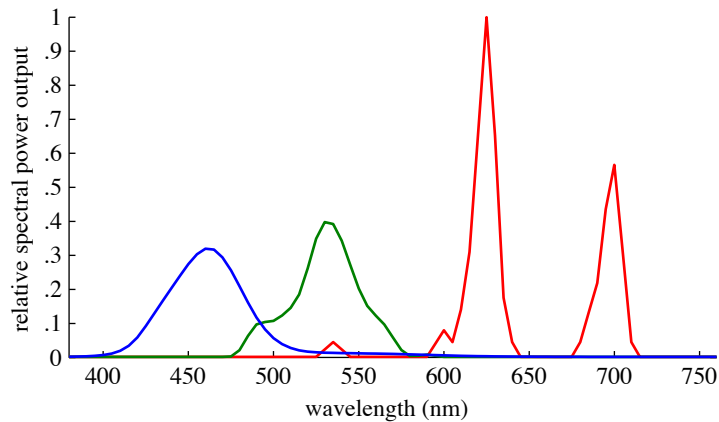# CS 465 Homework 10

out: Monday 20 November 2006
due: Friday 1 December 2006

Note: You are encouraged to use mathematical tools like MATLAB, Excel, graphing caluclators, or small programs in Java to come up with the answers for this homework.

Consider a CRT monitor M_1 with the following primary color spectra:

(The data for this plot are available in `m-rgb.txt`.) The maximum luminance of this monitor is 100; that is, the Y tristimulus value when the monitor is displaying pure white is 100. We consider this monitor to be an ideal monitor; that is, the gamma value is 1 and it accepts floating point numbers of arbitrary precision so that there are no quantization artifacts.

Part A

1. What is the RGB-to-XYZ transformation matrix for M_1?

   Answer:

   Intuitively, the monitor is given RGB values which it then converts into a spectrum, which is then measured in the XYZ color space. Thus, if xyz and m_rgb are the matrix data available on the website, we can represent the RGB-to-XYZ transform as

$$MRGB_{1_{temp}} = transpose(xyz(1:77, 2:4)) * m\_rgb(1:77, 2:4)$$

This matrix is correct up to an unknown scale factor. However, we also know that the maximum luminance of the monitor is 100, so we need to scale the matrix so that the Y component of $MRGB_{1_{temp}} * [1; 1; 1]$ is 100. The computed Y component of $MRGB_{1_{temp}} * [1; 1; 1]$ is 4.3581, so we scale by $100/4.3581 = 22.946$ to give us the final matrix of

$$MRGB_1 = \begin{bmatrix} 57.076 & 18.348 & 19.624 \\ 27.17 & 62.617 & 10.212 \\ 0.07146 & 7.2723 & 101.53 \end{bmatrix}$$

2. What are the chromaticities of the primaries?

   Answer: For red, take the vector $[1; 0; 0]$ and multiply it by $MRGB_1$ to obtain XYZ values. The chromaticity x, y is then just $x = X/(X + Y + Z)$ and $y = Y/(X + Y + Z)$. Repeat this for both $[0; 1; 0]$ and $[0; 0; 1]$ to obtain

red	$x = 0.67692$	$y = 0.32224$
green	$x = 0.20794$	$y = 0.70965$
blue	$x = 0.14939$	$y = 0.07774$

3. The white point of a monitor is the chromaticity of the displayed color when the monitor is outputting all white ($R_{M_1} = G_{M_1} = B_{M_1} = 1.0$). What is the white point of monitor M_1?

   Answer: Multiplying $MRGB_1$ by $[1; 1; 1]$ and then applying the above equations for chromaticity gives $x = 0.31274, y = 0.32904$ for the whitepoint.

Part B

Now consider a second LCD monitor M_2, again with maximum luminance of 100. Rather than measure the spectra, we have obtained from the manufacturer the chromaticities of the primaries as well as the white point, which are:

Red primary	$x = 0.640$	$y = 0.330$
Green primary	$x = 0.300$	$y = 0.600$
Blue primary	$x = 0.150$	$y = 0.060$
White point	$x = 0.313$	$y = 0.329$

(These are the values for the Dell 2407WFP display, as reported by the display's firmware.) Again, treat this as an ideal monitor with a gamma of 1 and no quantization performed.

1. What is the RGB-to-XYZ transformation matrix for M_2? *Hint:* With just the chromaticities given, there is an unknown scale factor for each of the primaries. The white point determines these factors, which you can recover using some linear equations.

   Answer: We need to recover the relative scaling of the three primaries by using the whitepoint. Given the x, y values, we can calculate z for each of the three primaries and the whitepoint via $z = 1 - x - y$. Now, we want to find w_r, w_g, w_b such that

   $$w_r x_r + w_g x_g + w_b x_b = x_w$$
   $$w_r y_r + w_g y_g + w_b y_b = y_w$$
 $$w_r z_r + w_g z_g + w_b z_b = z_w$$

   Plugging in the numbers and solving for w_r, w_g, w_b gives $w_r = 0.21272, w_g = 0.39179, w_b = 0.39549$. The unscaled matrix is then

   $$MRGB_{2_{temp}} = \begin{bmatrix} w_r x_r & w_g x_g & w_b x_b \\ w_r y_r & w_g y_g & w_b y_b \\ w_r z_r & w_g z_g & w_b z_b \end{bmatrix}$$

   Again we have to scale this so that the maximum luminance at $[1; 1; 1]$ is 100. Multiplying $MRGB_{2_{temp}} * [1; 1; 1]$ gives us a Y value of 0.329, so we scale by $100/0.329 = 303.95$ and get as the final answer

   $$MRGB_2 = \begin{bmatrix} 41.38 & 35.725 & 18.032 \\ 21.336 & 71.451 & 7.2126 \\ 1.9397 & 11.908 & 94.966 \end{bmatrix} \tag{1}$$

2. Consider the spectrum below, which is a medium skin tone under daylight illumination.
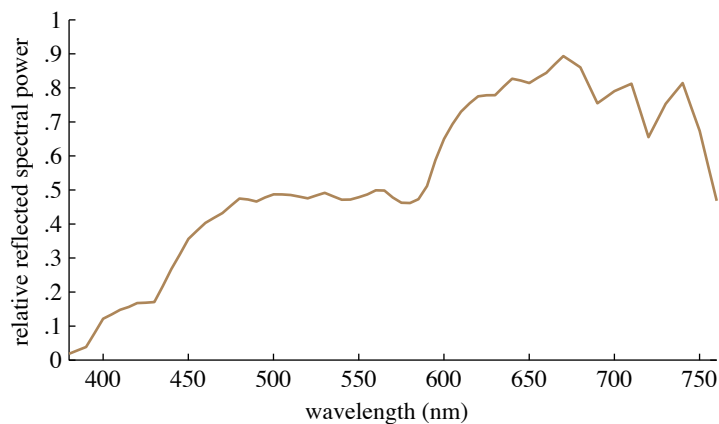
(The data for this plot are available in `skin-d65.txt`.) The plot gives the relative spectral distribution, and the luminance of the color is 50. What RGB values are needed to produce a metamer (in the XYZ color space) of this spectrum for monitor M_1? For monitor M_2?

Answer: We first need to compute what are the measured XYZ values for the skin spectrum. To do this we multiply our XYZ response matrix against the spectrum data:

$$skin\_xyz = transpose(xyz(1:77,2:4)) * skin\_d65(1:77,2) \qquad (2)$$

We need to now scale this so that the observed luminance is 50: i.e. scale it by $50/skin\_xyz(2)$. This gives us the observed XYZ values

$$\begin{bmatrix} 53.027 \\ 50 \\ 34.549 \end{bmatrix} \qquad (3)$$

We now need to compute what RGB values will produce these XYZ values for each of the monitors. This can be done by multiplying the above vector by the inverses of the matrices $MRGB_1$ and $MRGB_2$. Thus, we get for $M1$:

$$MRGB_1^{-1} * [53.027; 50; 34.549] = [0.67731; 0.45449; 0.30726]$$
$$MRGB_2^{-1} * [53.027; 50; 34.549] = [0.775; 0.43878; 0.29295]$$

So, the RGB values needed for $M1$ are $R_1 = 0.67731, G_1 = 0.45449 B_1 = 0.30726$, and for M_2 we need $R_2 = 0.775, G_2 = 0.43878, B_2 = 0.29295$

3. Because these are all linear transformations, we can compute an RGB transformation matrix through matrix products that, given an RGB triplet $(R_{M_1}, G_{M_1}, B_{M_1})$ to be displayed on monitor M_1, will produce the RGB triplet $(R_{M_2}, G_{M_2}, B_{M_2})$ that should be displayed on monitor M_2 in order to produce the same color response. Compute this matrix for transforming from M_1 to M_2, and show that it works as expected by plugging in your answers to (2).

Answer: This matrix is simply $MRGB_2^{-1}MRGB_1$; i.e. the matrix that takes RGB values in M_1 to XYZ and then takes those XYZ to RGB in M_2 through the inverse of M_2's RGB-XYZ transform. Thus, it works out to be

$$\begin{bmatrix} 1.4269 & -0.40291 & -0.027128 \\ -0.043511 & 1.0008 & 0.043604 \\ -0.022935 & -0.04069 & 1.0642 \end{bmatrix}$$

Multiplying this matrix by $[.67731; .45449; .30726]$ produces $[0.77499; 0.43878; 0.29294]$, as expected.