

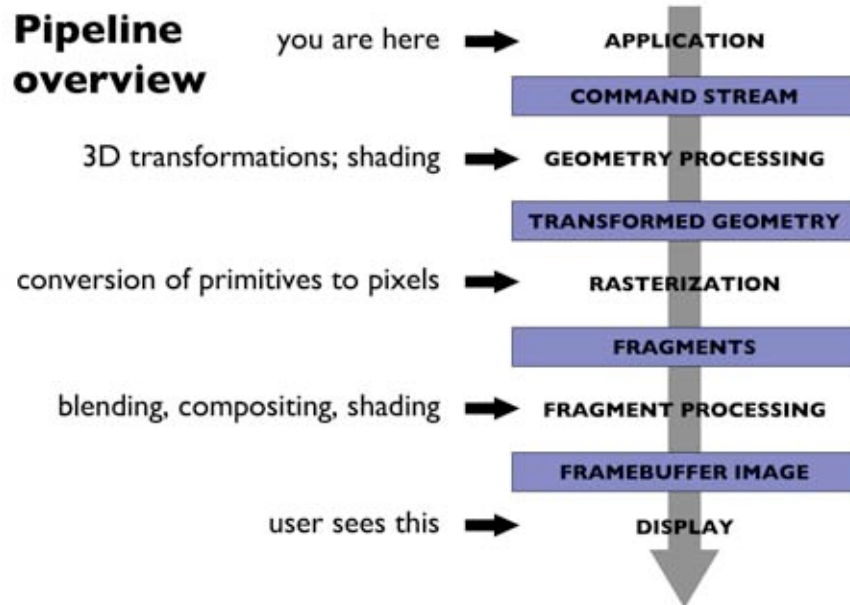
Pipeline and Rasterization

CS465 Lecture 15

The graphics pipeline

- The standard approach to object-order graphics
- Many versions exist
 - software, e.g. Pixar's REYES architecture
 - many options for quality and flexibility
 - hardware, e.g. graphics cards in PCs
 - amazing performance: millions of triangles per frame
- We'll focus on an abstract version of hardware pipeline
- "Pipeline" because of the many stages
 - very parallelizable
 - leads to remarkable performance of graphics cards (many times the flops of the CPU at ~1/5 the clock speed)

Pipeline overview



Primitives

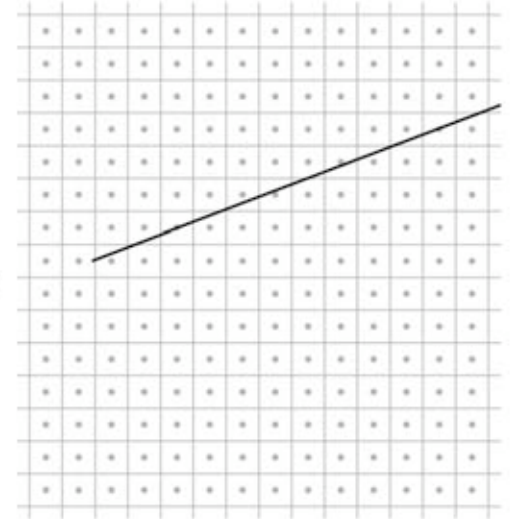
- Points
- Line segments
 - and chains of connected line segments
- Triangles
- And that's all!
 - Curves? Approximate them with chains of line segments
 - Polygons? Break them up into triangles
 - Curved regions? Approximate them with triangles
- Trend has been toward minimal primitives
 - simple, uniform, repetitive: good for parallelism

Rasterization

- First job: enumerate the pixels covered by a primitive
 - simple, aliased definition: pixels whose centers fall inside
- Second job: interpolate values across the primitive
 - e.g. colors computed at vertices
 - e.g. normals at vertices
 - will see applications later on

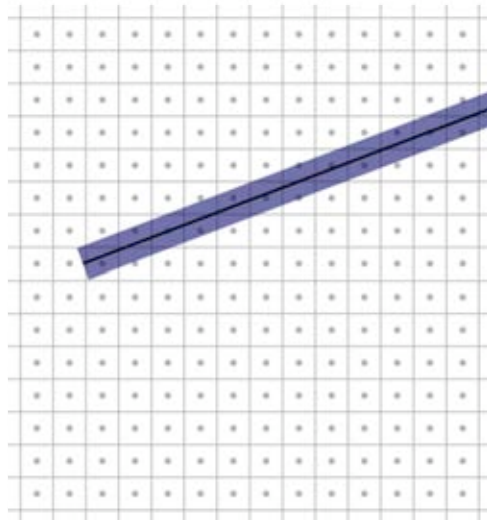
Rasterizing lines

- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside



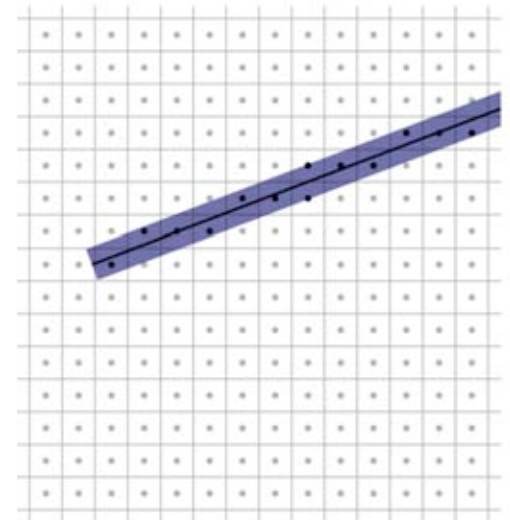
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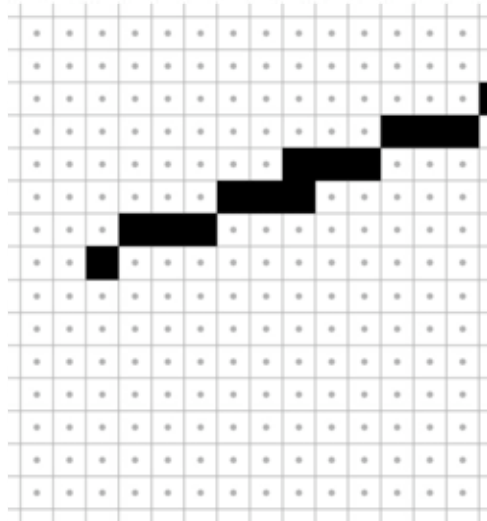
Point sampling

- Approximate rectangle by drawing all pixels whose centers fall within the line
- Problem: sometimes turns on adjacent pixels

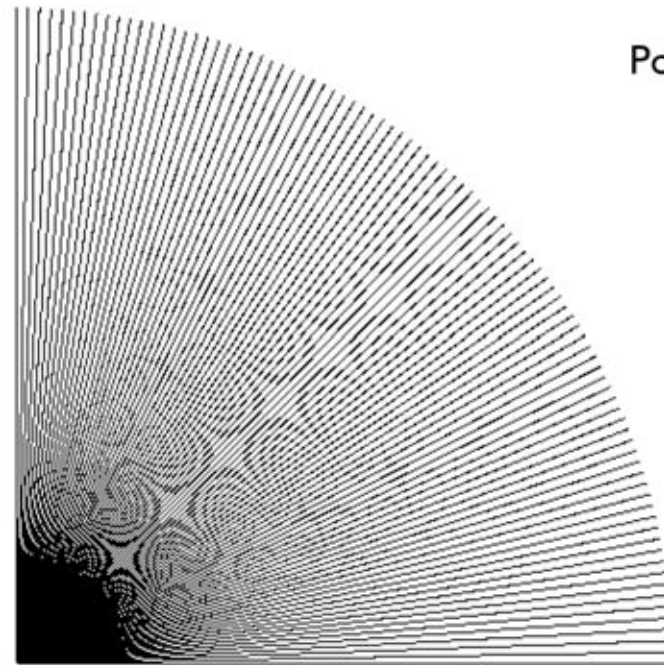


Point sampling

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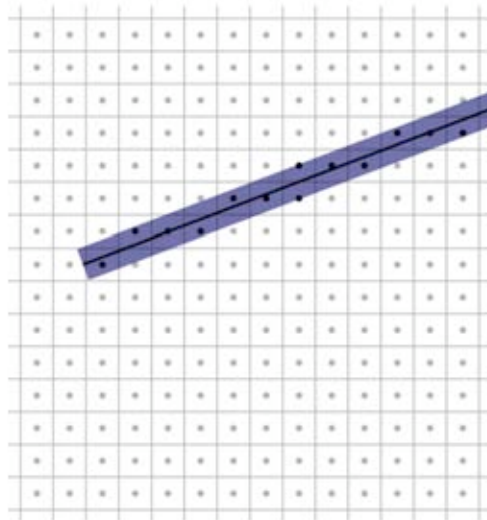


Point sampling in action



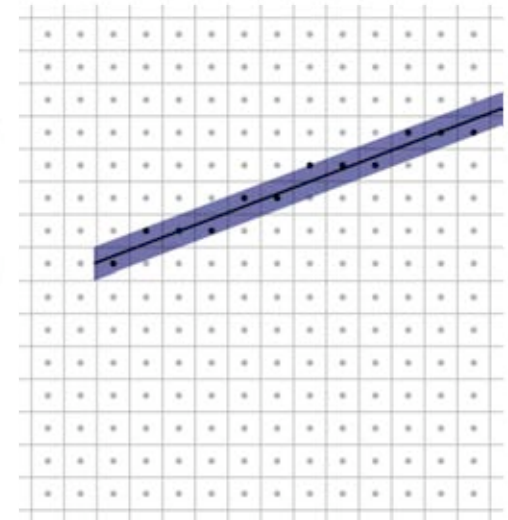
Bresenham lines (midpoint alg.)

- Point sampling unit width rectangle leads to uneven line width
- Define line width parallel to pixel grid
- That is, turn on the single nearest pixel in each column
- Note that 45° lines are now thinner



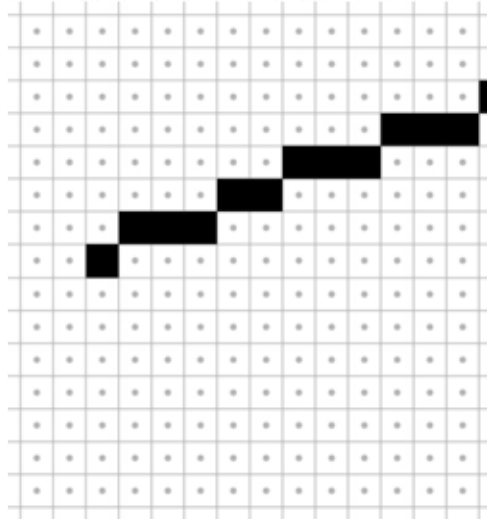
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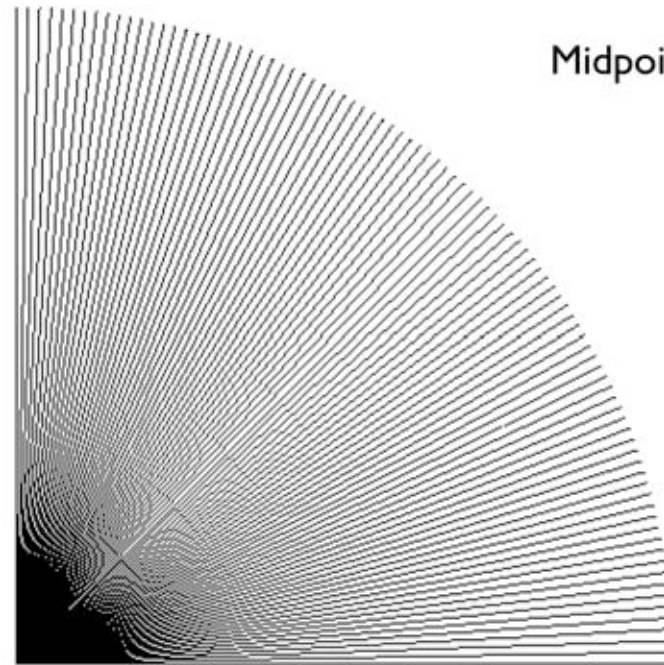


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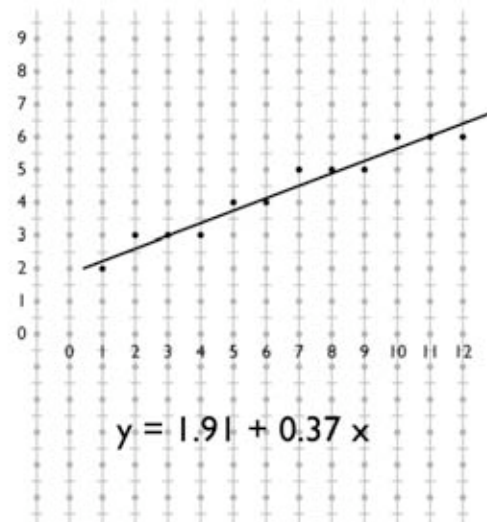
Midpoint algorithm in action



Algorithms for drawing lines

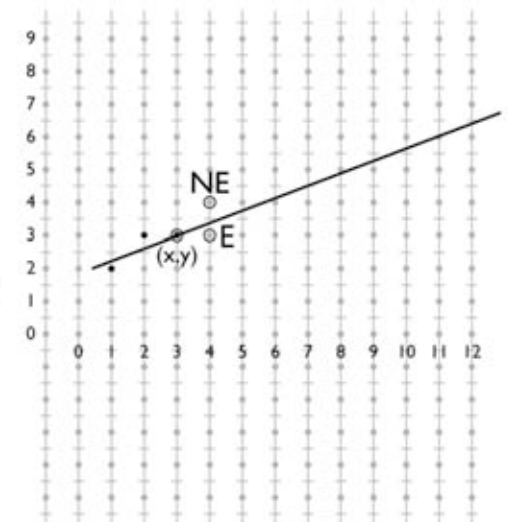
- line equation:
 $y = b + m x$
- Simple algorithm:
evaluate line equation per column
- W.l.o.g. $x_0 < x_1$;
 $0 \leq m \leq 1$

for $x = \text{ceil}(x_0)$ to $\text{floor}(x_1)$
 $y = b + m * x$
 output($x, \text{round}(y)$)



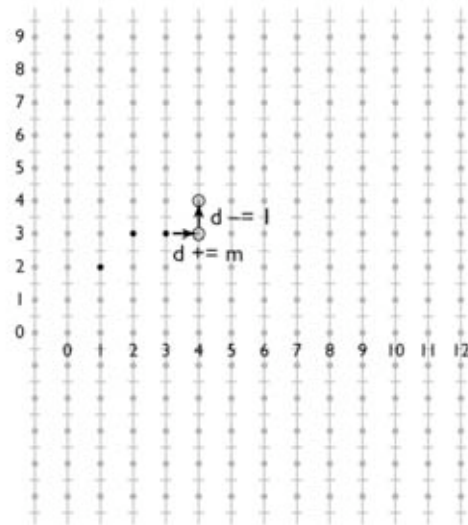
Optimizing line drawing

- Multiplying and rounding is slow
- At each pixel the only options are E and NE
- $d = m(x + 1) + b - y$
- $d > 0.5$ decides between E and NE



Optimizing line drawing

- $d = m(x + 1) + b - y$
- Only need to update d for integer steps in x and y
- Do that with addition
- Known as “DDA” (digital differential analyzer)

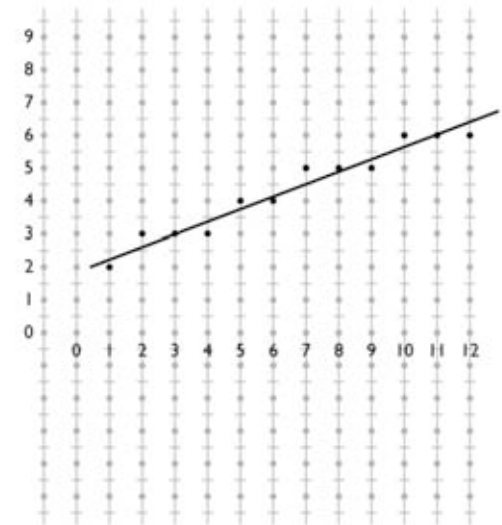


Midpoint line algorithm

```

x = ceil(x0)
y = round(m*x + b)
d = m*(x + 1) + b - y
while x < floor(x1)
  if d > 0.5
    y += 1
    d -= 1
  x += 1
  d += m
  output(x, y)

```

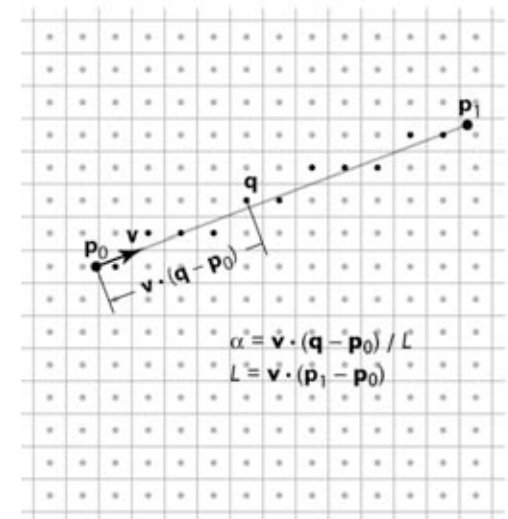


Linear interpolation

- We often attach attributes to vertices
 - e.g. computed diffuse color of a hair being drawn using lines
 - want color to vary smoothly along a chain of line segments
- Recall basic definition
 - 1D: $f(x) = (1 - \alpha) y_0 + \alpha y_1$
 - where $\alpha = (x - x_0) / (x_1 - x_0)$
- In the 2D case of a line segment, alpha is just the fraction of the distance from (x_0, y_0) to (x_1, y_1)

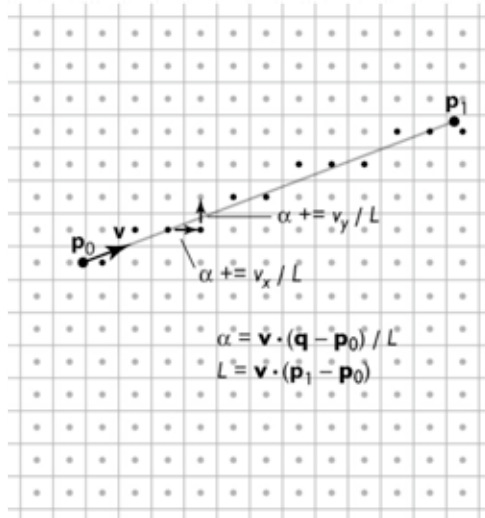
Linear interpolation

- Pixels are not exactly on the line
- Define 2D function by projection on line
 - this is linear in 2D
 - therefore can use DDA to interpolate



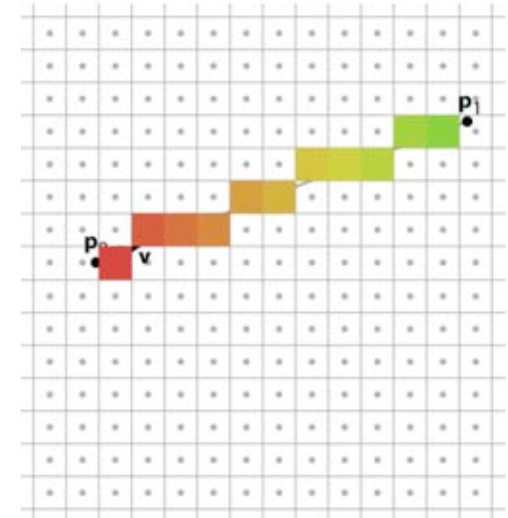
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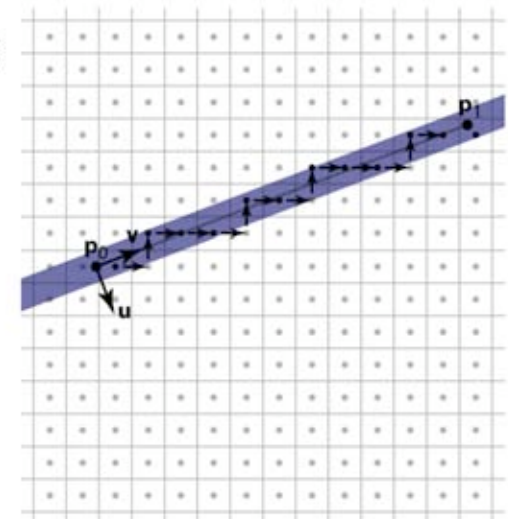


Alternate interpretation

- We are updating d and α as we step from pixel to pixel
 - d tells us how far from the line we are
 - α tells us how far along the line we are
- So d and α are coordinates in a coordinate system oriented to the line

Alternate interpretation

- View loop as visiting all pixels the line passes through
 - Interpolate d and α for each pixel
 - Only output frag. if pixel is in band
- This makes linear interpolation the primary operation

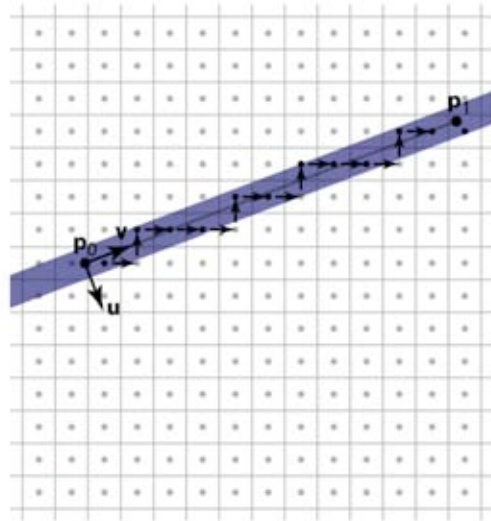


Pixel-walk line rasterization

```

x = ceil(x0)
y = round(m*x + b)
d = m*x + b - y
while x < floor(x1)
  if d > 0.5
    y += 1; d -= 1;
  else
    x += 1; d += m;
  if -0.5 < d ≤ 0.5
    output(x, y)

```



Rasterizing triangles

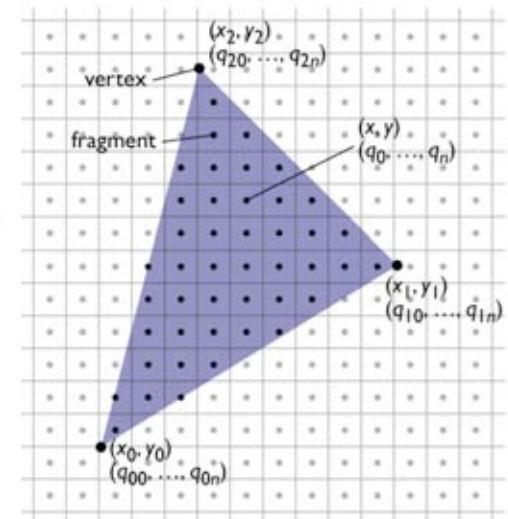
- The most common case in most applications
 - with good antialiasing can be the only case
 - some systems render a line as two skinny triangles
- Triangle represented by three vertices
- Simple way to think of algorithm follows the pixel-walk interpretation of line rasterization
 - walk from pixel to pixel over (at least) the polygon's area
 - evaluate linear functions as you go
 - use those functions to decide which pixels are inside

Rasterizing triangles

- Input:
 - three 2D points (the triangle's vertices in pixel space)
 - $(x_0, y_0); (x_1, y_1); (x_2, y_2)$
 - parameter values at each vertex
 - $q_{00}, \dots, q_{0n}; q_{10}, \dots, q_{1n}; q_{20}, \dots, q_{2n}$
- Output: a list of fragments, each with
 - the integer pixel coordinates (x, y)
 - interpolated parameter values q_0, \dots, q_n

Rasterizing triangles

- Summary
 - 1 evaluation of linear functions on pixel grid
 - 2 functions defined by parameter values at vertices
 - 3 using extra parameters to determine fragment set



Incremental linear evaluation

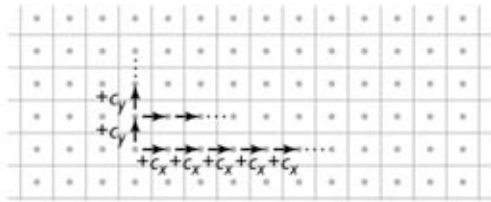
- A linear (affine, really) function on the plane is:

$$q(x, y) = c_x x + c_y y + c_k$$

- Linear functions are efficient to evaluate on a grid:

$$q(x + 1, y) = c_x(x + 1) + c_y y + c_k = q(x, y) + c_x$$

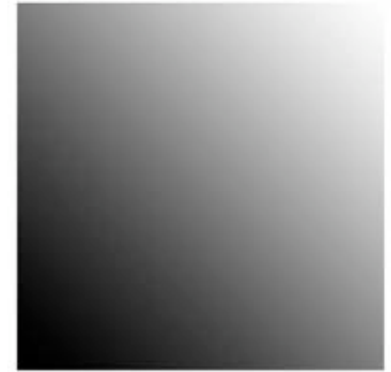
$$q(x, y + 1) = c_x x + c_y(y + 1) + c_k = q(x, y) + c_y$$



Incremental linear evaluation

```
linEval(xl, xh, yl, yh, cx, cy, ck) {
    // setup
    qRow = cx*xl + cy*yl + ck;

    // traversal
    for x = yl to yh {
        qPix = qRow;
        for x = xl to xh {
            output(x, y, qPix);
            qPix += cx;
        }
        qRow += cy;
    }
}
```

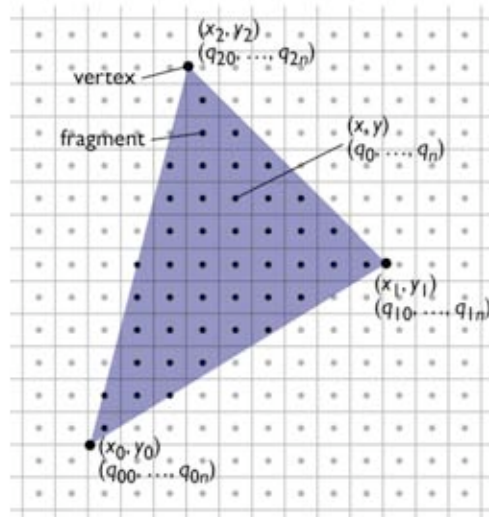


$c_x = .005; c_y = .005; c_k = 0$
(image size 100x100)

Rasterizing triangles

- Summary

- 1 evaluation of linear functions on pixel grid
- 2 functions defined by parameter values at vertices
- 3 using extra parameters to determine fragment set



Defining parameter functions

- To interpolate parameters across a triangle we need to find the c_x , c_y , and c_k that define the (unique) linear function that matches the given values at all 3 vertices

- this is 3 constraints on 3 unknown coefficients:

$$\begin{aligned} c_x x_0 + c_y y_0 + c_k &= q_0 \\ c_x x_1 + c_y y_1 + c_k &= q_1 \\ c_x x_2 + c_y y_2 + c_k &= q_2 \end{aligned} \quad \begin{array}{l} \text{(each states that the function} \\ \text{agrees with the given value} \\ \text{at one vertex)} \end{array}$$

- leading to a 3x3 matrix equation for the coefficients:

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_k \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix} \quad \begin{array}{l} \text{(singular iff triangle} \\ \text{is degenerate)} \end{array}$$

Defining parameter functions

- More efficient version: shift origin to (x_0, y_0)

$$q(x, y) = c_x(x - x_0) + c_y(y - y_0) + q_0$$

$$q(x_1, y_1) = c_x(x_1 - x_0) + c_y(y_1 - y_0) + q_0 = q_1$$

$$q(x_2, y_2) = c_x(x_2 - x_0) + c_y(y_2 - y_0) + q_0 = q_2$$

- now this is a 2x2 linear system (since q_0 falls out):

$$\begin{bmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} = \begin{bmatrix} q_1 - q_0 \\ q_2 - q_0 \end{bmatrix}$$

- solve using Cramer's rule (see Shirley):

$$c_x = (\Delta q_1 \Delta y_2 - \Delta q_2 \Delta y_1) / (\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

$$c_y = (\Delta q_2 \Delta x_1 - \Delta q_1 \Delta x_2) / (\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

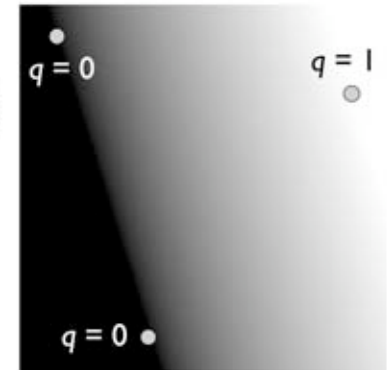
Defining parameter functions

```
linInterp(xl, xh, yl, yh, x0, y0, q0,
          x1, y1, q1, x2, y2, q2) {
```

```
// setup
det = (x1-x0)*(y2-y0) - (x2-x0)*(y1-y0);
cx = ((q1-q0)*(y2-y0) - (q2-q0)*(y1-y0)) / det;
cy = ((q2-q0)*(x1-x0) - (q1-q0)*(x2-x0)) / det;
qRow = cx*(x1-x0) + cy*(y1-y0) + q0;
```

```
// traversal (same as before)
```

```
for x = yl to yh {
  qPix = qRow;
  for x = xl to xh {
    output(x, y, qPix);
    qPix += cx;
  }
  qRow += cy;
}
```

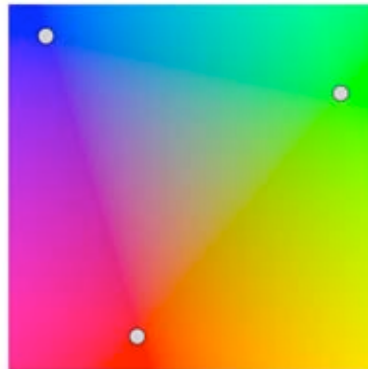


Interpolating several parameters

```
linInterp(xl, xh, yl, yh, n, x0, y0, q0[],
          x1, y1, q1[], x2, y2, q2[]) {
```

```
// setup
for k = 0 to n-1
  // compute cx[k], cy[k], qRow[k]
  // from q0[k], q1[k], q2[k]
```

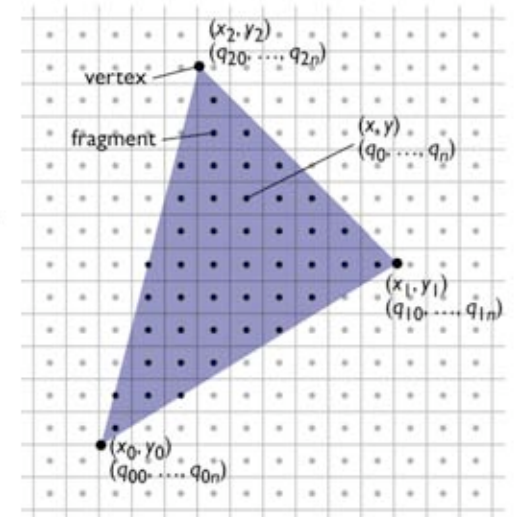
```
// traversal
for x = yl to yh {
  for k = 1 to n, qPix[k] = qRow[k];
  for x = xl to xh {
    output(x, y, qPix);
    for k = 1 to n, qPix[k] += cx[k];
  }
  for k = 1 to n, qRow[k] += cy[k];
}
```



Rasterizing triangles

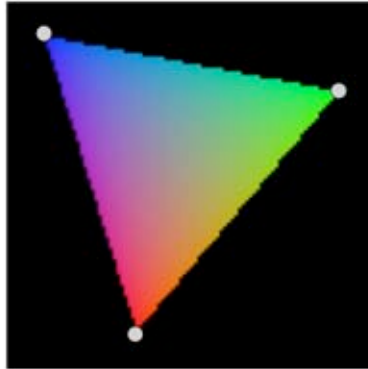
- Summary

- evaluation of linear functions on pixel grid
- functions defined by parameter values at vertices
- using extra parameters to determine fragment set



Clipping to the triangle

- Interpolate three *barycentric coordinates* across the plane
 - each barycentric coord is 1 at one vert. and 0 at the other two
- Output fragments only when all three are > 0 .



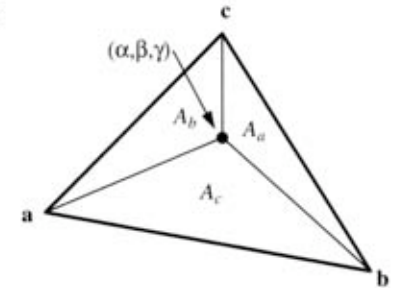
Barycentric coordinates

- A coordinate system for triangles
 - algebraic viewpoint:

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

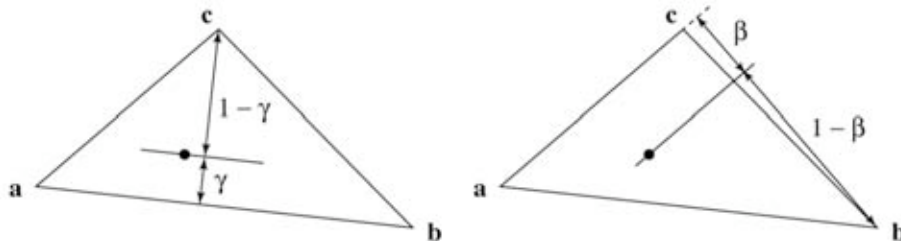
$$\alpha + \beta + \gamma = 1$$
 - geometric viewpoint (areas):
- Triangle interior test:

$$\alpha > 0; \quad \beta > 0; \quad \gamma > 0$$



Barycentric coordinates

- A coordinate system for triangles
 - geometric viewpoint: distances



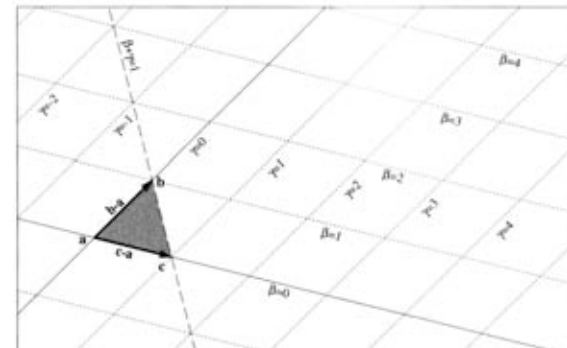
- linear viewpoint: basis of edges

$$\alpha = 1 - \beta - \gamma$$

$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

Barycentric coordinates

- Linear viewpoint: basis for the plane



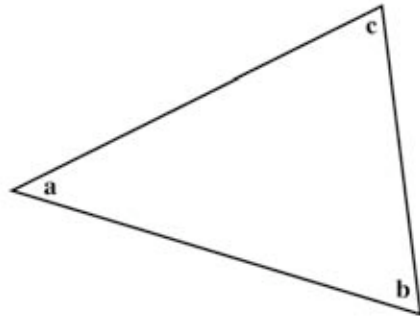
- in this view, the triangle interior test is just

$$\beta > 0; \quad \gamma > 0; \quad \beta + \gamma < 1$$

Edge equations

- In plane, triangle is the intersection of 3 half spaces

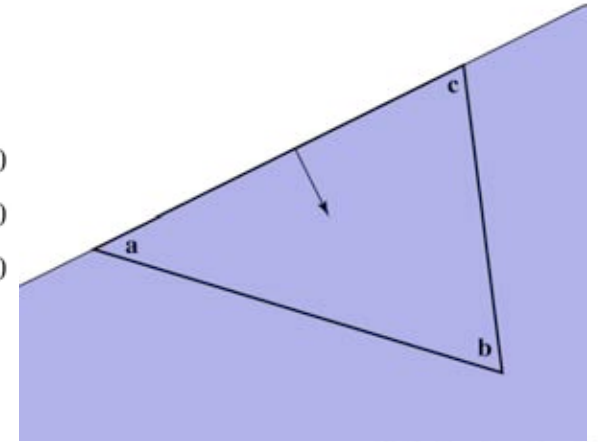
$$\begin{aligned}(x - a) \cdot (b - a)^\perp &> 0 \\(x - b) \cdot (c - b)^\perp &> 0 \\(x - c) \cdot (a - c)^\perp &> 0\end{aligned}$$



Edge equations

- In plane, triangle is the intersection of 3 half spaces

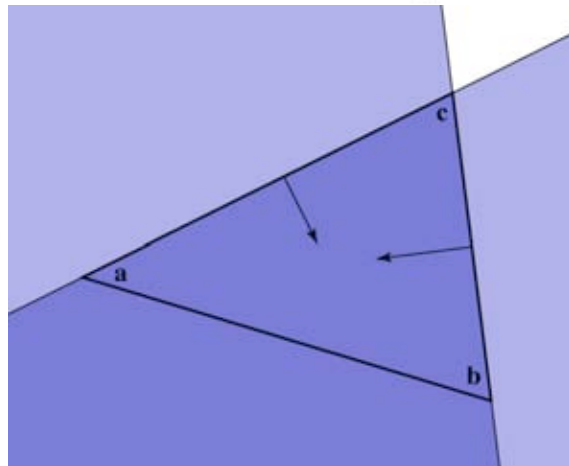
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Edge equations

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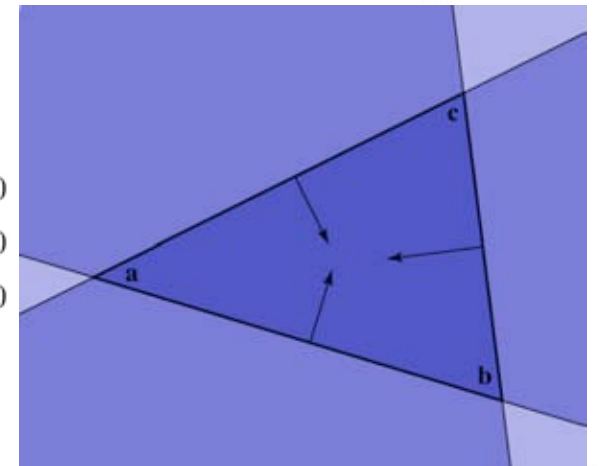
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Edge equations

- In plane, triangle is the intersection of 3 half spaces

$$\begin{aligned}(x - a) \cdot (b - a)^\perp &> 0 \\(x - b) \cdot (c - b)^\perp &> 0 \\(x - c) \cdot (a - c)^\perp &> 0\end{aligned}$$

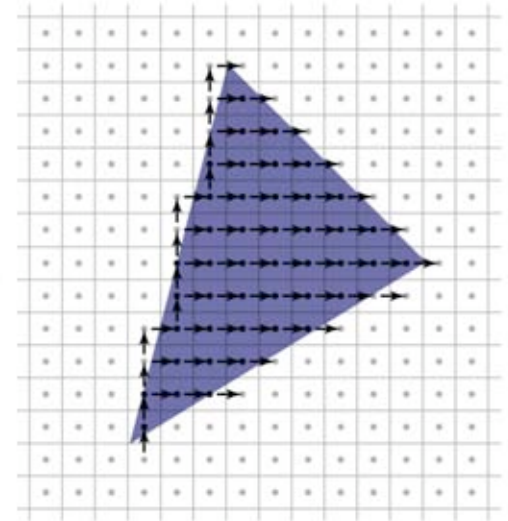


Walking edge equations

- We need to update values of the three edge equations with single-pixel steps in x and y
- Edge equation already in form of dot product
- components of vector are the increments

Pixel-walk (Pineda) rasterization

- Conservatively visit a superset of the pixels you want
- Interpolate linear functions
- Use those functions to determine when to emit a fragment



Rasterizing triangles

- Exercise caution with rounding and arbitrary decisions
 - need to visit these pixels once
 - but it's important not to visit them twice!

