Modeling Sequence Data

CS4780 – Machine Learning
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Thorsten Joachims
Cornell University

Reading:
Leeds Online HMM Tutorial
(except Forward and Forward/Backward Algorithm)

Outline

- **Markov Models in Classification**
  - A "less naïve" Bayes for text classification
- **Hidden Markov Models**
  - Part-of-speech tagging
  - Viterbi Algorithm
  - Estimation with fully observed training data

"Less Naïve" Bayes Classifier

- **Example: Classify sentences as insulting/not insulting**

text	Insult?
$\vec{x}_1 = (Peter, is, nice, and, not, stupid)$	-1
$\vec{x}_2 = (Peter, is, not, nice, and, stupid)$	+1

- **Assumption (*l* words in document)**

$$P(X=\vec{x}|Y=+1) = P(W=w_i|Y=+1)\prod_{i=2}^{l} P(W=w_i|W_{prev}=w_{i-1}, Y=+1)$$

$$P(X=\vec{x}|Y=-1) = P(W=w_i|Y=-1)\prod_{i=2}^{l} P(W=w_i|W_{prev}=w_{i-1}, Y=-1)$$

- **Decision Rule**

$$h_{less}(\vec{x}) = argmax_{y\in\{+1,-1\}}\left\{P(Y=y)P(W=w_i|Y=y)\prod_{i=2}^{l} P(W=w_i|W_{prev}=w_{i-1}, Y=y)\right\}$$

Markov Model

- **Definition**
  - Set of States: $s_1,\ldots,s_k$
  - Start probabilities: *P(S=s)*
  - Transition probabilities: *P(S=s | S_{prev}=s')*
- **Random walk on graph**
  - Start in state *s* with probability *P(S=s)*
  - Move to next state with probability *P(S=s | S_{prev}=s')*
- **Assumptions**
  - Limited dependence: Next state depends only on previous state, but no other state (i.e. first order Markov model)
  - Stationary: *P(S=s | S_{prev}=s')* does not change

Part-of-Speech Tagging Task

- **Assign the correct part of speech (word class) to each word in a document**

  "The/DT planet/NN Jupiter/NNP and/CC its/PRP moons/NNS are/VBP in/IN effect/NN a/DT mini-solar/JJ system/NN ,/, and/CC Jupiter/NNP itself/PRP is/VBZ often/RB called/VBN a/DT star/NN that/IN never/RB caught/VBN fire/NN ./."
- **Needed as an initial processing step for a number of language technology applications**
  - Information extraction
  - Answer extraction in QA
  - Base step in identifying syntactic phrases for IR systems
  - Critical for word-sense disambiguation (WordNet apps)
  - …

Why is POS Tagging Hard?

- **Ambiguity**
  - He will race/VB the car.
  - When will the race/NOUN end?
  - I bank/VB at CFCU.
  - Go to the bank/NOUN!
- **Average of ~2 parts of speech for each word**
- **The number of tags used by different systems varies a lot. Some systems use < 20 tags, while others use > 400.**

The POS Learning Problem

- **Example**

sentence	POS
$\vec{x}_1 = (I, bank, at, CFCU)$	$\vec{y}_1 = (PRP, V, PREP, N)$
$\vec{x}_2 = (Go, to, the, bank)$	$\vec{y}_2 = (V, PREP, DET, N)$

Hidden Markov Model for POS Tagging

- **States**
  - Think about as nodes of a graph
  - One for each POS tag
  - special start state (and maybe end state)
- **Transitions**
  - Think about as directed edges in a graph
  - Edges have transition probabilities
- **Output**
  - Each state also produces a word of the sequence
  - Sentence is generated by a walk through the graph

Hidden Markov Model

- **States: $y \in \{s_1,\ldots,s_k\}$**
- **Outputs symbols: $x \in \{o_1,\ldots,o_m\}$**
- **Starting probability $P(Y_1 = y_1)$**
  - Specifies where the sequence starts
- **Transition probability $P(Y_i = y_i \mid Y_{i-1} = y_{i-1})$**
  - Probability that one states succeeds another
- **Output/Emission probability $P(X_i = x_i \mid Y_i = y_i)$**
  - Probability that word is generated in this state

=> Every output + state sequence has a probability

$$P(x_1,\ldots,x_n,y_1,\ldots,y_n) = \left[P(y_1)P(x_1 \mid y_1)\prod_{i=2}^{l} P(x_i \mid y_i)P(x_i \mid y_{i-1})\right]$$

HMM Decoding: Viterbi Algorithm

- **Question: What is the most likely state sequence given an output sequence**
  - Given fully specified HMM:
    - $P(Y_1 = y_1)$,
    - $P(Y_i = y_i \mid Y_{i-1} = y_{i-1})$,
    - $P(X_i = x_i \mid Y_i = y_i)$
  - Find

$$\max P(y_1,\ldots,y_l,x_1,\ldots,x_l) = \max_{(y_1,\ldots,y_l)}\left[P(y_1)P(x_1 \mid y_1)\prod_{i=2}^{l} P(x_i \mid y_i)P(y_i \mid y_{i-1})\right]$$

  - "Viterbi" algorithm has runtime linear in length of sequence
  - Example: find the most likely tag sequence for a given sequence of words

Viterbi Example

P(X=x|Y=y)	I	bank	at	CFCU	go	to	the
DET	0.01	0.01	0.01	0.01	0.01	0.01	0.94
PRP	0.94	0.01	0.01	0.01	0.01	0.01	0.01
N	0.01	0.4	0.01	0.4	0.16	0.01	0.01
PREP	0.01	0.01	0.48	0.01	0.01	0.47	0.01
V	0.01	0.4	0.01	0.01	0.55	0.01	0.01

P(Y=y)	
DET	0.3
PRP	0.3
N	0.1
PREP	0.1
V	0.2

$P(Y\|Y_{prev})$	DET	PRP	N	PREP	V
DET	0.01	0.01	0.96	0.01	0.01
PRP	0.01	0.01	0.01	0.2	0.77
N	0.01	0.2	0.3	0.3	0.19
PREP	0.3	0.2	0.3	0.19	0.01
V	0.2	0.19	0.3	0.3	0.01

Estimating the Probabilities

- **Given: Fully observed data**
  - Pairs of output sequence with their state sequence
- **Estimating transition probabilities $P(S_t \mid S_{t-1})$**

$$P(y_a \mid y_b) = \frac{\#ofTimesStateAFollowsStateB}{\#ofTimesStateBOccurs}$$

- **Estimating mission probabilities $P(W_t \mid S_t)$**

$$P(x_a \mid y_b) = \frac{\#ofTimesOutputAIsObservedInStateB}{\#ofTimesStateBOccurs}$$

- **Smoothing the estimates**
  - Laplace smoothing -> uniform prior
  - See naïve Bayes for text classification
- **Partially observed data: Expectation Maximization (EM)**

HMM's for POS Tagging

- **Design HMM structure (vanilla)**
  - States: one state per POS tag
  - Transitions: fully connected
  - Emissions: all words observed in training corpus
- **Estimate probabilities**
  - Use corpus, e.g. Treebank
  - Smoothing
  - Unseen words?
- **Tagging new sentences**
  - Use Viterbi to find most likely tag sequence

Experimental Results

Tagger	Accuracy	Training time	Prediction time
HMM	96.80%	20 sec	18.000 words/s
TBL Rules	96.47%	9 days	750 words/s

- **Experiment setup**
  - WSJ Corpus
  - Trigram HMM model
  - Lexicalized
  - from [Pla and Molina, 2001]

Discriminative vs. Generative

Bayes Rule

$$h_{bayes}(x) = argmax_{y \in Y} [P(Y = y|X = x)]$$
$$= argmax_{y \in Y} [P(X = x|Y = y)P(Y = y)]$$

Generative:

- Make assumptions about $P(X = x|Y = y), P(Y = y)$
- Estimate parameters of the two distributions

Discriminative:

- Define set of prediction rules (i.e. hypotheses) H
- Find h in H that best approximates

$$h_{bayes}(x) = argmax_{y \in Y} [P(Y = y|X = x)]$$

Question: Can we train HMM's discriminately?

Idea for Discriminative Training of HMM

Bayes Rule

$$h_{bayes}(x) = argmax_{y \in Y} [P(Y = y|X = x)]$$
$$= argmax_{y \in Y} [P(X = x|Y = y)P(Y = y)]$$

Model $P(Y = y|X = x)$ **with** $\vec{w} \cdot \Phi(x, y)$ **so that**

$$\left(argmax_{y \in Y} [P(Y = y|X = x)]\right) = \left(argmax_{y \in Y} [\vec{w} \cdot \Phi(x, y)]\right)$$

Intuition:

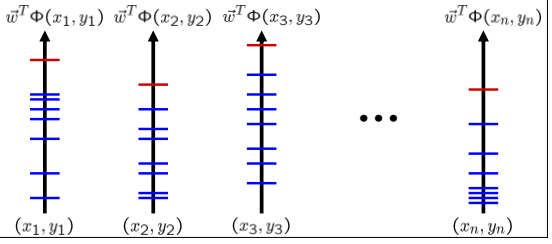
- Tune $\vec{w}$ so that correct y has the highest value of $\vec{w} \cdot \Phi(x, y)$
- $\Phi(x, y)$ is a feature vector that describes the match between x and y

Training HMMs with Structural SVM

- **Define** $\Phi(x, y)$ **so that model is isomorphic to HMM**
  - One feature for each possible start state
  - One feature for each possible transition
  - One feature for each possible output in each possible state
  - Feature values are counts

Structural Support Vector Machine

- **Joint features** $\Phi(x, y)$ **describe match between** *x* **and** *y*
- **Learn weights** $\vec{w}$ **so that** $\vec{w}^T \Phi(x, y)$ **is max for correct** *y*



Structural SVM Training Problem

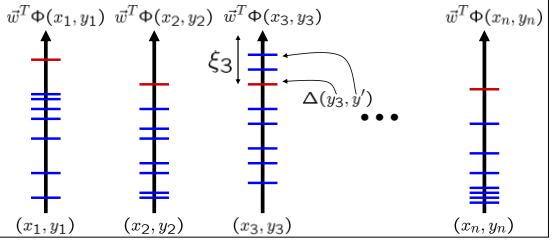
Hard-margin optimization problem:

$$\min_{\vec{w}} \quad \frac{1}{2}\vec{w}^T\vec{w}$$
$$s.t. \quad \forall y \in Y \setminus y_1 : \vec{w}^T\Phi(x_1,y_1) \geq \vec{w}^T\Phi(x_1,y) + 1$$
$$\ldots$$
$$\forall y \in Y \setminus y_n : \vec{w}^T\Phi(x_n,y_n) \geq \vec{w}^T\Phi(x_n,y) + 1$$

- **Training Set:** $(x_1,y_1),\ldots,(x_n,y_n) \sim P(X,Y)$
- **Prediction Rule:** $h_{svm}(x) = argmax_{y\in Y}\,[\vec{w}\cdot\Phi(x,y)]$
- **Optimization:**
  - Correct label y_i must have higher value of $\vec{w}\cdot\Phi(x_i,y_i)$ than any incorrect label y
  - Find weight vector with smallest norm
  - Polynomial time algorithm (e.g. SVM-struct)

Loss Functions: Soft-Margin Struct SVM

- **Loss function $\Delta(y_i,y)$ measures match between target and prediction.**



Soft-Margin Structural SVM

Soft-margin optimization problem:

$$\min_{\vec{w},\vec{\xi}} \quad \frac{1}{2}\vec{w}^T\vec{w} + C\sum_{i=1}^{n}\xi_i$$
$$s.t. \quad \forall y\in Y\setminus y_1 : \vec{w}^T\Phi(x_1,y_1) \geq \vec{w}^T\Phi(x_1,y) + \Delta(y_1,y) - \xi_1$$
$$\ldots$$
$$\forall y\in Y\setminus y_n : \vec{w}^T\Phi(x_n,y_n) \geq \vec{w}^T\Phi(x_n,y) + \Delta(y_n,y) - \xi_n$$

Lemma: The training loss is upper bounded by

$$Err_S(h) = \frac{1}{n}\sum_{i=1}^{n}\Delta(y_i,h(\vec{x}_i)) \leq \frac{1}{n}\sum_{i=1}^{n}\xi_i$$

Sparse Approximation Algorithm for Structural SVM

- **Input:** $(x_1,y_1),\ldots,(x_n,y_n),C,\epsilon$
- $S\leftarrow\emptyset, \vec{w}\leftarrow 0, \vec{\xi}\leftarrow 0$
- **REPEAT**
  - FOR $i = 1,\ldots,n$
    - compute $\hat{y} = argmax_{y\in Y}\{\Delta(y_i,y) + \vec{w}^T\Phi(x_i,y)\}$
    - IF($\Delta(y_i,\hat{y}) - \vec{w}^T[\Phi(x_i,y_i) - \Phi(x_i,\hat{y})]) > \xi_i + \epsilon$
      - $S \leftarrow S \cup \{\vec{w}^T[\Phi(x_i,y_i) - \Phi(x_i,\hat{y})] \geq \Delta(y_i,\hat{y}) - \xi_i\}$
      - $[\vec{w},\vec{\xi}] \leftarrow$ optimize StructSVM over S
    - ENDIF
  - ENDFOR
- **UNTIL** S **has not changed during iteration**

Find most violated constraint

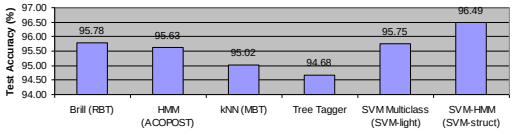
Violated by more than ε ?

Add constraint to working set

Experiment: Part-of-Speech Tagging

- **Task**
  - Given a sequence of words *x*, predict sequence of tags *y*.

  x: The dog chased the cat → y: Det→N→V→Det→N
  - Dependencies from tag-tag transitions in Markov model.
- **Model**
  - Markov model with one state per tag and words as emissions
  - Each word described by ~250,000 dimensional feature vector (all word suffixes/prefixes, word length, capitalization …)
- **Experiment (by Dan Fleisher)**
  - Train/test on 7966/1700 sentences from Penn Treebank



NE Identification

- **Identify all named locations, named persons, named organizations, dates, times, monetary amounts, and percentages.**

The delegation, which included the commander of the U.N.'s troops in Bosnia, Lt. Gen. Sir Michael Rose, went to the Serb stronghold of Pale, near Sarajevo, for talks with Bosnian Serb leader Radovan Karadzic.

Este ha sido el primer comentario publico del presidente Clinton respecto a la crisis de Oriente Medio desde que el secretario de Estado, Warren Christopher, decidiera regresar precipitadamente a Washington para impedir la ruptura del proceso de paz tras la violencia desatada en el sur de Libano.

1. Locations
2. Persons
3. Organizations

Figure 1.1 Examples. Examples of correct labels for English text and for Spanish text.

Experiment: Named Entity Recognition

- **Data**
  - Spanish Newswire articles
  - 300 training sentences
  - 9 tags
    - no-name,
    - beginning and continuation of person name, organization, location, misc name
  - Output words are described by features (e.g. starts with capital letter, contains number, etc.)
- **Error on test set (% mislabeled tags):**
  - Generative HMM: 9.36%
  - Support Vector Machine HMM: 5.08%

General Problem: Predict Complex Outputs

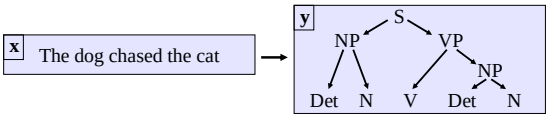
- **Supervised Learning from Examples**
  - Find function from input space *X* to output space *Y*

$$h : X \longrightarrow Y$$

    such that the prediction error is low.
- **Typical**
  - Output space is just a single number
    - Classification: -1,+1
    - Regression: some real number
- **General**
  - Predict outputs that are complex objects

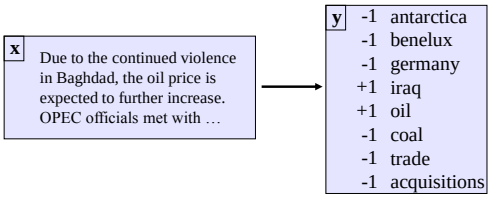
Examples of Complex Output Spaces

- **Natural Language Parsing**
  - Given a sequence of words *x*, predict the parse tree *y*.
  - Dependencies from structural constraints, since *y* has to be a tree.



Examples of Complex Output Spaces

- **Multi-Label Classification**
  - Given a (bag-of-words) document x, predict a set of labels *y*.
  - Dependencies between labels from correlations between labels ("iraq" and "oil" in newswire corpus)



Examples of Complex Output Spaces

- **Noun-Phrase Co-reference**
  - Given a set of noun phrases *x*, predict a clustering *y*.
  - Structural dependencies, since prediction has to be an equivalence relation.
  - Correlation dependencies from interactions.