Kalman Filters: Examples

CS 4758

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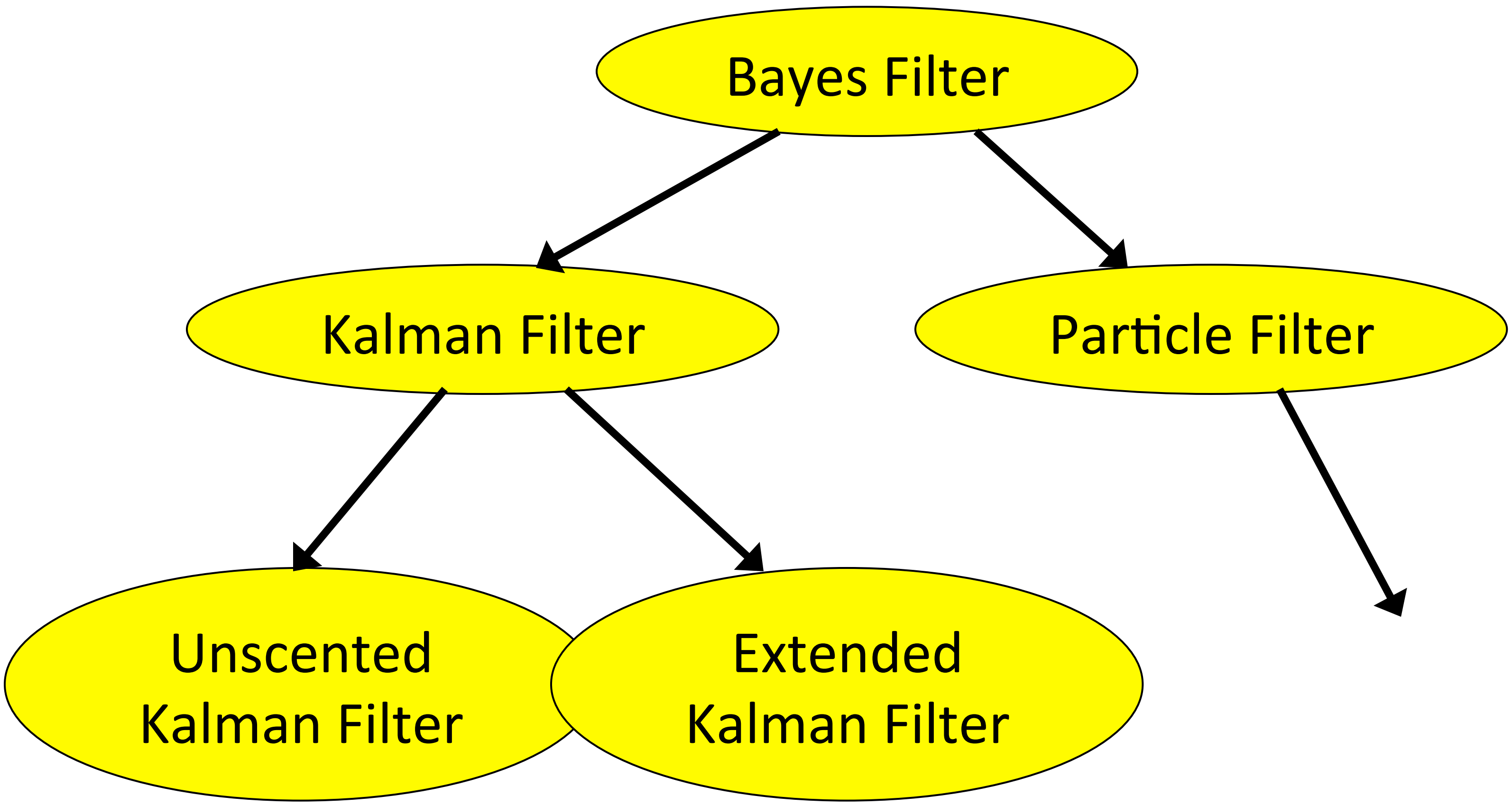
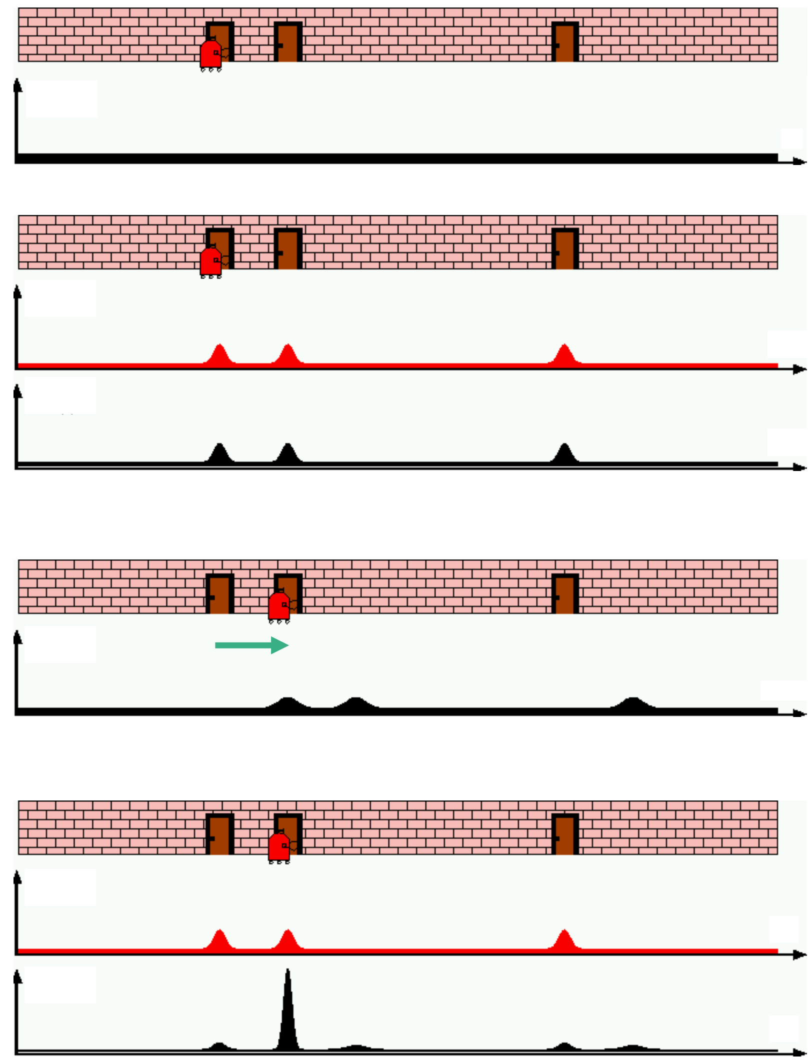
Methods



Bayes Filter
Kalman Filter
Particle Filter
Unscented Kalman Filter
Extended Kalman Filter

PR2 localization: Door Detector



Example 2: Simple 1D Linear System with Erroneous Start

Given: F=G=H=1, u=cos(t/5)
Initial state estimate = 20

Linear system:

$$x_{t+1} = x_t + \cos(t/5) + w_t$$

$$z_{t+1} = x_{t+1} + n_{t+1}$$

Unknown noise parameters

Propagation:

$$\hat{x}_{t+1} = \hat{x}_t + \cos(t/5)$$

$$P_{t+1}^{-} = P_t + Q_t$$

Update: *(no change)*

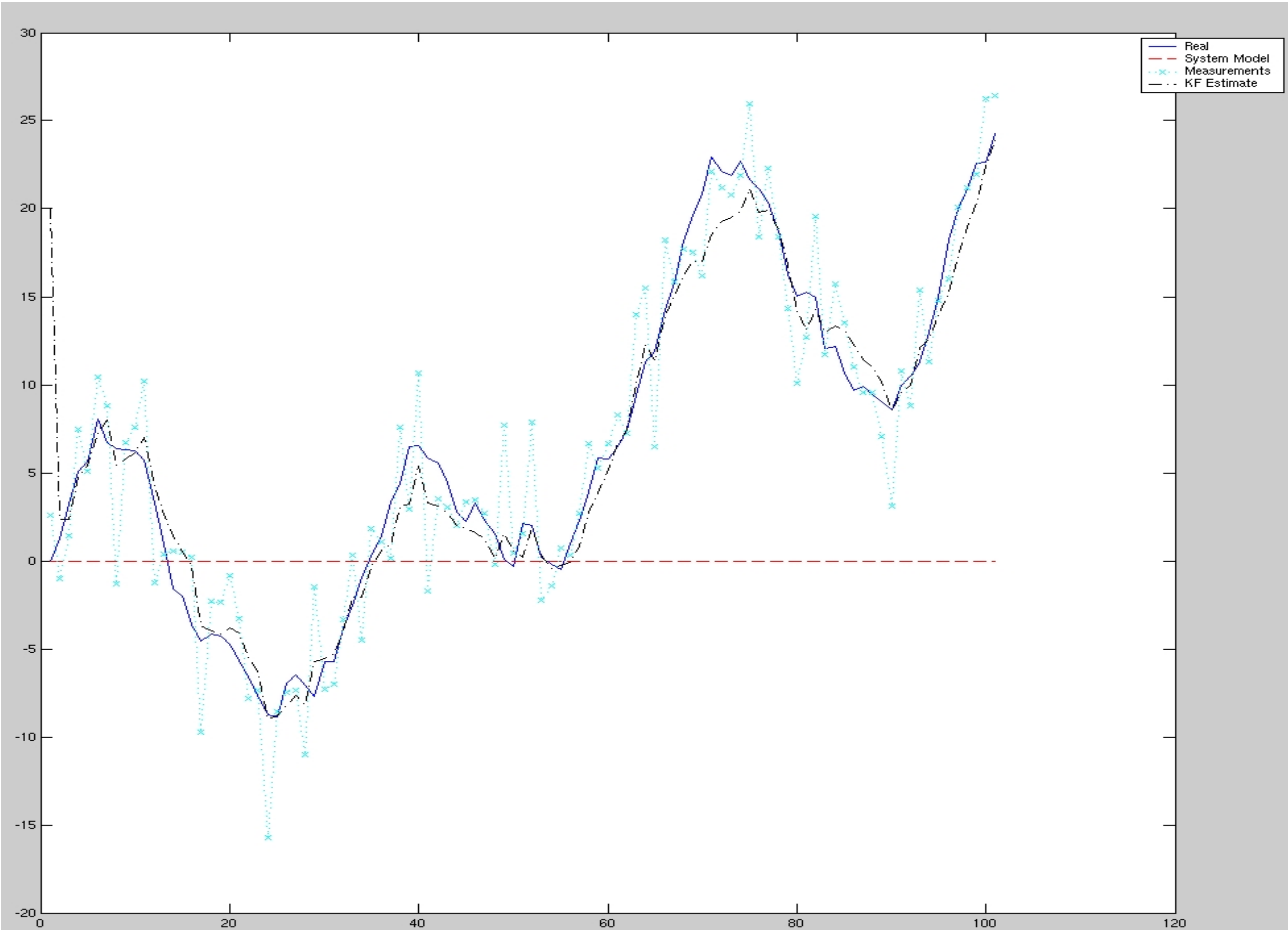
$$\hat{z}_{t+1} = \hat{x}_{t+1}$$

$$K_{t+1} = P_{t+1}^{-}(P_{t+1}^{-} + R_{t+1})^{-1}$$

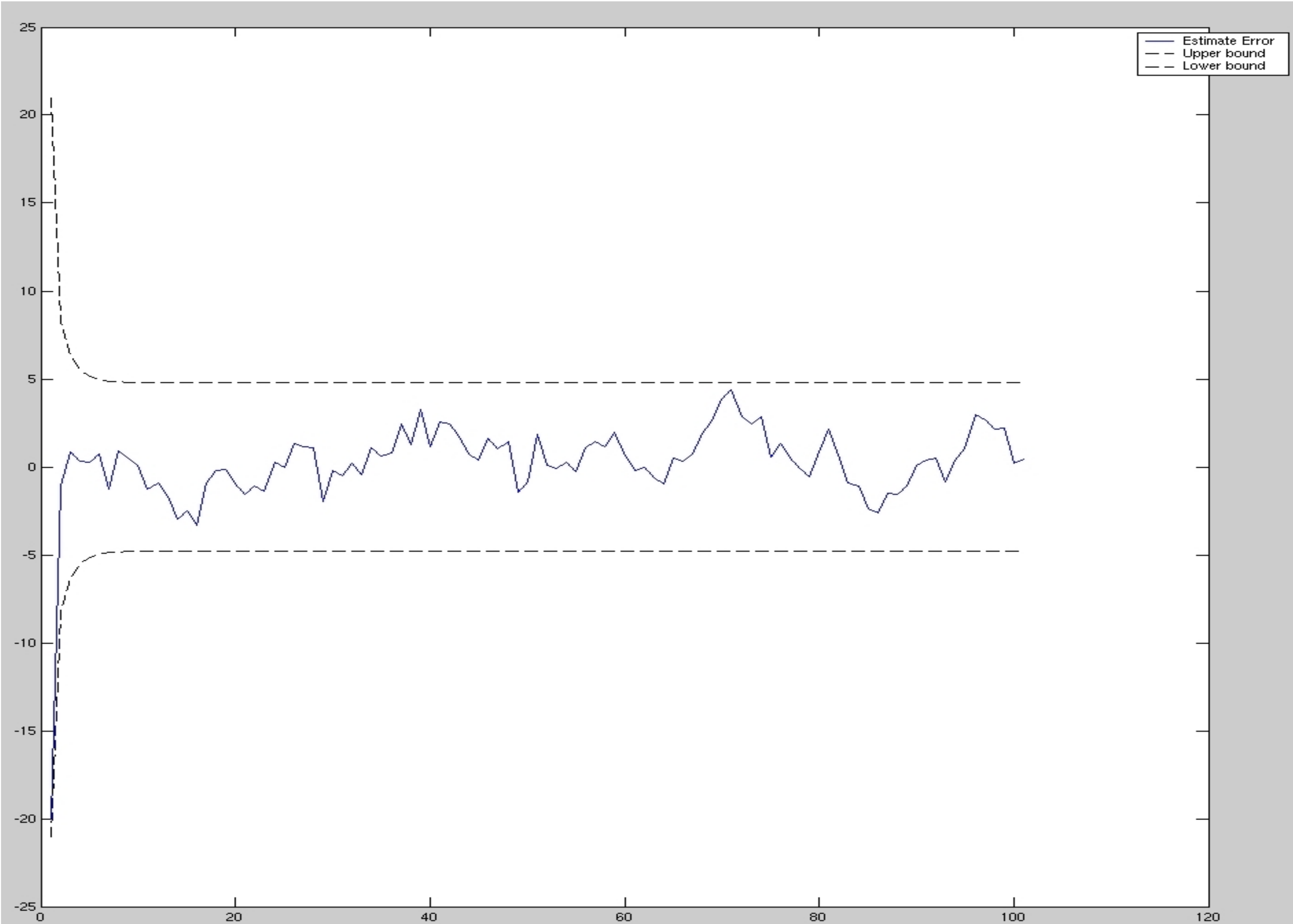
$$\hat{x}_{t+1} = \hat{x}_{t+1}^{-} + K_{t+1}(z_{t+1} - \hat{x}_{t+1}^{-})$$

$$P_{t+1} = (I - K_{t+1})P_{t+1}^{-}$$

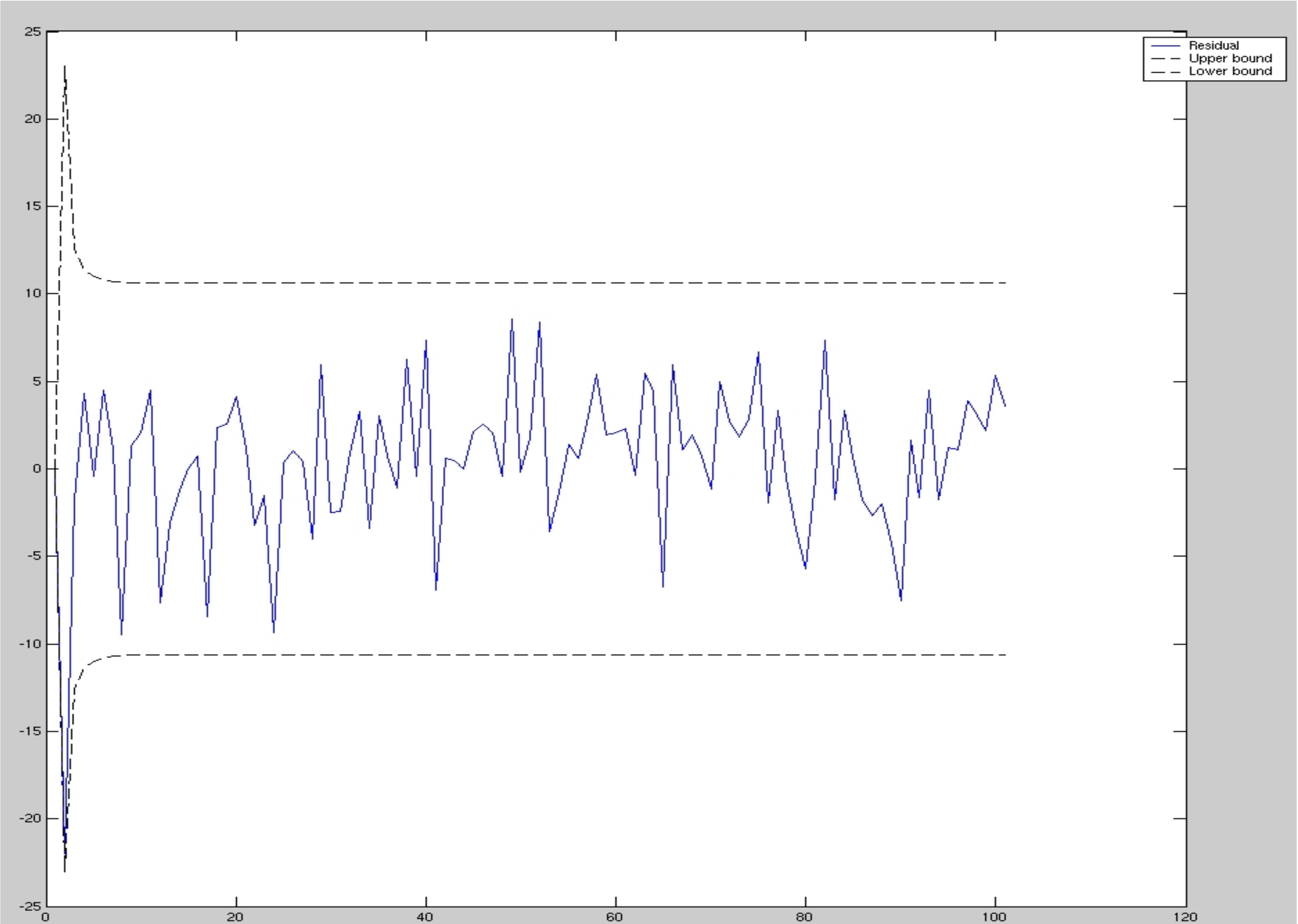
State Estimate



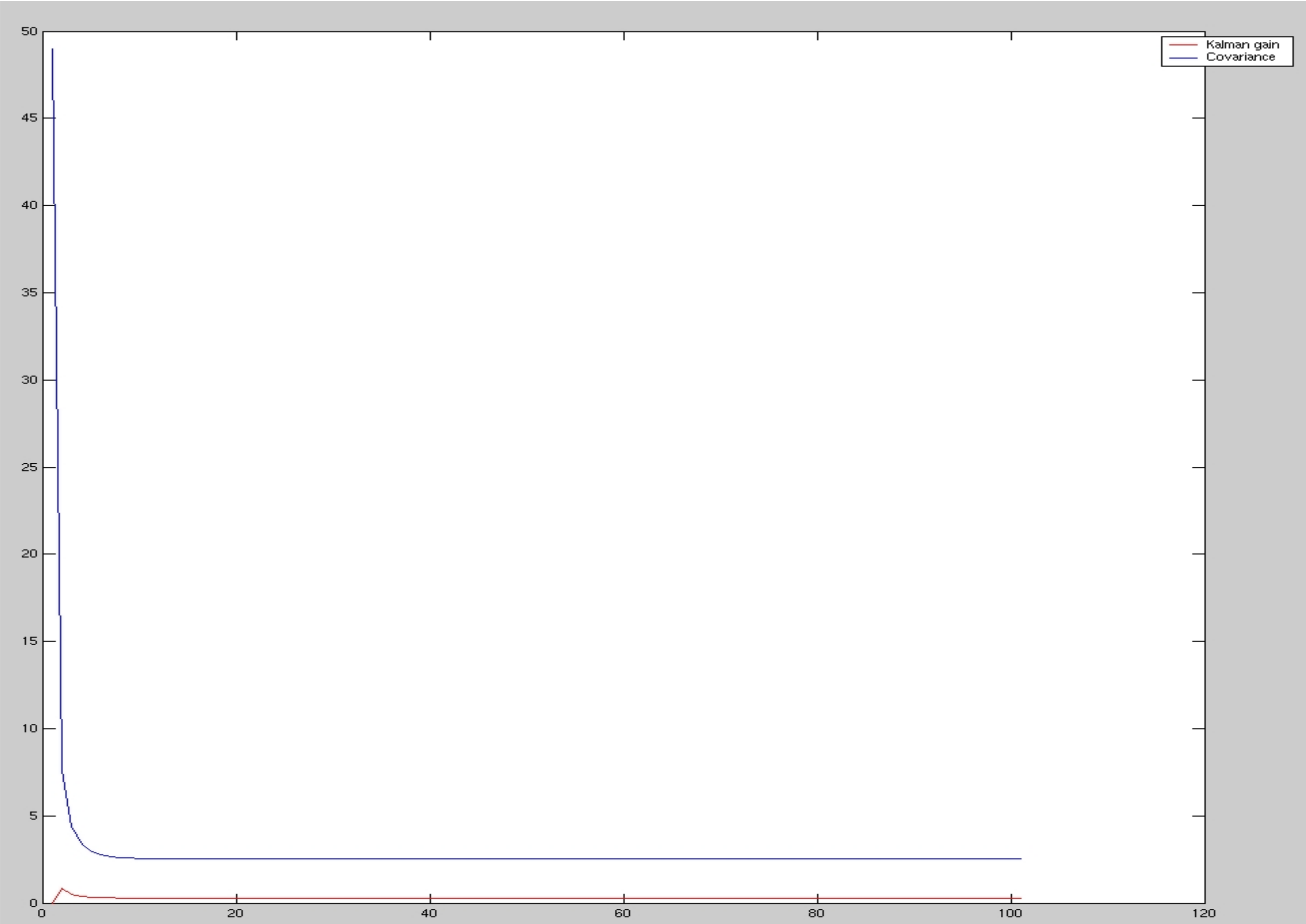
State Estimation Error vs 3σ Region of Confidence



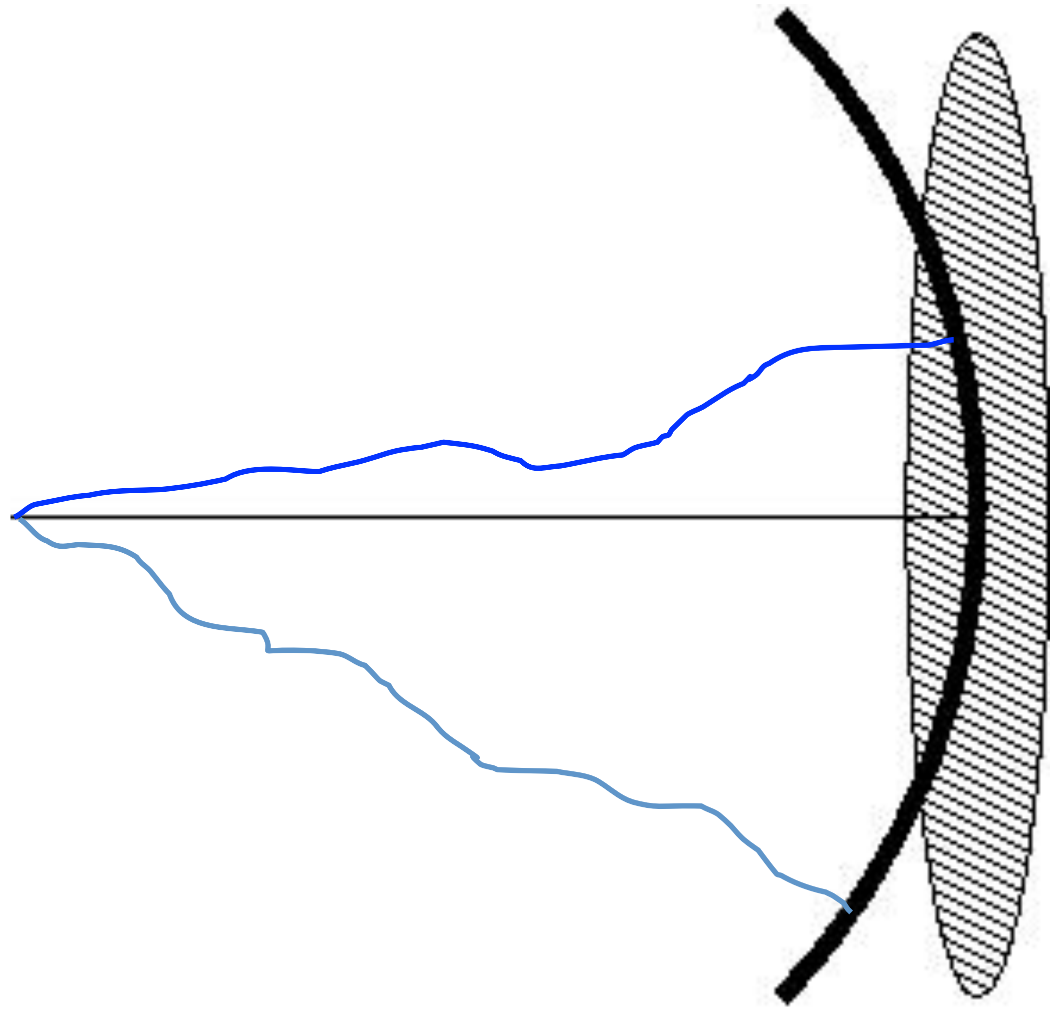
Sensor Residual vs 3σ Region of Confidence



Kalman Gain and State Covariance



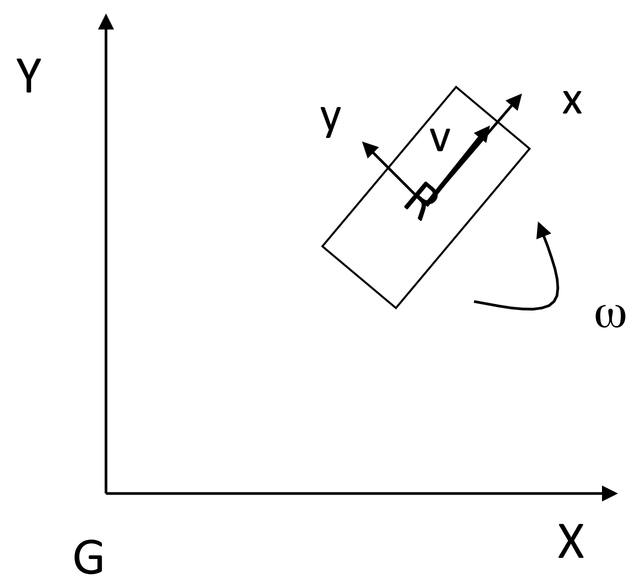
Approximating Robot Motion Uncertainty with a Gaussian



Robot Motions are not linear

- Extended Kalman Filter

Linearized Motion Model for PR2 / aerial robot.



From a robot-centric perspective, the velocities look like this:

$$\dot{x}_t = V_t$$
$$\dot{y}_t = 0$$
$$\dot{\phi}_t = \omega_t$$

From the global perspective, the velocities look like this:

$$\dot{x}_t = V_t \cos\phi_t$$
$$\dot{y}_t = V_t \sin\phi_t$$
$$\dot{\phi}_t = \omega_t$$

The discrete time state estimate (including noise) looks like this:

$$\hat{x}_{t+1} = \hat{x}_t + (V_t + w_{V_t})\delta t \cos\hat{\phi}_t$$
$$\hat{y}_{t+1} = \hat{y}_t + (V_t + w_{V_t})\delta t \sin\hat{\phi}_t$$
$$\hat{\phi}_{t+1} = \hat{\phi}_t + (\omega_t + w_{\omega_t})\delta t$$

Problem! We don't know linear and rotational velocity errors. The state estimate will rapidly diverge if this is the only source of information!

Linearized Motion Model for PR2/aerial robot

Now, we have to compute the covariance matrix propagation equations.

The indirect Kalman filter derives the pose equations from the estimated error of the state:

$$x_{t+1} - \hat{x}_{t+1} = \tilde{x}_{t+1}$$
$$y_{t+1} - \hat{y}_{t+1} = \tilde{y}_{t+1}$$
$$\phi_{t+1} - \hat{\phi}_{t+1} = \tilde{\phi}_{t+1}$$

In order to linearize the system, the following small-angle assumptions are made:

$$\cos\tilde{\phi} \cong 1$$
$$\sin\tilde{\phi} \cong \tilde{\phi}$$

Linearized Motion Model for PR2/aerial robot

From the error-state propagation equation, we can obtain the State propagation and noise input functions *F* and *G* :

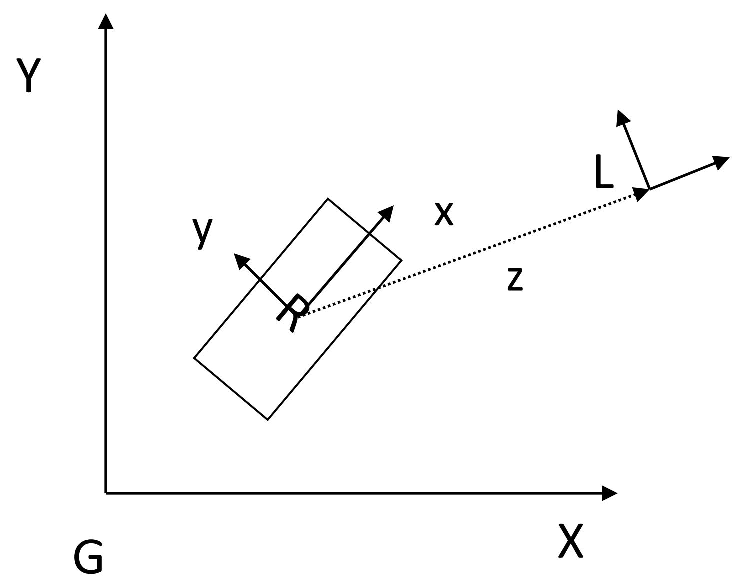
$$\begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{y}_{t+1} \\ \tilde{\varphi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -V_m \delta t \sin\hat{\varphi} \\ 0 & 1 & V_m \delta t \cos\hat{\varphi} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_t \\ \tilde{y}_t \\ \tilde{\varphi}_t \end{bmatrix} + \begin{bmatrix} -\delta t \cos\varphi_R & 0 \\ -\delta t \sin\varphi_R & 0 \\ 0 & -\delta t \end{bmatrix} \begin{bmatrix} w_{V_t} \\ w_{\omega_t} \end{bmatrix}$$

$$\tilde{X}_{t+1} = A_t \tilde{X}_t + G_t W_t$$

From these values, we can easily compute the standard covariance propagation equation:

$$P_{t+1}^{-} = A_t P_t A_t^T + G_t Q_t G_t^T$$

Sensor Model for a Robot with a Perfect Map



From the robot, the measurement looks like this:

$$z_{t+1} = \begin{bmatrix} x_{L_{t+1}} \\ y_{L_{t+1}} \\ \phi_{L_{t+1}} \end{bmatrix} + \begin{bmatrix} n_x \\ n_y \\ n_\phi \end{bmatrix}$$

From a global perspective, the measurement looks like:

$$z_{t+1} = \begin{bmatrix} \cos\phi_{t+1} & -\sin\phi_{t+1} & 0 \\ \sin\phi_{t+1} & \cos\phi_{t+1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{L_{t+1}} - x_{t+1} \\ y_{L_{t+1}} - y_{t+1} \\ \phi_{L_{t+1}} - \phi_{t+1} \end{bmatrix} + \begin{bmatrix} n_x \\ n_y \\ n_\phi \end{bmatrix}$$

The measurement equation is nonlinear and must also be linearized!

Sensor Model for a Robot with a Perfect Map

Now, we have to compute the linearized sensor function. Once again, we make use of the indirect Kalman filter where the error in the reading must be estimated.

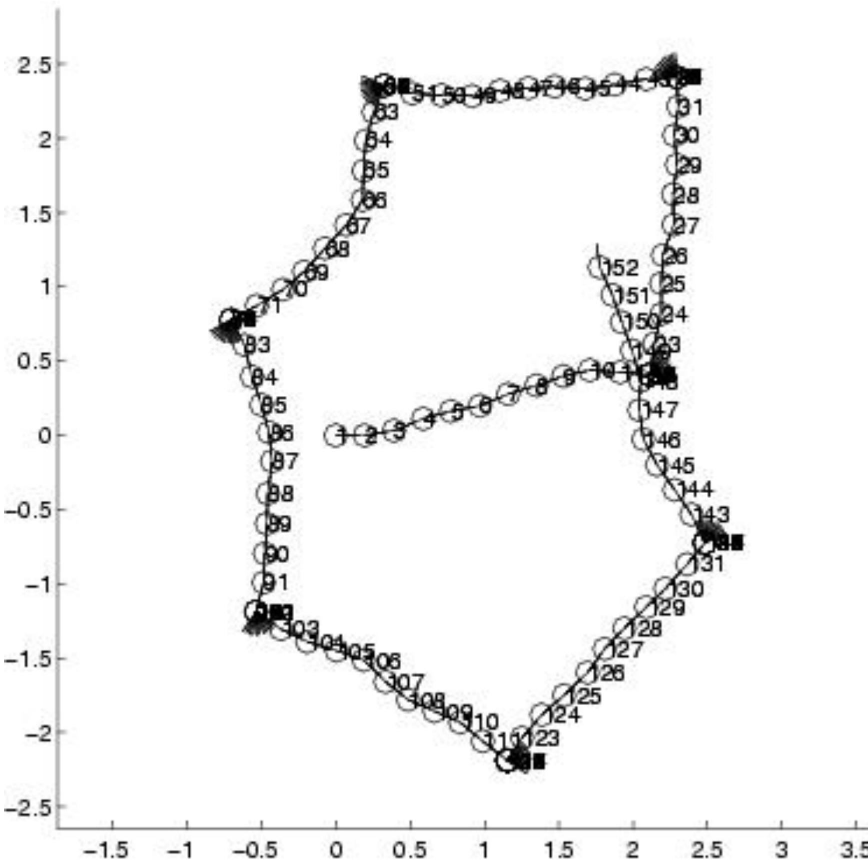
In order to linearize the system, the following small-angle assumptions are made:

$$\cos\tilde{\phi} \cong 1$$
$$\sin\tilde{\phi} \cong \tilde{\phi}$$

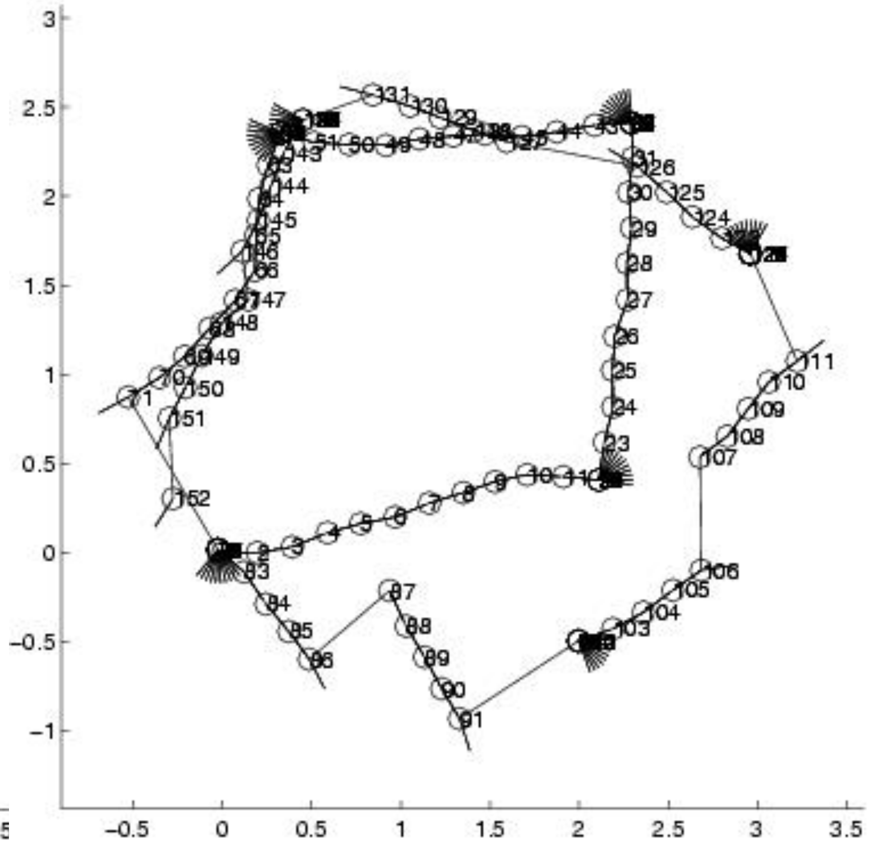
The final expression for the error in the sensor reading is:

$$\begin{bmatrix} \tilde{x}_{L_{t+1}} \\ \tilde{y}_{L_{t+1}} \\ \tilde{\phi}_{L_{t+1}} \end{bmatrix} = \begin{bmatrix} -\cos\hat{\phi}_{t+1} & -\sin\hat{\phi}_{t+1} & -\sin\hat{\phi}_{t+1}(x_L - \hat{x}_{t+1}) + \cos\hat{\phi}_t(y_L - \hat{y}_{t+1}) \\ \sin\hat{\phi}_{t+1} & -\cos\hat{\phi}_{t+1} & -\cos\hat{\phi}_{t+1}(x_L - \hat{x}_{t+1}) - \sin\hat{\phi}_t(y_L - \hat{y}_{t+1}) \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \tilde{x}_{t+1} \\ \tilde{y}_{t+1} \\ \tilde{\phi}_{t+1} \end{bmatrix} + \begin{bmatrix} n_x \\ n_y \\ n_\phi \end{bmatrix}$$

Updating the State Vector



Propagation only



Propagation and update