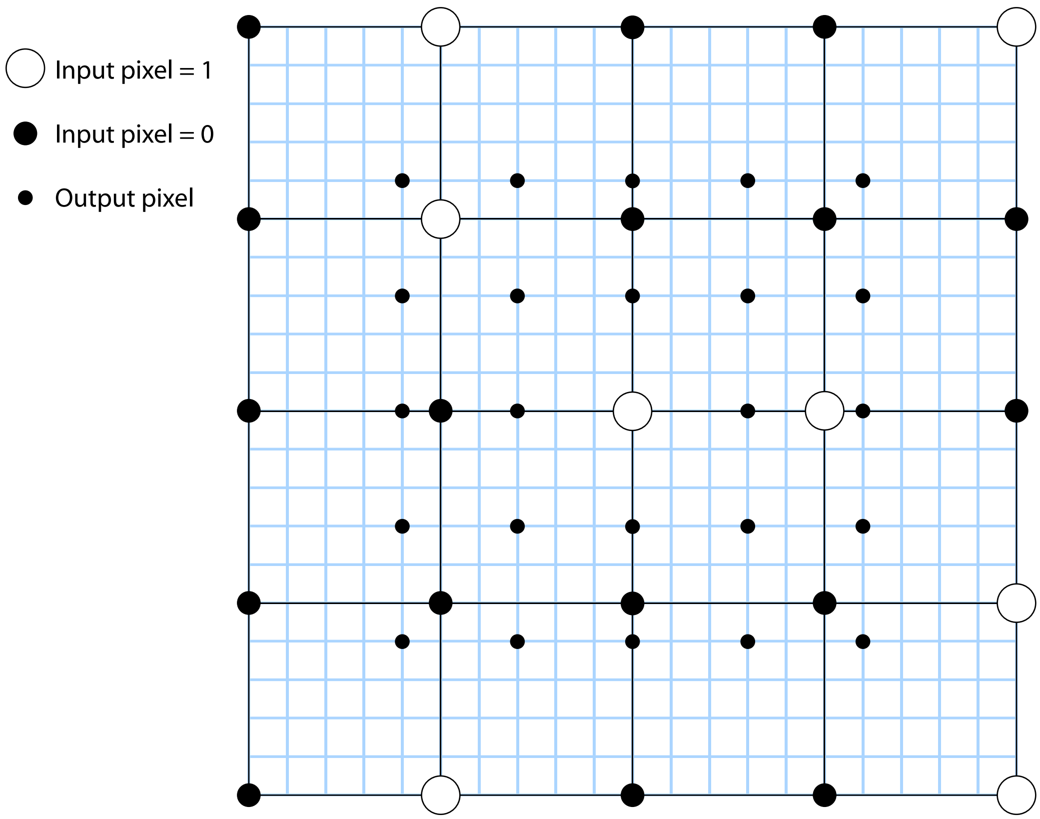
CS 465 HW2 Solution

(revised September 21, 2005)

Problem 4: Resampling The original image was 27x9, implying that upsampling to 45x15 would give us a new frequency $f = \frac{5}{3} \frac{\text{new pixels}}{\text{old pixels}}$. The new wavelength then, or distance between samples, is 0.6 units.



In the above diagram, the center pixel is (12,7) on the output image. It maps to (7,4) on the input image if you rescape the 45x15 grid of pixels so as to preserve the boundaries of the image. (0,0) on the output image maps to (-0.2, -0.2) on the input image, where (0,0) is the center of the lower-left pixel. Notice that if we measure in input image coordinates, the output pixel positions are $\frac{3}{5}$ unit apart. The grid above is laid out to make it easier to measure distances between output pixel centers and input pixel centers.

1. **5x5, Box Filter, radius=0.5**:

   The grid below indicates the values of the pixels in the output image. The grid is oriented in the same manner as the image — x increases left to right and y increases bottom to top.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. **5x5, Tent Filter, radius=1**:

   Notice that for the purposes of this assignment, "radius" is the radius of support of a one-dimensional filter, NOT the radius of support of a two-dimensional radially defined filter. To filter in two dimensions, we use the product of a horizontal and vertical filter.

$$\frac{1}{25}\begin{bmatrix} 20 & 25 & 0 & 0 & 1 \\ 12 & 13 & 10 & 10 & 8 \\ 0 & 10 & 25 & 25 & 20 \\ 0 & 4 & 10 & 10 & 11 \\ 4 & 3 & 0 & 0 & 5 \end{bmatrix}$$

3. **3x3, B-Spline, radius=2**:

   The B-Spline case took more from a computational point of view, but the mechanics of it could be simplified in a number of ways. Taking advantage of the seperable construction of the filter, we need only compute the value of the B-Spline function at a few values, and re-use these values in weighting our input samples. Because relatively few pixels are "on" (contribute nonzero amount to the output pixel), we only need sum weights for those pixels.

   Here are the values of the B-spline filter we need to compute:

$$\begin{aligned} f(0) &= \tfrac{2}{3} \\ f\left(\frac{2}{5}\right) &= \tfrac{202}{375} \\ f\left(\frac{3}{5}\right) &= \tfrac{311}{750} \\ f\left(\frac{5}{5}\right) &= \tfrac{1}{6} \\ f\left(\frac{7}{5}\right) &= \tfrac{9}{250} \\ f\left(\frac{8}{5}\right) &= \tfrac{4}{375} \end{aligned}$$

Here is the rounded table of correct answers.

$$\begin{bmatrix} 0.4859 & 0.4413 & 0.4031 \\ 0.3733 & 0.5833 & 0.6433 \\ 0.2015 & 0.3533 & 0.4165 \end{bmatrix}$$