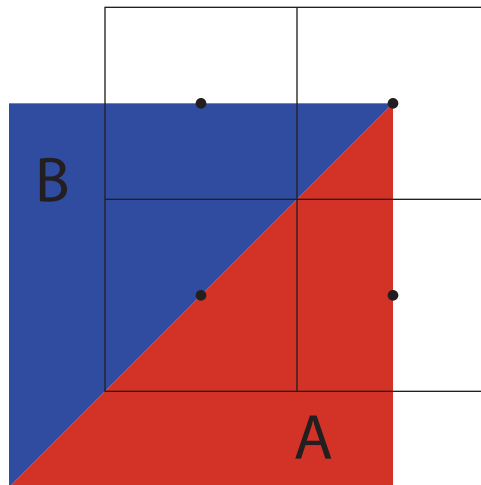# CS 465 Homework 9

out: Friday 10 November 2006
**due: Friday 17 November 2006**



For the following problems consider the above picture: we have two triangles $A$ and $B$, with screen space coordinates $A_1 = (1,1)$, $A_2 = (-1,-1)$, $A_3 = (1,-1)$ and $B_1 = (-1,-1)$, $B_2 = (1,1)$, $B_3 = (-1,1)$. Our image size is $2 \times 2$ pixels, following the normal conventions of $(0,0)$ being the bottom leftmost pixel and integral centers for pixels. The triangle $A$ has color $(210, 50, 30)$, the triangle $B$ has color $(10, 60, 190)$, and the background of the image is $(255, 255, 255)$. We assume in all questions below that we are performing alpha-compositing on the resultant scene using the **over** operator and box filtering.

1. What is the color of pixel $(1,0)$ using compositing? What is its true value (i.e. the value you would get using ideal antialiasing directly on the complete scene)?

   **Answer:** For compositing, the precomputed pixel value is $0.5 * [210, 50, 30] = [105, 25, 15]$

   Thus, for alpha compositing, we get
   $[105, 25, 15] + (1 - 0.5) * [255, 255, 255] = [232.5, 152.5, 142.5]$

   The true value using ideal antialiasing is just the color of each object weighted by the percentage that it is visible in the pixel, or $0.5 * [210, 50, 30] + 0.5 * [255, 255, 255] = [232.5, 152.5, 142.5]$, i.e. the same answer as alpha compositing.

2. What is the color of pixel $(1, 1)$ using compositing (compositing $A$ first and then $B$)? What is its true value (using the same definition as above)?

   **Answer:** Again, for compositing the precomputed value for A is $1/8*[210, 50, 30] = [26.25, 6.25, 3.75]$ and the precomputed value for B is $1/8*[10, 60, 190] = [1.25, 7.5, 23.75]$.

   Thus, for alpha compositing, we get

   $$\begin{aligned} C &= B' + (1 - \frac{1}{8})(A' + (1 - \frac{1}{8})bg) \\ &= [1.25, 7.5, 23.75] + \frac{7}{8}([26.25, 6.25, 3.75] + \frac{7}{8}[255, 255, 255]) \\ &= [219.45, 208.2, 222.26] \end{aligned}$$

   The true antialiased value is again the colors of the objects weighted by their proportional visibility, or

   $$\begin{aligned} C_{true} &= 1/8 * A + 1/8 * B + 3/4 * bg \\ &= (218.75, 205, 218.75) \end{aligned}$$

   We note that these values are, in fact, different.

3. Suppose we make $n$ copies of triangle $A$ and composite the resultant image. What is the color of pixel $(1, 0)$ as $n \rightarrow \infty$?

   **Answer:** As we composite more and more copies of $A$, the background color vanishes. and we end up with a pure color of $A$ in the pixel, i.e. $[210, 50, 30]$. This can be seen by writing out the first few composites:

   $$\begin{aligned} C_1 &= 1/2 * A + 1/2 * bg \\ C_2 &= 1/2 * A + 1/2 * (1/2 * A + 1/2 * bg) = 3/4 * A + 1/4 * bg \\ C_3 &= 1/2 * A + 1/2 * (3/4 * A + 1/4 * bg) = 7/8 * A + 1/8 * bg \end{aligned}$$

   In fact, you can show that if $A$ has **any** nonzero contribution to a pixel, as you composite $A$ infinitely the color of that pixel will tend to $A$ regardless of what the initial color was.

4. Suppose we make $n$ copies of triangle $A$ and $n$ copies of triangle $B$ and composite the image by first compositing all the copies of $A$ and then all the copies of $B$. What is the color of pixel $(1, 1)$ as $n \rightarrow \infty$?

   **Answer:** The same principle holds as in the previous problem. After we have composited a large number of copies of $A$, the color of the pixel will converge towards

the color of $A$. However, this color doesn't matter because we then composite a large number of copies of $B$, and the argument above showed that it doesn't matter at all what color the pixel was initially, so the color of the pixel will converge towards $B$, or $[10, 60, 190]$

5. Again, suppose we make $n$ copies of triangles $A$ and $B$, except this time we alternate between compositing an $A$ copy and a $B$ copy. What happens to the color of pixel $(1, 1)$ as $n \to \infty$? Be as specific as possible.

   **Answer:** The previous arguments do not work for this, because we are now alternating compositions of color. As the number of compositions go to infinity, the color of the pixel will converge to an alternation between two colors, namely $[103.33, 55.33, 115.33]$ when $B$ is composited last and $[116.67, 54.67, 104.67]$ when $A$ is composited last. There are a few ways of determining these values: experimentally, with geometric series, and with linear equations.

   Experimentally is straightforward: run a small program in MATLAB or Java and you will see the convergence to the two numbers. For geometric series, you can derive an expression for the color of the pixel after $B$ was composited last and another expression for the color after $A$ was composited last and compute the value as $n \to \infty$.

   For linear equations, we first hypothesize that we will see this alternation of colors. We are then looking for two colors $x_1$ and $x_2$ such that the following system is satisfied:

$$A' + 7/8x_1 = x_2$$
$$B' + 7/8x_2 = x_1$$

   Solving this system for $x_1$ and $x_2$ with respect to the $R$, $G$, and $B$ channels gives the same answer as above