# CS 4758/6758: Robot Learning

Spring 2010.

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- Periodically visit for announcements, lecture notes and homeworks.

http://www.cs.cornell.edu/courses/CS4758/2010sp

- Homework 1 will be posted today on the website) and is due on Feb 10.

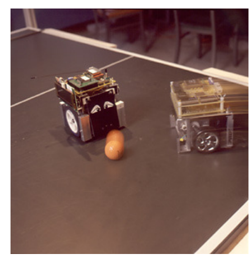


## Pre-reqs

- Knowledge of basic computer science principles and skills, at a level **sufficient to write a reasonably non-trivial computer program**. (E.g., CS 1114 or CS 2110 or CS 3110 or equivalent.) Knowledge of **C/C++/C#** is not a pre-requisite, but it is strongly desirable.
- A course in **probability/statistics** (e.g. CS 2800, ECE 2200, ECE 3100, or ENGRD 2700 or equivalent).
- Familiarity with the basic **linear algebra**. (E.g., MATH 2210 is sufficient but not necessary.) **Strong mathematical skills** are required in this course.
- A course in Artificial Intelligence or Robotics is desirable, but not essential.
- Motivation and patience to **hack** for **long hours**.



## Representation



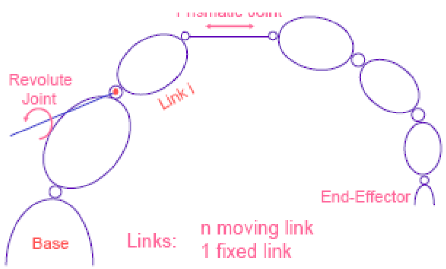


## Robots



## Robots

## Manipulators: Kinematics




Links: n moving link
1 fixed link

Joints: Revolute (1 DOF)
Prismatic (1 DOF)

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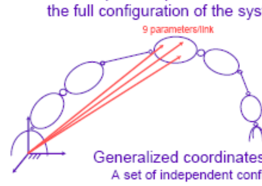
## Configuration Parameters

A set of position parameters that describes the full configuration of the system.




Generalized coordinates
A set of independent configuration parameters

Degrees of Freedom
Number of generalized coordinates

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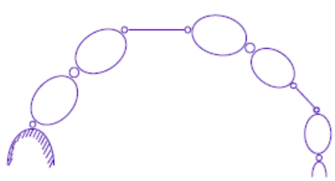
## Generalized Coordinates



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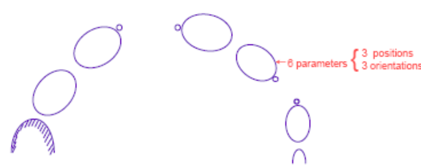
## Generalized Coordinates




n moving links: 6n parameters

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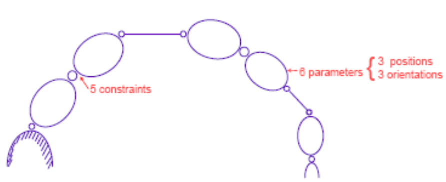
## Generalized Coordinates




n moving links: 6n parameters
n 1 d.o.f. joints: 5n constraints
d.o.f. (system): 6n - 5n = n

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## Question

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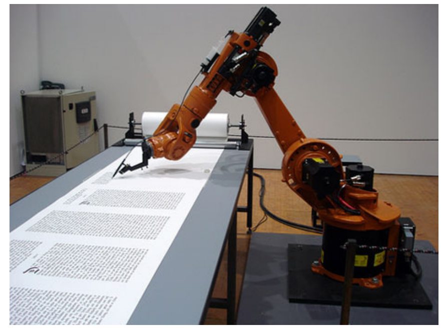
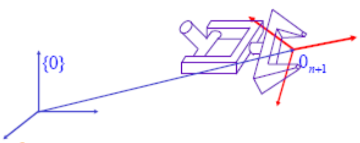
## How many degrees of freedom?







## End-Effector Configuration Parameters




A set of m parameters:

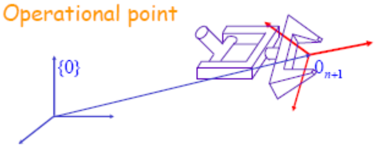
$(x_1, x_2, x_3, \ldots, x_m)$

that completely specifies the end-effector position and orientation with respect to $\{0\}$



## Operational Coordinates

$O_{n+1}$ : Operational point

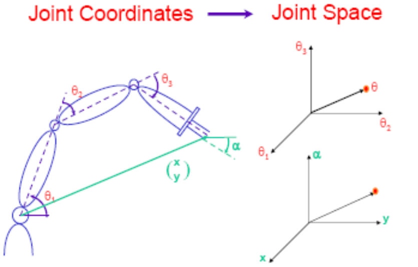



A set $x_1, x_2, \ldots, x_{m_0}$
of $m_0$ independent configuration parameters

$m_0$ : number of degrees of freedom of the end-effector.



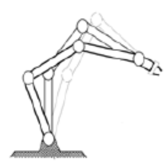
Joint Coordinates → Joint Space



Operational Coordinates → Operational Space



## Redundancy

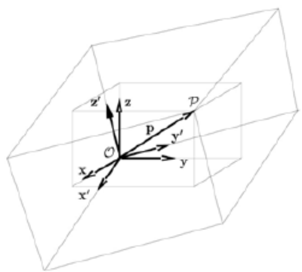


A robot is said to be redundant if

$n > m_0$

Degrees of redundancy: $n - m_0$

## Coordinate Frames

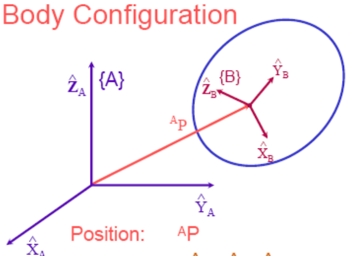




## Why Coordinate Transformations?




## Rigid Body Configuration




Position: $^{A}P$

Orientation: $\{^{A}\hat{X}_B, {}^{A}\hat{Y}_B, {}^{A}\hat{Z}_B\}$

Describes rotations of {B} with respect to {A}

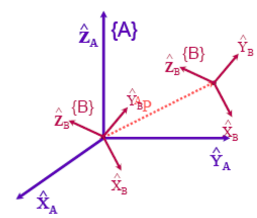
→ **Rotation Matrix**



## Rotation Matrix

$$^{A}_{B}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$^{A}\hat{X}_B = {}^{A}_{B}R \, {}^{B}\hat{X}_B$$




$$^{A}\hat{X}_B = {}^{A}_{B}R \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad {}^{A}\hat{Y}_B = {}^{A}_{B}R \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad {}^{A}\hat{Z}_B = {}^{A}_{B}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow {}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix}$$



## Rotation Matrix

$$^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_A^T \\ {}^{B}\hat{Y}_A^T \\ {}^{B}\hat{Z}_A^T \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_A & {}^{B}\hat{Y}_A & {}^{B}\hat{Z}_A \end{bmatrix}^T = {}^{B}_{A}R^T$$

$$^{A}_{B}R = {}^{B}_{A}R^T$$

### Inverse of Rotation Matrices

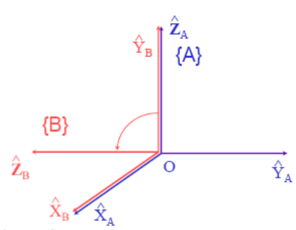
$$^{A}_{B}R^{-1} = {}^{B}_{A}R = {}^{A}_{B}R^T$$

$$^{A}_{B}R^{-1} = {}^{A}_{B}R^T$$

Orthonormal Matrix



## Example



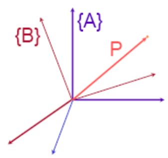

$$^{A}_{B}R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{matrix} \leftarrow {}^{B}\hat{X}_A^T \\ \leftarrow {}^{B}\hat{Y}_A^T \\ \leftarrow {}^{B}\hat{Z}_A^T \end{matrix}$$

$$\uparrow {}^{A}\hat{X}_B \quad \uparrow {}^{A}\hat{Y}_B \quad \uparrow {}^{A}\hat{Z}_B$$

## Mapping

changing descriptions from frame to frame

### Rotations



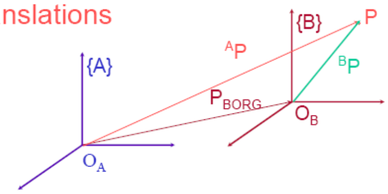

If P is given in {B}: $^{B}P$

$$^{A}P = \begin{pmatrix} {}^{B}\hat{X}_A \cdot {}^{B}P \\ {}^{B}\hat{Y}_A \cdot {}^{B}P \\ {}^{B}\hat{Z}_A \cdot {}^{B}P \end{pmatrix} = \begin{pmatrix} {}^{B}\hat{X}_A^T \\ {}^{B}\hat{Y}_A^T \\ {}^{B}\hat{Z}_A^T \end{pmatrix} {}^{B}P$$

$$^{A}P = {}^{A}_{B}R \; {}^{B}P$$



## Translations




changing the position description of a point P

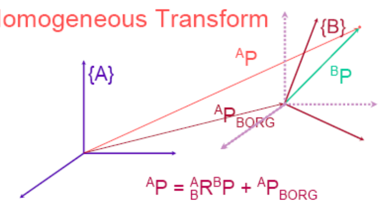
$\overrightarrow{O_BP} \Rightarrow \overrightarrow{O_AP}$ (Two different vectors)

$P_{BORG}$ : $P_{O_B} \Rightarrow P_{O_A}$

$$^{A}P_{O_A} = {}^{A}P_{O_B} + {}^{A}P_{BORG}$$



## Homogeneous Transform



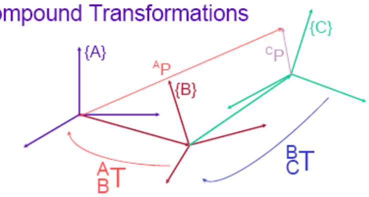

$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

$$\begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{Borg} \\ 0 \;\; 0 \;\; 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix}$$

$$\underset{(4x1)}{^{A}P} = \underset{(4x4)}{{}^{A}_{B}T} \; \underset{(4x1)}{^{B}P}$$



## Compound Transformations




$$^{B}P = {}^{B}_{C}T \; {}^{C}P$$

$$^{A}P = {}^{A}_{B}T \; {}^{B}P$$

$$^{A}P = {}^{A}_{B}T \; {}^{B}_{C}T \; {}^{C}P \Rightarrow {}^{A}_{C}T = {}^{A}_{B}T \; {}^{B}_{C}T$$

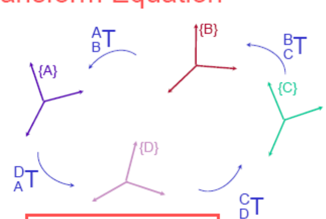


$$^{A}_{C}T = {}^{A}_{B}T \; {}^{B}_{C}T$$

$$^{A}_{C}T = \begin{bmatrix} {}^{A}_{B}R\,{}^{B}_{C}R & {}^{A}_{B}R\,{}^{B}P_{Corg} + {}^{A}P_{Borg} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$



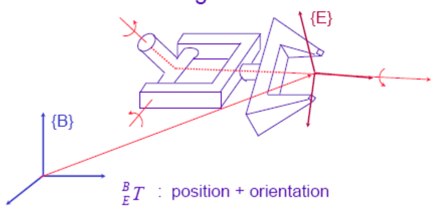
## Transform Equation




$$^{A}_{B}T \; {}^{B}_{C}T \; {}^{C}_{D}T \; {}^{D}_{A}T = I$$

$$\Rightarrow \; {}^{B}_{A}T = {}^{B}_{C}T \; {}^{C}_{D}T \; {}^{D}_{A}T$$

## End-Effector Configuration



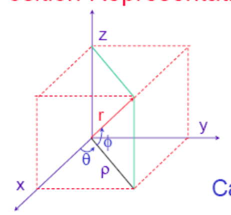

${}^B_E T$ : position + orientation

End-Effector Configuration Parameters

$$X = \begin{bmatrix} X_P \\ X_R \end{bmatrix} \begin{matrix} \text{position} \\ \text{orientation} \end{matrix}$$



## Position Representations



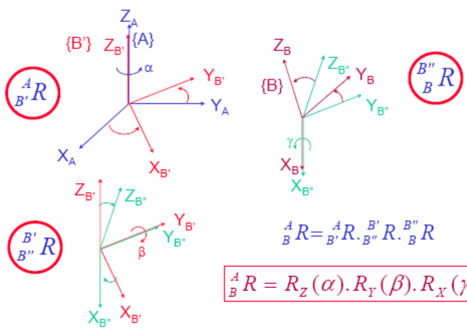

Cartesian: (x, y, z)

Cylindrical: (ρ, θ, z)

Spherical: (r, θ, ϕ)



## Euler Angles (Z-Y-X)




$${}^A_B R = {}^A_{B'}R.{}^{B'}_{B''}R.{}^{B''}_B R$$

$${}^A_B R = R_Z(\alpha).R_Y(\beta).R_X(\gamma)$$



## Z-Y-X Euler Angles

$${}^A_B R = R_{Z'}(\alpha).R_{Y'}(\beta).R_{X'}(\gamma)$$

$$\begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$$

$${}^A_B R = {}^A_B R_{Z'Y'X'}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha.c\beta & X & X \\ s\alpha.c\beta & X & X \\ -s\beta & c\beta.s\gamma & c\beta.c\gamma \end{bmatrix}$$



## Z-Y-Z Euler Angles

$${}^A_B R = R_{Z'}(\alpha).R_{Y'}(\beta).R_{Z'}(\gamma)$$

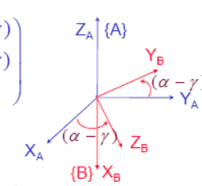
$${}^A_B R = {}^A_B R_{Z'Y'Z'}(\alpha,\beta,\gamma) = \begin{bmatrix} X & X & c\alpha.s\beta \\ X & X & s\alpha.s\beta \\ -s\beta.c\gamma & s\beta.s\gamma & c\beta \end{bmatrix}$$



## Singularities - Example ( $R_{Z'Y'X'}$ )

$c\beta = 0,\ s\beta = +1$

$${}^A_B R = \begin{pmatrix} 0 & -s(\alpha-\gamma) & c(\alpha-\gamma) \\ 0 & c(\alpha-\gamma) & s(\alpha-\gamma) \\ -1 & 0 & 0 \end{pmatrix}$$




$c\beta = 0,\ s\beta = -1$

$${}^A_B R = \begin{pmatrix} 0 & -s(\alpha+\gamma) & -c(\alpha+\gamma) \\ 0 & c(\alpha+\gamma) & -s(\alpha+\gamma) \\ 1 & 0 & 0 \end{pmatrix}$$

## Inverse Kinematics

- Given (x,y,z) of the end-effector, solve for the angles.
- Not straight-forward to solve.



## Nearest Neighbors

- Find the point closest to the query point.



## k-Nearest Neighbors