}$$

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However $W^2[i,1] = 1/K = 1$ when $K = 1$

$$= \sum_{i=1}^{d} \tilde{\mathbf{x}}^2[i] + \sum_{i'>i} \left(W[i,1] \cdot W[i',1]\right) \cdot \left(\tilde{\mathbf{x}}[i] \cdot \tilde{\mathbf{x}}[i']\right)$$

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Hence,

$$\mathbb{E}\left[\tilde{\mathbf{y}}^2\right] = \sum_{i=1}^{d} \tilde{\mathbf{x}}^2[i] + \sum_{i'>i} \mathbb{E}\left[W[i,1] \cdot W[i',1]\right] \cdot \left(\tilde{\mathbf{x}}[i] \cdot \tilde{\mathbf{x}}[i']\right)$$

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However $W[i,1]$ and $W[i',1]$ are independent and so

$$\mathbb{E}\left[W[i,1] \cdot W[i',1]\right] = \mathbb{E}\left[W[i,1]\right] \cdot \mathbb{E}\left[W[i',1]\right] = 0$$

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Using this we conclude that

$$\mathbb{E}\left[\tilde{\mathbf{y}}^2\right] = \sum_{i=1}^{d} \tilde{\mathbf{x}}^2[i] = \|\tilde{\mathbf{x}}\|^2$$

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Lets try this in Matlab ...

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This is like taking an average of K independent measurements whose expectations are $\|\tilde{\mathbf{x}}\|_2^2$

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For any $\epsilon > 0$, if $K \approx \log(n/\delta)/\epsilon^2$, with probability $1 - \delta$ over draw of W, for all pairs of data points $i, j \in \{1, \ldots, n\}$,

$$(1 - \epsilon)\left\|\mathbf{y}_i - \mathbf{y}_j\right\|_2^2 \leq \left\|\mathbf{x}_i - \mathbf{x}_j\right\|_2 \leq (1 + \epsilon)\left\|\mathbf{y}_i - \mathbf{y}_j\right\|_2^2$$

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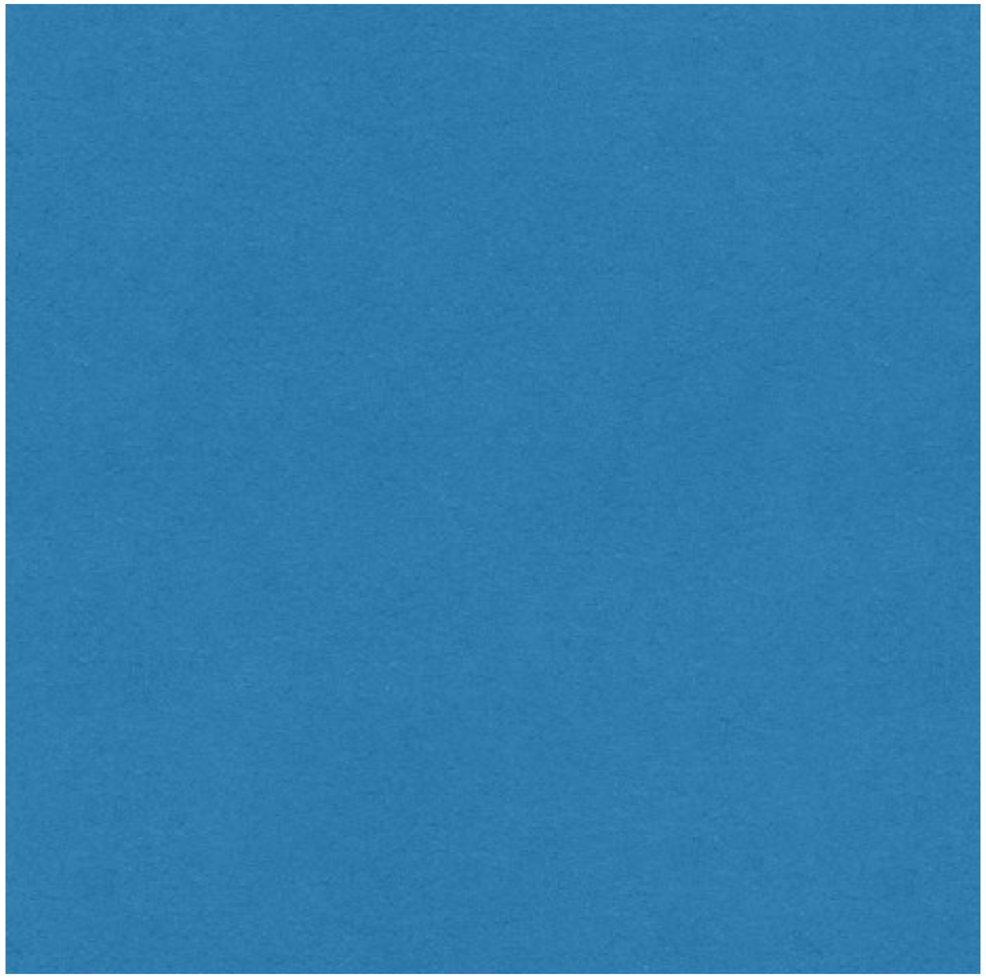
This is called the Johnson-Lindenstrauss lemma or JL lemma for short.

Why is this so Ridiculously Magical?



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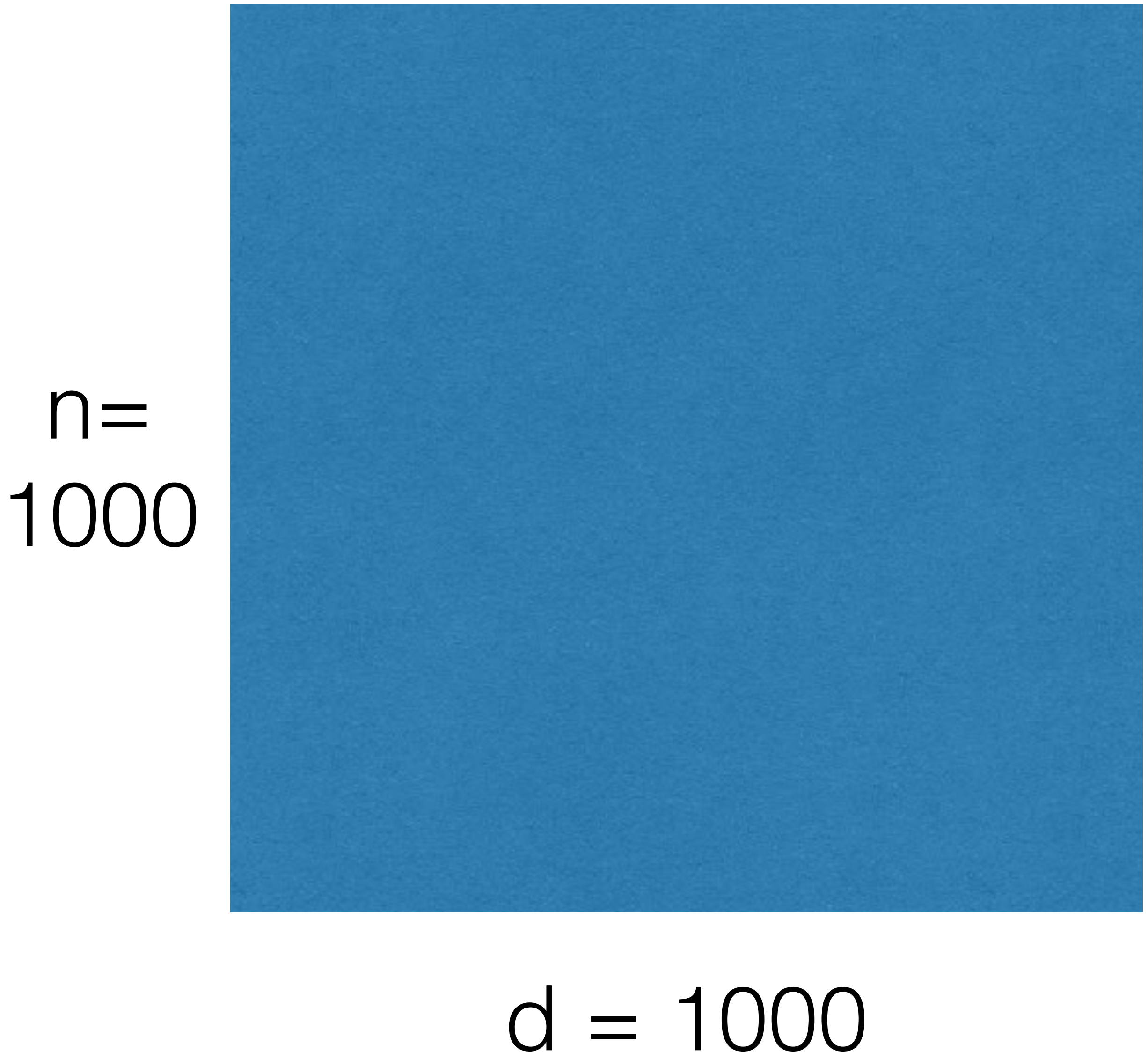

If we take $K = 69.1/\epsilon^2$, with probability 0.99 distances are preserved to accuracy ϵ

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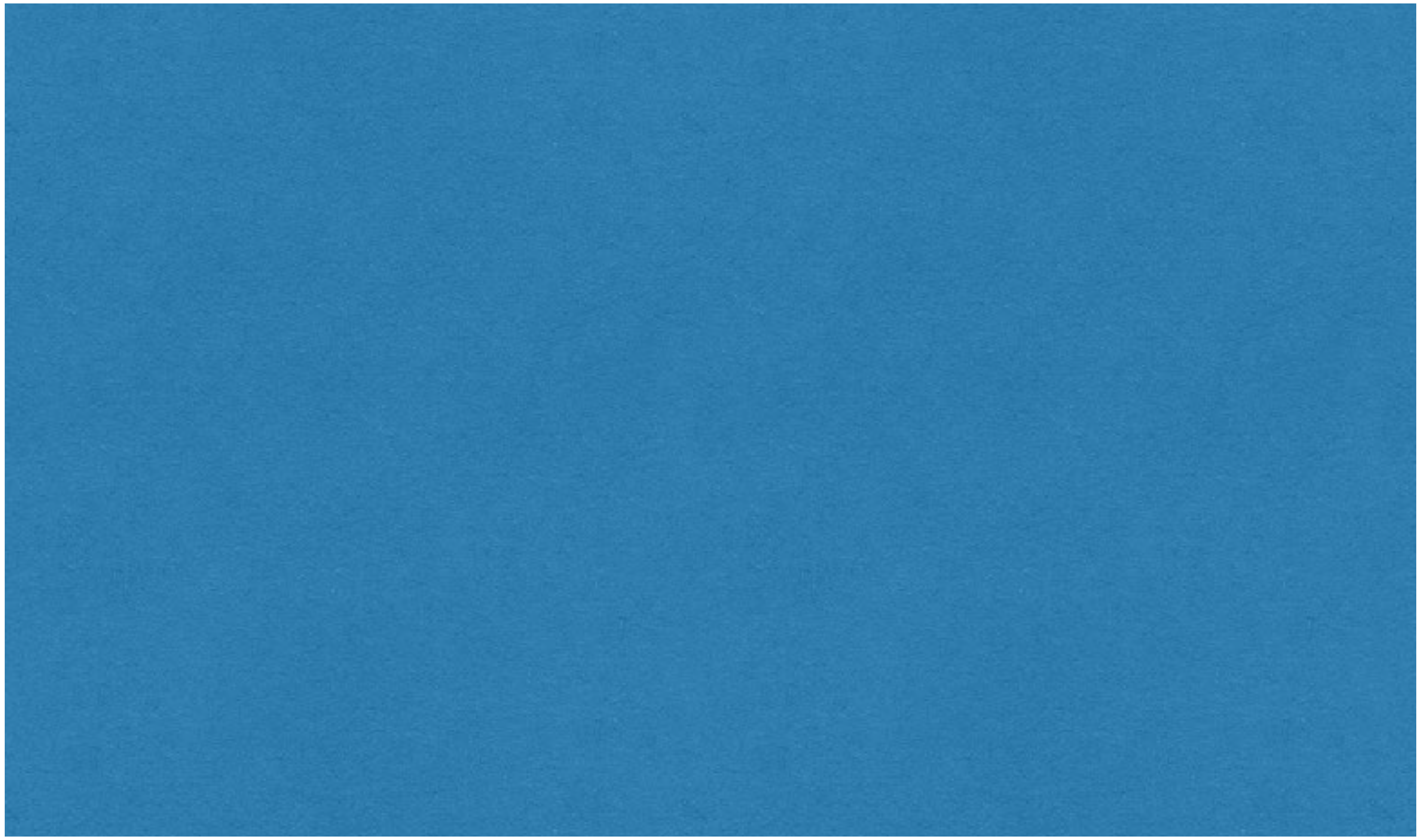
n=
1000



d = 10000

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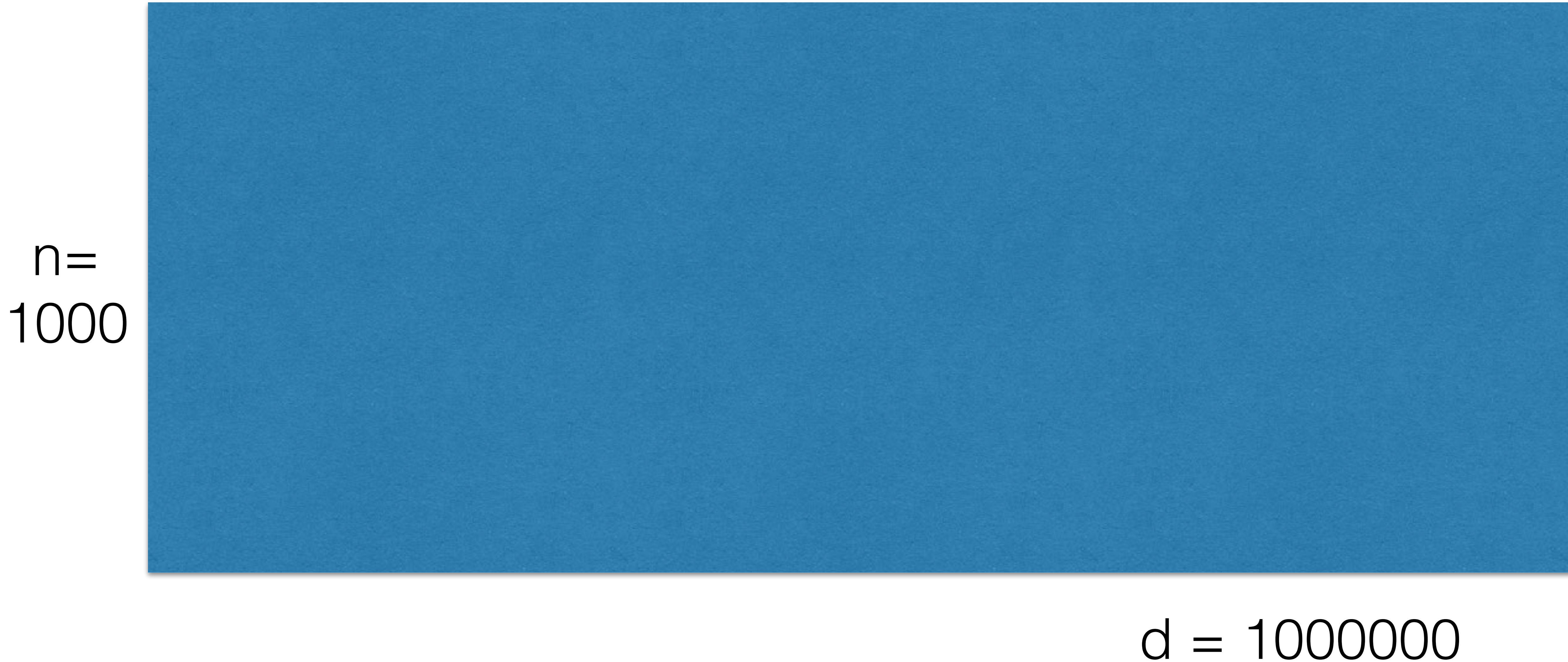

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Two View Dimensionality Reduction

- Data comes in pairs $(\mathbf{x}_1, \mathbf{x}_1'), \ldots, (\mathbf{x}_n, \mathbf{x}_n')$ where $\mathbf{x}_t$'s are d dimensional and $\mathbf{x}_t'$'s are d' dimensional
- Goal: Compress say view one into $\mathbf{y}_1, \ldots, \mathbf{y}_n$, that are K dimensional vectors
  - Retain information redundant between the two views
  - Eliminate "noise" specific to only one of the views

Canonical Correlation Analysis

Canonical Correlation Analysis

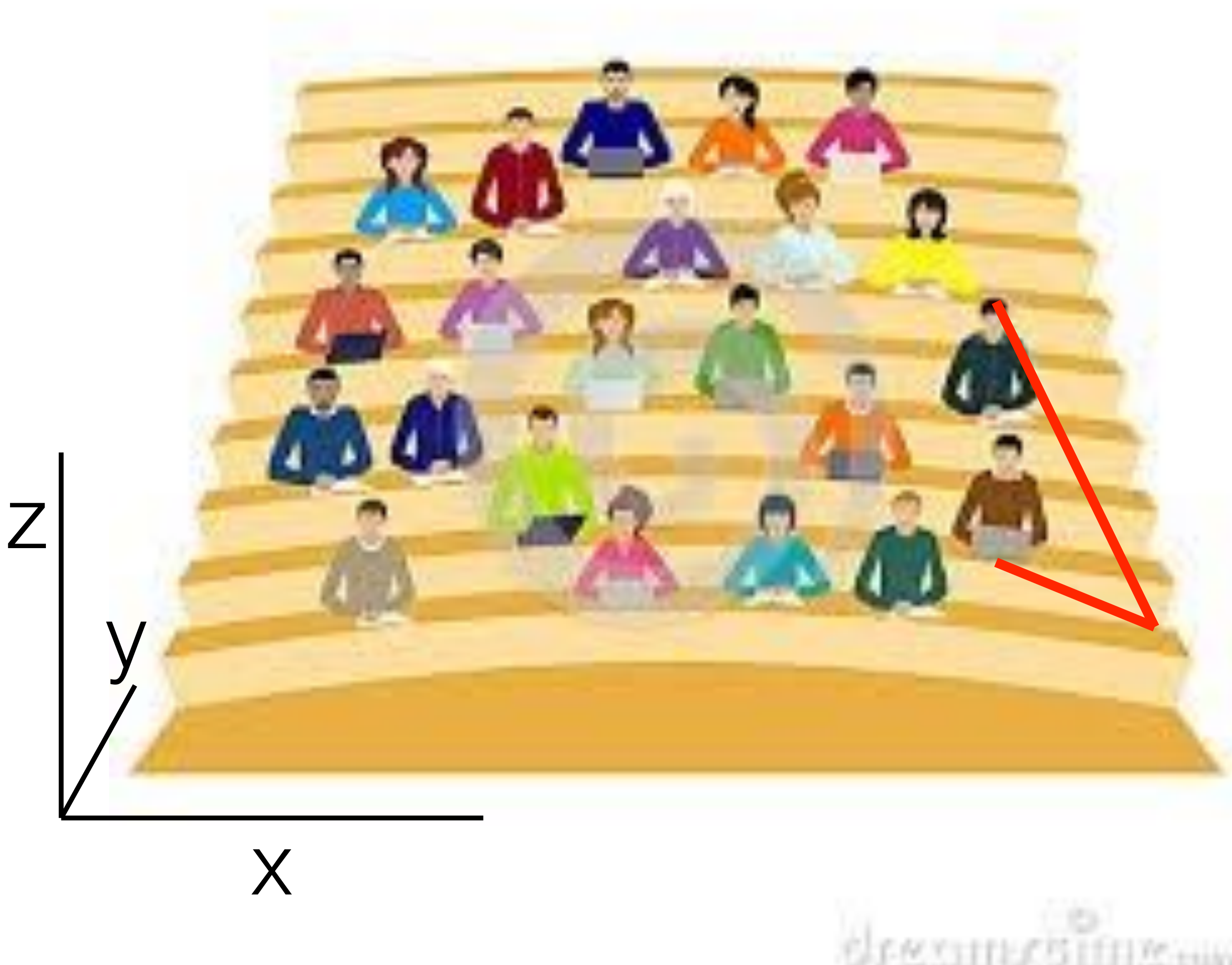
+

Age

Gender

Angle

Canonical Correlation Analysis



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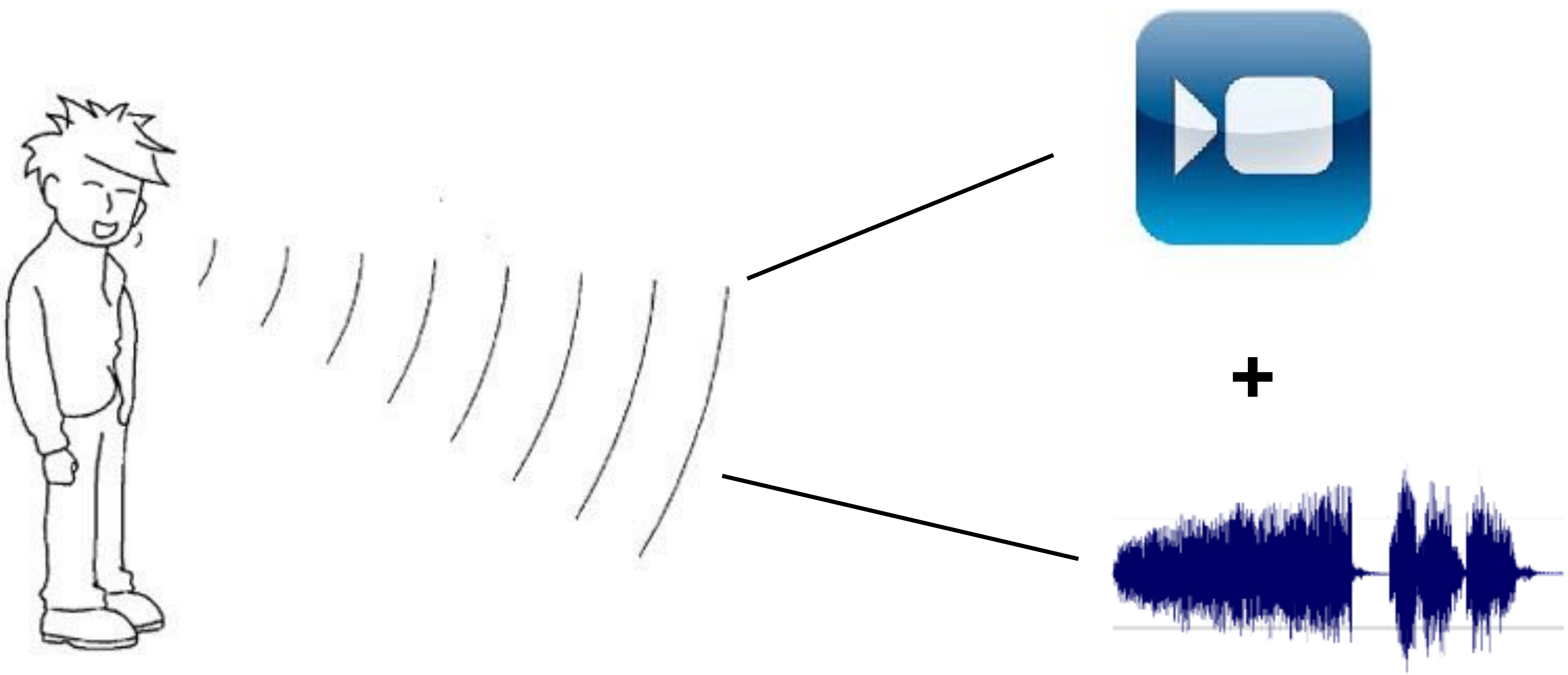
Example I: Speech Recognition



- Audio might have background sounds uncorrelated with video
- Video might have lighting changes uncorrelated with audio
- Redundant information between two views: the speech

Example II: Combining Feature Extractions

- Method A and Method B are both equally good feature extraction techniques
- Concatenating the two features blindly yields large dimensional feature vector with redundancy
- Applying techniques like CCA extracts the key information between the two methods
- Removes extra unwanted information

How do we get the right direction? (say K = 1)

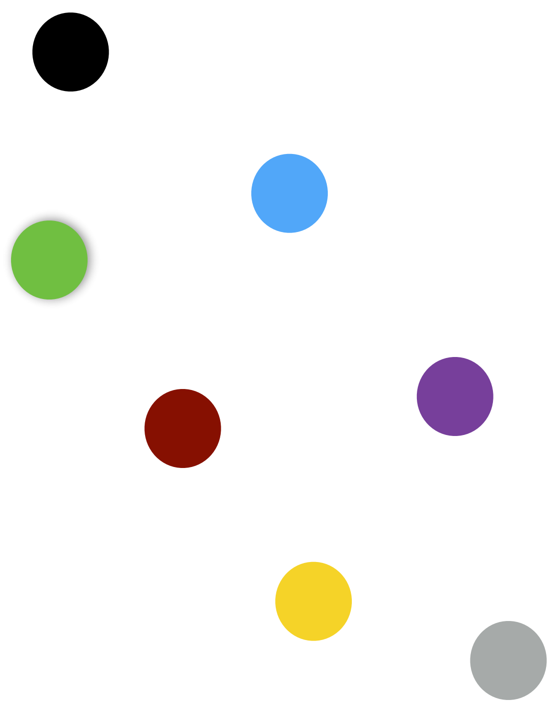
+

Age

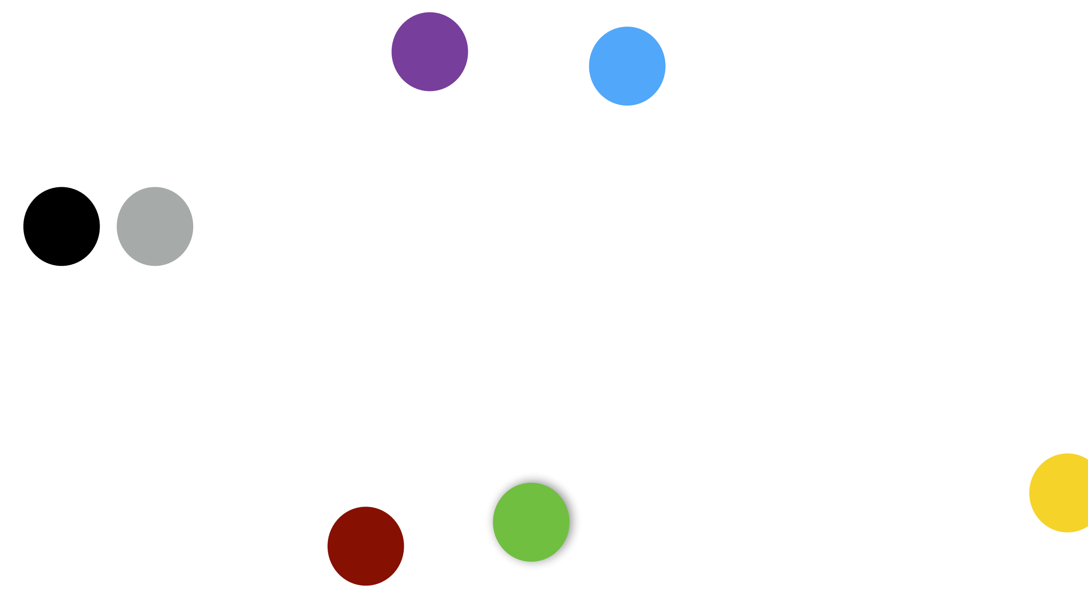
Gender

Angle

Which Direction to Pick?



View I



View II

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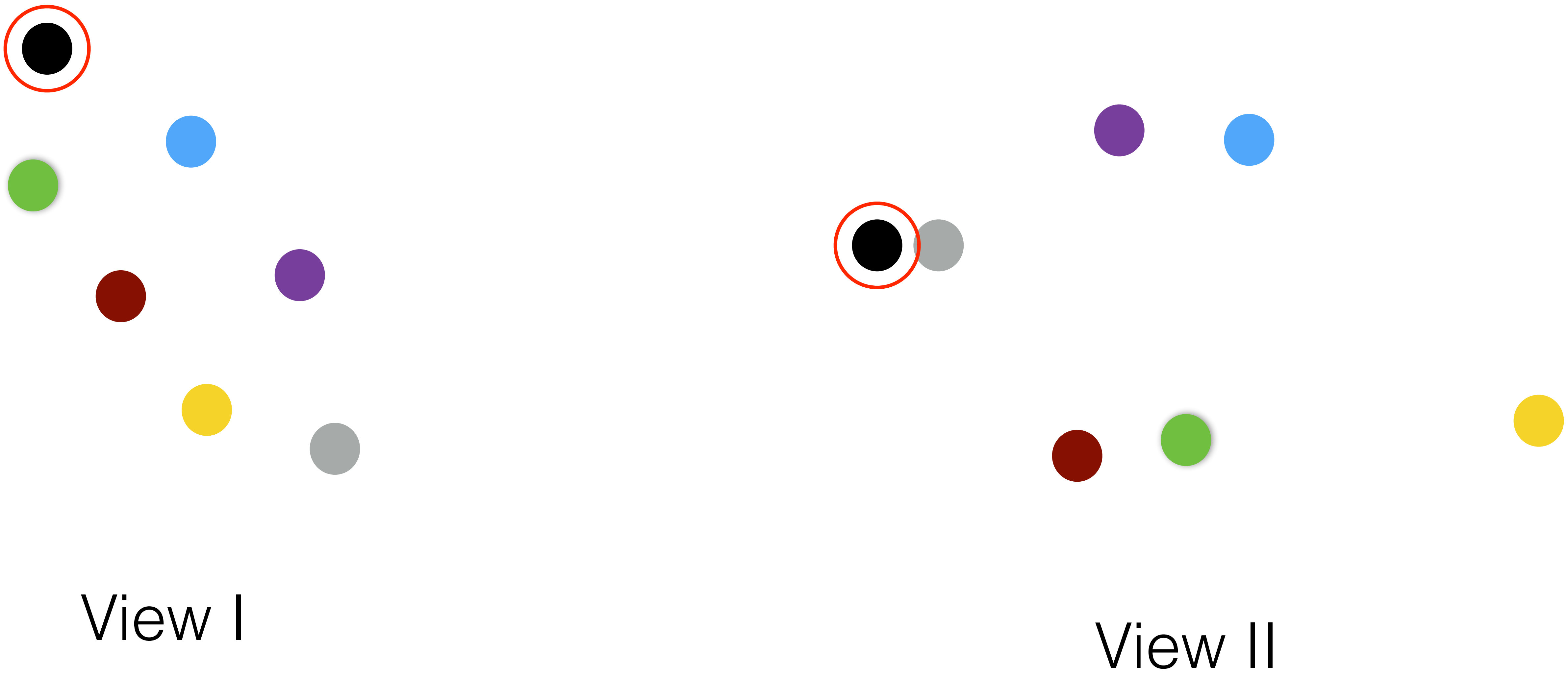

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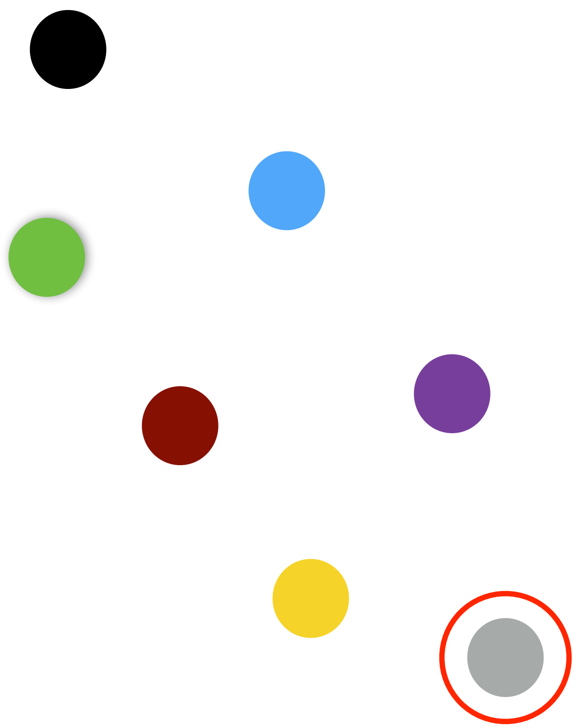

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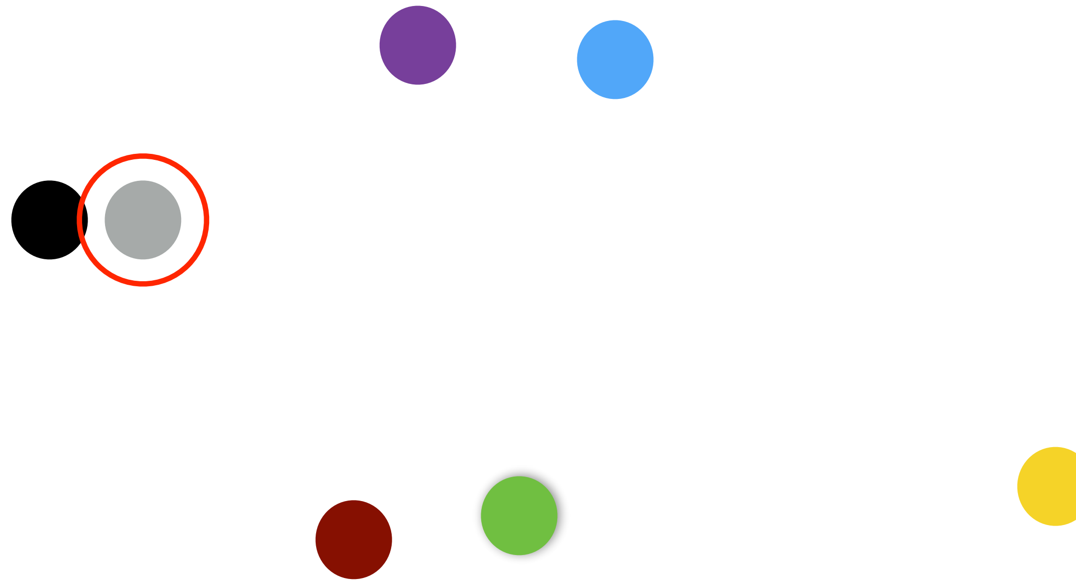

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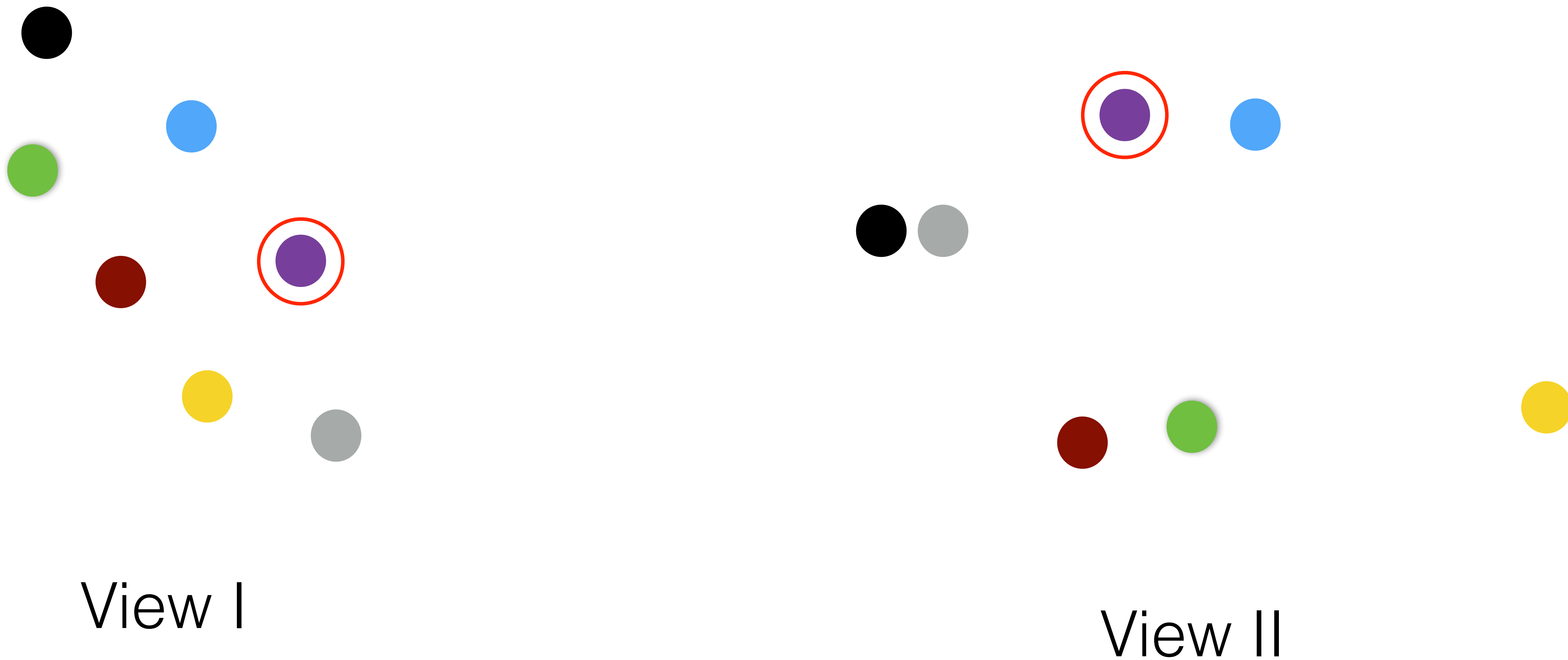

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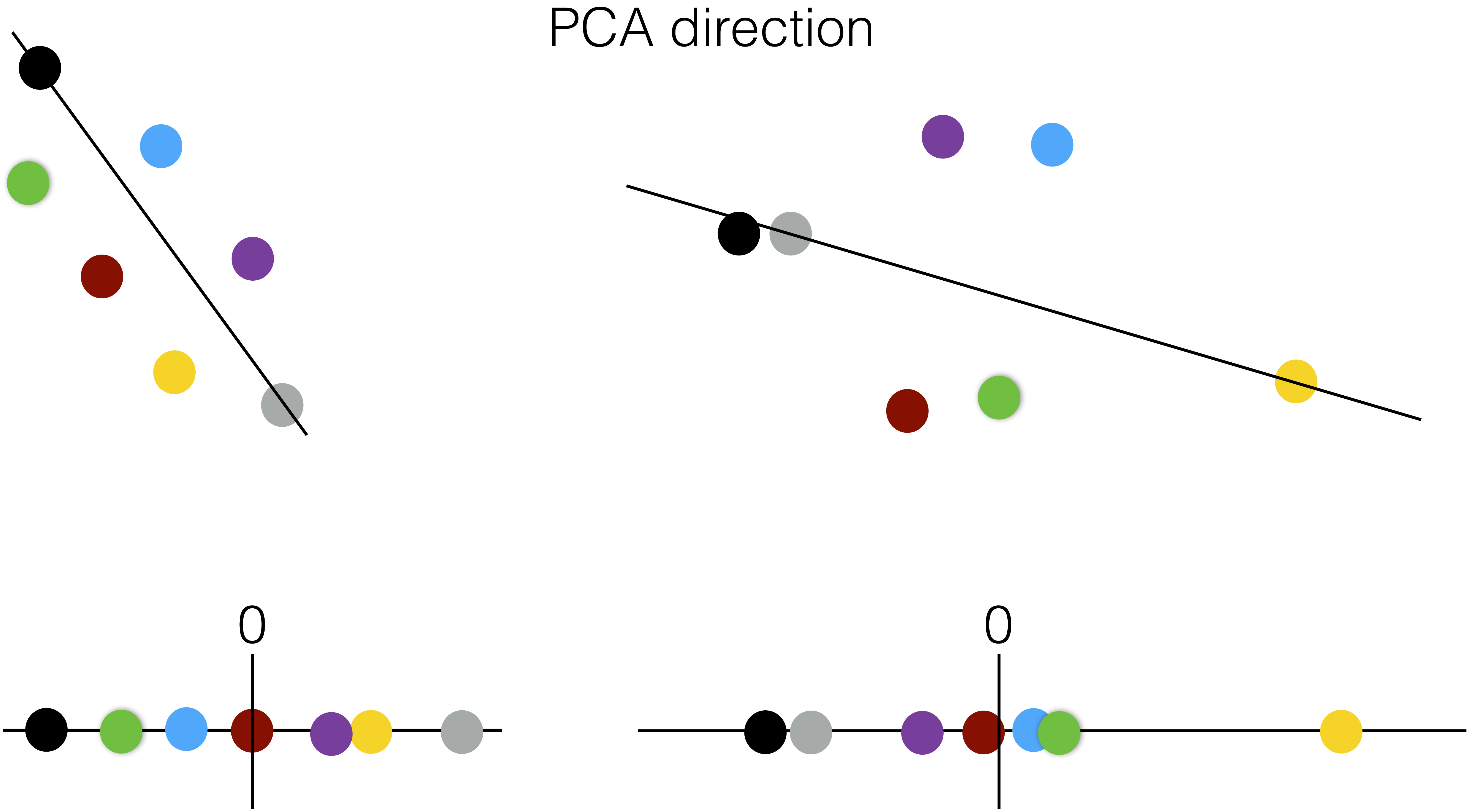

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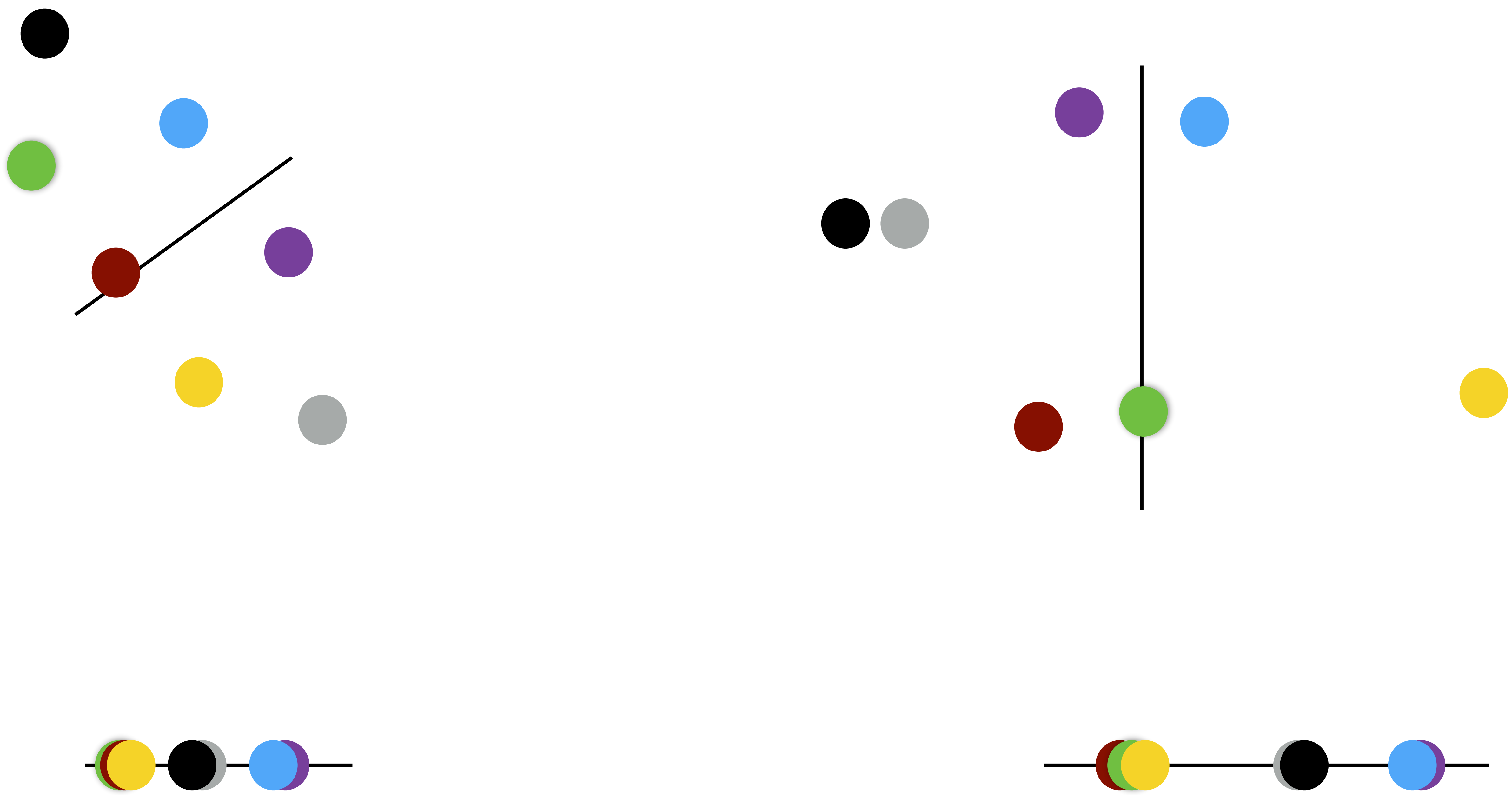


Direction has large covariance

How do we pick the right direction to project to?

Maximizing Correlation Coefficient

- Say $\mathbf{w}_1$ and $\mathbf{v}_1$ are the directions we choose to project in views 1 and 2 respectively we want these directions to maximize,

$$\frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{y}_t[1]-\frac{1}{n}\sum_{t=1}^{n}\mathbf{y}_t[1]\right)\cdot\left(\mathbf{y}'_t[1]-\frac{1}{n}\sum_{t=1}^{n}\mathbf{y}'_t[1]\right)$$

where $\mathbf{y}_t[1]=\mathbf{w}_1^\top\mathbf{x}_t$ and $\mathbf{y}'_t[1]=\mathbf{v}_1^\top\mathbf{x}'_t$

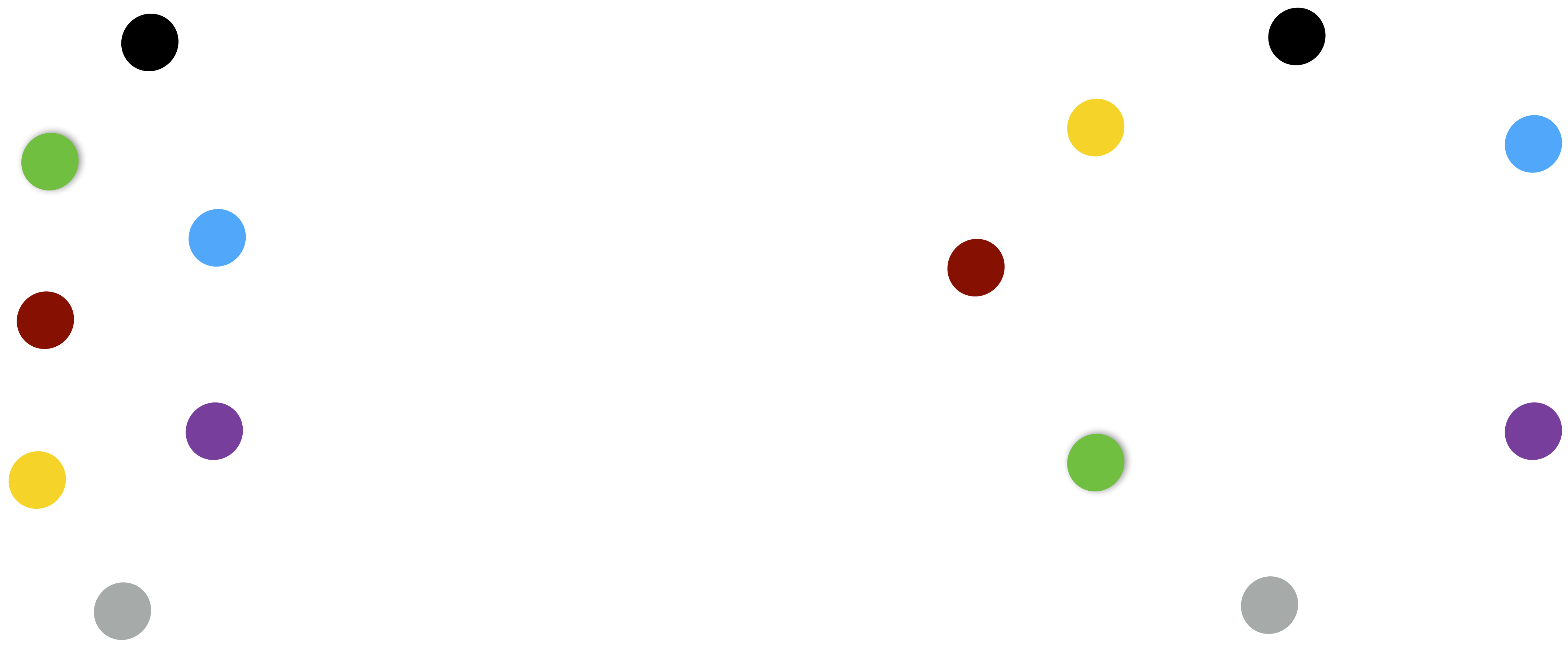
What is the problem with the above?

Why not Maximize Covariance

$$\text{Say } \frac{1}{n}\sum_{t=1}^{n} \mathbf{x}_t[2] \cdot \mathbf{x}'_t[2] > 0$$

Scaling up this coordinate we can blow up covariance

Why not Maximize Covariance



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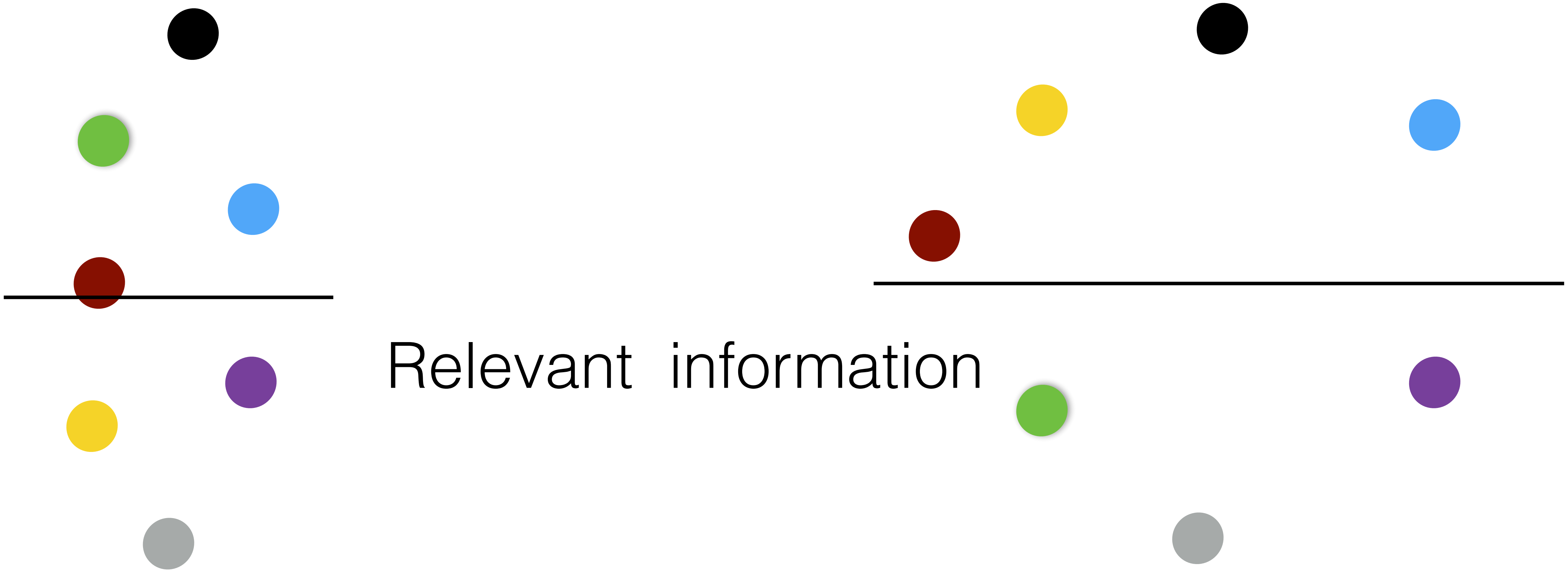

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$$\frac{\frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{y}_t[1] - \frac{1}{n}\sum_{t=1}^{n}\mathbf{y}_t[1]\right)\cdot\left(\mathbf{y}'_t[1] - \frac{1}{n}\sum_{t=1}^{n}\mathbf{y}'_t[1]\right)}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{y}_t[1] - \frac{1}{n}\sum_{t=1}^{n}\mathbf{y}_t[1]\right)^2}\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{y}'_t[1] - \frac{1}{n}\sum_{t=1}^{n}\mathbf{y}'_t[1]\right)}}$$

Basic Idea of CCA

- Normalize variance in chosen direction to be constant (say 1)
- Then maximize covariance
- This is same as maximizing "correlation coefficient"

Covariance Vs Correlation

- $\text{Covariance}(A, B) = \mathbb{E}[(A - \mathbb{E}[A]) \cdot (B - \mathbb{E}[B])]$

  Depends on the scale of A and B. If B is rescaled, covariance shifts.

- $\text{Corelation}(A, B) = \frac{\mathbb{E}[(A - \mathbb{E}[A]) \cdot (B - \mathbb{E}[B])]}{\sqrt{\text{Var}(A)}\sqrt{\text{Var}(B)}}$

  Scale free.

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s.t. $\frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{y}_t[1]-\frac{1}{n}\sum_{t=1}^{n}\mathbf{y}_t[1]\right)^2 = \frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{y}'_t[1]-\frac{1}{n}\sum_{t=1}^{n}\mathbf{y}'_t[1]\right) = 1$

where $\mathbf{y}_t[1] = \mathbf{w}_1^\top\mathbf{x}_t$ and $\mathbf{y}'_t[1] = \mathbf{v}_1^\top\mathbf{x}'_t$

Canonical Correlation Analysis

- Hence we want to solve for projection vectors $\mathbf{w}_1$ and $\mathbf{v}_1$ that

$$\text{maximize } \frac{1}{n}\sum_{t=1}^{n} \mathbf{w}_1^\top(\mathbf{x}_t - \mu) \cdot \mathbf{v}_1^\top(\mathbf{x}'_t - \mu')$$

$$\text{subject to } \frac{1}{n}\sum_{t=1}^{n}(\mathbf{w}_1^\top(\mathbf{x}_t - \mu))^2 = \frac{1}{n}\sum_{t=1}^{n}(\mathbf{v}_1^\top(\mathbf{x}'_t - \mu'))^2 = 1$$

where $\mu = \frac{1}{n}\sum_{t=1}^{n}\mathbf{x}_t$ and $\mu' = \frac{1}{n}\sum_{t=1}^{n}\mathbf{x}'_t$

Canonical Correlation Analysis

- Hence we want to solve for projection vectors $\mathbf{w}_1$ and $\mathbf{v}_1$ that

$$\begin{aligned}&\text{maximize } \mathbf{w}_1^\top \Sigma_{1,2} \mathbf{v}_1 \\ &\text{subject to } \mathbf{w}_1^\top \Sigma_{1,1} \mathbf{w}_1 = \mathbf{v}_1^\top \Sigma_{2,2} \mathbf{v}_1 = 1\end{aligned}$$

CANONICAL CORRELATION ANALYSIS

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$$\Sigma = \begin{matrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix} = \mathrm{cov}\left(X \; X' \right)$$

Solution

Solution

$$W_1 = \text{eigs}\left(\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}, K\right)$$

$$W_2 = \text{eigs}\left(\Sigma_{22}^{-1}\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}, K\right)$$

CCA Algorithm

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1. $$X = \Big({}^{n}\underbrace{X_1}_{d_1},\ \underbrace{X_2}_{d_2} \Big)$$

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CCA Algorithm

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3. $W_1 = \mathrm{eigs}\left(\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}, K \right)$

CCA Algorithm

1. $X = \left({}^{n} X_1 , X_2 \right)$ with widths d_1, d_2

2. $\Sigma = \begin{matrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix} = \mathrm{cov}\left(X \right)$

3. $W_1 = \mathrm{eigs}\left(\Sigma_{11}^{-1}\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}, K \right)$

4. $Y_1 = X_1 - \mu_1 \times W_1$

CCA Demo