CS 465 Homework 1

out: Friday 26 August 2005
due: Friday 2 September 2005

Problem 1: Image formats

Many of the answers to this assignment make use of the following conversions:

$$\begin{aligned} 1 \text{ byte} &= 8 \text{ bits} \\ 1 \text{ MB} &= 1024 \times 1024 \text{ bytes} = 1048576 \text{ bytes} \\ 1 \text{ MB} &= 8388608 \text{ bits} \end{aligned}$$

1. Suppose a PC is connected digitally to a 1600 by 1200 pixel color LCD display. The display is operating at 60Hz, has a gamma of 2.2, and always takes 8 bits per color channel per pixel.

   (a) What data rate is flowing across the cable to the display?

   $$\left(1600 \times 1200 \, \frac{\text{pixel}}{\text{frame}}\right)\left(60 \, \frac{\text{frame}}{\text{second}}\right)\left(3 \, \frac{\text{channel}}{\text{pixel}}\right)\left(8 \, \frac{\text{bits}}{\text{channel}}\right)$$

   $$\begin{aligned} &= \left(2764800000 \, \frac{\text{bit}}{\text{second}}\right) \\ &\approxeq \left(2.57 \, \frac{\text{Gbit}}{\text{second}}\right) \end{aligned}$$

   (b) How much of the graphics card's memory is occupied by the framebuffer for each of the following pixel formats:

   i. 16 bits per pixel (4 red, 5 green, 4 blue)?

   $$\left(1600 \times 1200 \, \frac{\text{pixel}}{\text{frame}}\right)\left(16 \, \frac{\text{bit}}{\text{pixel}}\right) = \left(30720000 \, \frac{\text{bit}}{\text{frame}}\right) \approxeq \left(3.7 \, \frac{\text{MB}}{\text{frame}}\right)$$

   ii. 8 bit RGB?

   $$\left(1600 \times 1200 \, \frac{\text{pixel}}{\text{frame}}\right)\left(3 \, \frac{\text{channel}}{\text{pixel}}\right)\left(8 \, \frac{\text{bit}}{\text{channel}}\right)$$

   $$\begin{aligned} &= \left(46080000 \, \frac{\text{bit}}{\text{frame}}\right) \\ &\approxeq \left(5.5 \, \frac{\text{MB}}{\text{frame}}\right) \end{aligned}$$

iii. 8 bit RGB with 8 bit alpha?

$$\begin{aligned}
&\left(1600 \times 1200\ \frac{\text{pixel}}{\text{frame}}\right)\left(4\ \frac{\text{channel}}{\text{pixel}}\right)\left(8\ \frac{\text{bit}}{\text{channel}}\right) \\
&= \left(69120000\ \frac{\text{bit}}{\text{frame}}\right) \\
&\approxeq \left(7.32\ \frac{\text{MB}}{\text{frame}}\right)
\end{aligned}$$

2. The image on the screen was computed by a ray tracer that (like the Ray I assignment) stores the RGB image it is computing as an array of double-precision floating point numbers, with the value of 1.0 corresponding to the maximum displayable intensity I_{max}, and lower values mapping linearly to fractions of that intensity.

   (a) How much of the PC's main memory is occupied by the image?

$$\begin{aligned}
&\left(1600 \times 1200\ \frac{\text{pixel}}{\text{frame}}\right)\left(3\ \frac{\text{channel}}{\text{pixel}}\right)\left(1\ \frac{\text{double}}{\text{channel}}\right)\left(8\ \frac{\text{byte}}{\text{double}}\right) \\
&= \left(46080000\ \frac{\text{byte}}{\text{frame}}\right) \\
&\approxeq \left(43.9\ \frac{\text{MB}}{\text{frame}}\right)
\end{aligned}$$

   (b) What processing has to be done to the pixels before they are written into the frame-buffer? Give a specific expression that could appear in the code used to do the processing.

   The each component of the output pixels should be gamma corrected for the display gamma scaled to fit in the range $[0, 1]$. In Java let `cMax` hold the maximum value of `c` and `cMin` hold the minimum:

```
c = Math.pow(c, 2.2);              \\ Gamma correction
c = (c - cMin)\(cMax - cMin);      \\ Scaling
```

   (c) Suppose the software thinks the display gamma is 1.5. Would the displayed image look darker or lighter than it should?

   The image will appear darker. The software should gamma correct the image for a gamma of 1.5 and then the display will will correct for its non-linearity and apply a gamma of 2.2. Consider a pixel component with value, c. Let d and g be the displayed and gamma corrected values respectively:

$$\begin{aligned}
g &= c^{\frac{1}{1.5}} \\
d &= I_{\text{max}} \cdot g^{2.2} \\
&= I_{\text{max}} \cdot c^{\frac{2.2}{1.5}} \\
&\approxeq I_{\text{max}} \cdot c^{1.47}
\end{aligned}$$

   For all values of $c \in [0, 1]$, $c^{1.47} \leq c$ and making the image appear darker.

(d) Assuming a black level of 1% of $I_{\max}$, what will be the observed intensities corresponding to the five darkest gray values that can be sent to the display? Which steps are visibly different from one another?

There is a misunderstanding about the meaning of this question between the TA's and Professor Marschner.

We thought that you were supposed to calculate the values as if you were incorrectly gamma correcting for a value of 1.5 and the monitor had a display gamma of 2.2 (or 1.0 in part (e)). This means you would use:

$$d = I_{max}(c^{\frac{\gamma_d}{\gamma_s}} + 0.01)$$

to calculate the requested values. Here d *is the display value,* c *is the computed value and* γ_d *and* γ_s *are the display and corrected gammas respectively.*

Professor Marschner thought that you were just to compute the display values for a monitor with a certain gamma using a different equation:

$$d = I_{max}(c^{\gamma_d} + 0.01)$$

We have accepted both answers as correct. *Finally there is some question as to whether the first 5 display values include black (the 0 value) or not. As long as you gave us 5 values we didn't care which you chose. For the answers, we give all 6 possible values.*

TA Values for values (0,1,2,3,4,5):

$$0.0100 \quad 0.0103 \quad 0.0108 \quad 0.0115 \quad 0.0123 \quad 0.0131$$

All steps are visibly different.

Professor Marschner's values (0,1,2,3,4,5):

$$0.0100 \quad 0.0100 \quad 0.0100 \quad 0.0101 \quad 0.0102 \quad 0.0103$$

No steps are visibly different.

(e) Suppose the display is linear (i.e. its gamma is 1.0), and answer the previous two questions.
See the comment in part(d)

TA Values for values (0,1,2,3,4,5):

$$0.0100 \quad 0.0349 \quad 0.0495 \quad 0.0617 \quad 0.0727 \quad 0.0927$$

All steps are visibly different.

Professor Marschner's values (0,1,2,3,4,5):

$$0.0100 \quad 0.0139 \quad 0.0178 \quad 0.0218 \quad 0.0257 \quad 0.0296$$

All steps are visibly different.

3. The same image is to be printed on a black-and-white laser printer that has a resolution of 600 dots per inch. It will appear on the page at the size 8 inches by 6 inches. There are two steps of processing that have do be done on the image before it can be printed.

   (a) What are the two steps? Give a specific expression for the first step, and name a general class of algorithms for the second step.

   The first step is to convert the image to gray scale. A good choice for this conversion is:

   $$\mathrm{gray} = 0.2\mathrm{R} + 0.7\mathrm{G} + 0.1\mathrm{B}$$

   The second step is to dither the image to produce the dot pattern for printer (select a dithering algorithm).

   (b) How much data needs to be transmitted over the network to the printer if:

   i. both steps are done in the computer?

   If both steps are done in the computer, the printer would have to be sent which dots are on and off.

   $$\begin{aligned}\left(8 \times 6\ \frac{\mathrm{inch}^2}{\mathrm{frame}}\right)\left(600^2\ \frac{\mathrm{dot}}{\mathrm{inch}^2}\right)\left(1\ \frac{\mathrm{bit}}{\mathrm{dot}}\right) \\ = \left(17280000\ \frac{\mathrm{bit}}{\mathrm{frame}}\right) \\ \approxeq \left(2.1\ \frac{\mathrm{MB}}{\mathrm{frame}}\right)\end{aligned}$$

   ii. one step is done in the computer and one in the printer?

   In this case, the printer must be sent the gray value of each pixel. Assuming 256 shades of gray (1 byte).

   $$\begin{aligned}\left(1600 \times 1200\ \frac{\mathrm{pixel}}{\mathrm{frame}}\right)\left(1\ \frac{\mathrm{channel}}{\mathrm{pixel}}\right)\left(1\ \frac{\mathrm{byte}}{\mathrm{channel}}\right) \\ = \left(1920000\ \frac{\mathrm{byte}}{\mathrm{frame}}\right) \\ \approxeq \left(1.8\ \frac{\mathrm{MB}}{\mathrm{frame}}\right)\end{aligned}$$

   iii. both are done in the printer?

   In this case, the original image needs to be sent to the printer. Assuming 256 values (1 byte) per channel.

   $$\begin{aligned}\left(1600 \times 1200\ \frac{\mathrm{pixel}}{\mathrm{frame}}\right)\left(3\ \frac{\mathrm{channel}}{\mathrm{pixel}}\right)\left(1\ \frac{\mathrm{byte}}{\mathrm{channel}}\right) \\ = \left(5760000\ \frac{\mathrm{byte}}{\mathrm{frame}}\right) \\ \approxeq \left(5.5\ \frac{\mathrm{MB}}{\mathrm{frame}}\right)\end{aligned}$$

Assume no data compression is used (this is rather unrealistic for option (a)).

Give data sizes in kilobytes, megabytes, or gigabytes as appropriate; give data rates in kilobits, megabits, or gigabits per second. Recall that these units traditionally refer to multiples of 1024, not multiples of 1000.