

Animation

CS 4620 Lecture 33

Announcements

- Grading A5 (and A6) on Monday after TG
- 4621: one-on-one sessions with TA this Friday

Quaternions

- Remember that
 - Orientations can be expressed as rotation
 - Why?
 - Start in a default position (say aligned with z axis)
 - New orientation is rotation from default position
 - Rotations can be expressed as (axis, angle)
- Quaternions let you express (axis, angle)

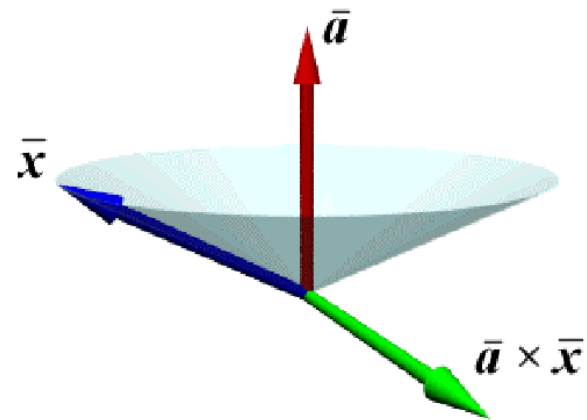
Quaternion for Rotation

- Rotate about axis \mathbf{a} by angle θ

$$q = (s, \mathbf{v}) = (s, v_1, v_2, v_3)$$

$$s = \cos\left(\frac{\theta}{2}\right)$$

$$\mathbf{v} = \sin\left(\frac{\theta}{2}\right) \hat{\mathbf{a}}$$



Rotation Using Quaternion

- A point in space is a quaternion with 0 scalar

$$X = (0, \vec{x})$$

- Rotation is computed as follows

$$x_{rotated} = qXq^{-1} = qXq'$$

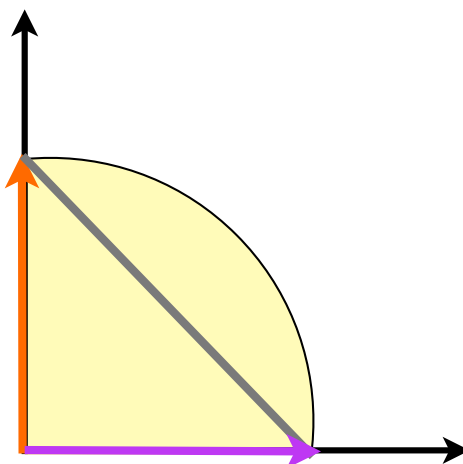
- See Buss 3D CG: A mathematical introduction with OpenGL, Chapter 7

Why Quaternions?

- Fast, few operations, not redundant
- Numerically stable for incremental changes
- Composes rotations nicely
- Convert to matrices at the end
- Biggest reason: spherical interpolation

Interpolating between quaternions

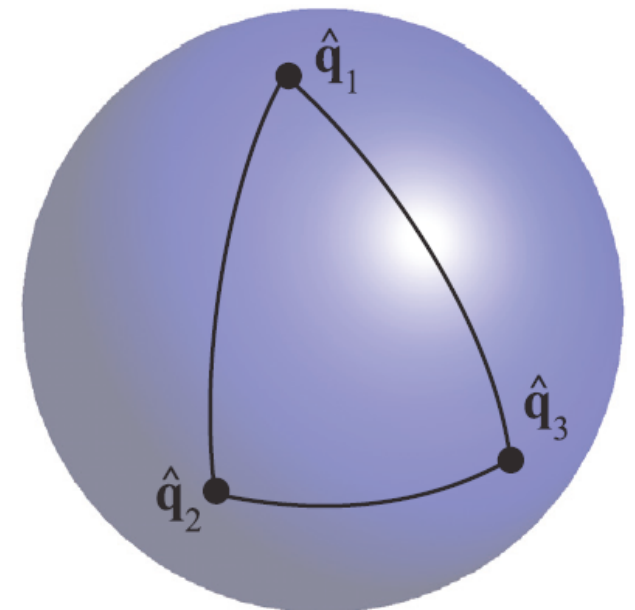
- Why not linear interpolation?
 - Need to be normalized
 - Does not have a constant rate of rotation



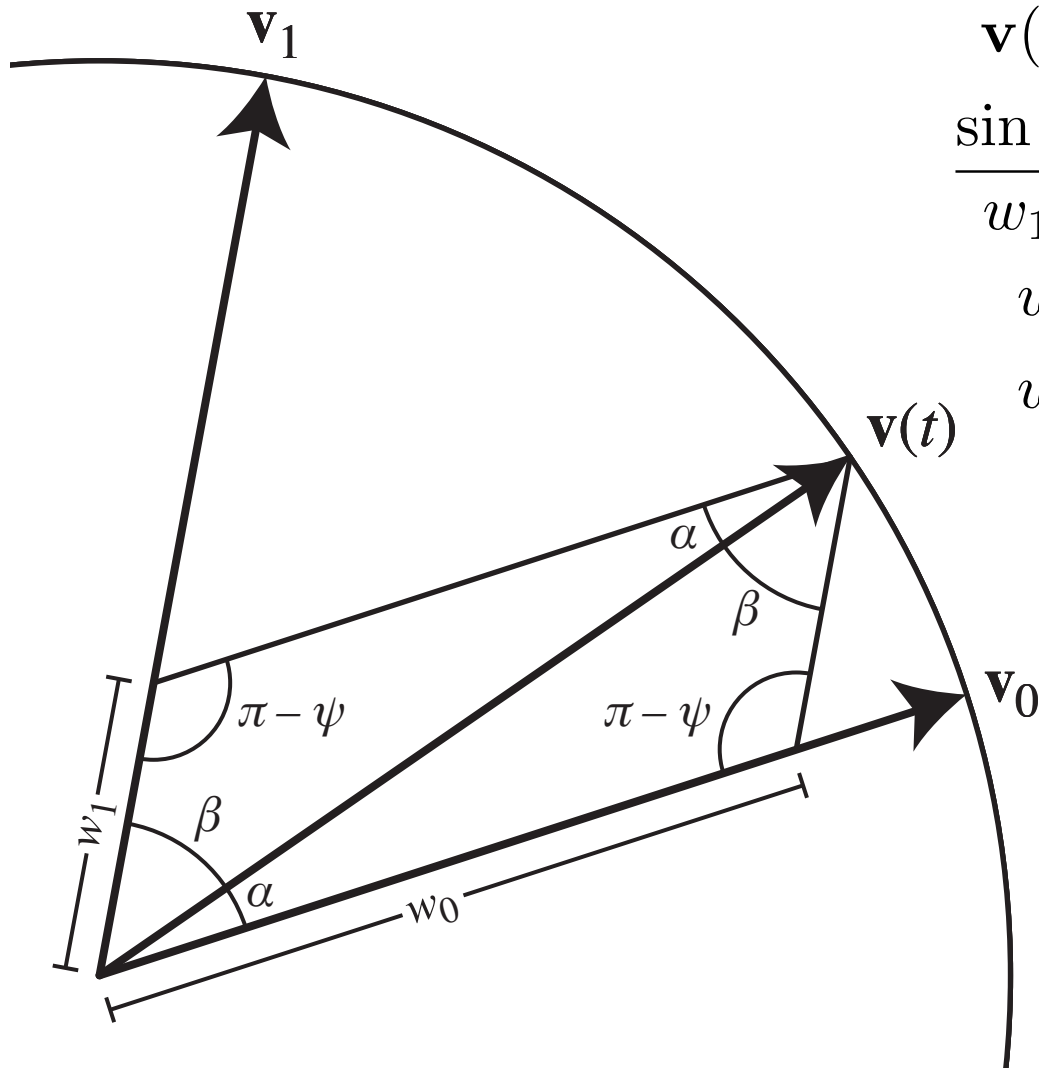
$$\frac{(1 - \alpha)x + \alpha y}{\|(1 - \alpha)x + \alpha y\|}$$

Spherical Linear Interpolation

- Intuitive interpolation between different orientations
 - Nicely represented through quaternions
 - Useful for animation
 - Given two quaternions, interpolate between them
- Shortest path between two points on sphere
 - Geodesic, on Great Circle



Spherical linear interpolation (“slerp”)



$$\alpha + \beta = \psi$$

$$\mathbf{v}(t) = w_0 \mathbf{v}_0 + w_1 \mathbf{v}_1$$

$$\frac{\sin \alpha}{w_1} = \frac{\sin \beta}{w_0} = \frac{\sin(\pi - \psi)}{1} = \sin \psi$$

$$w_0 = \sin \beta / \sin \psi$$

$$w_1 = \sin \alpha / \sin \psi$$

$$\psi = \cos^{-1}(\mathbf{v}_0 \cdot \mathbf{v}_1)$$

Quaternion Interpolation

- Spherical linear interpolation naturally works in any dimension
- Traverses a great arc on the sphere of unit quaternions
Uniform angular rotation velocity about a fixed axis

$$\psi = \cos^{-1}(q_0 \cdot q_1)$$
$$q(t) = \frac{q_0 \sin(1-t)\psi + q_1 \sin t\psi}{\sin \psi}$$

Practical issues

- When angle gets close to zero, use small angle approximation
 - degenerate to linear interpolation
- When angle close to 180, there is no shortest geodesic, but can pick one
- q is same rotation as $-q$
 - if q_1 and q_2 angle < 90 , slerp between them
 - else, slerp between q_1 and $-q_2$

Interpolating transformations

- Linear interpolation of matrices is not effective
 - leads to shrinkage when interpolating rotations
- One approach: always keep transformations in a canonical form (e.g. translate-rotate-scale)
 - then the pieces can be interpolated separately
 - rotations stay rotations, scales stay scales, all is good
- But you might be faced with just a matrix. What then?

Decomposing transformations

- A product $M = TRS$ is not hard to take apart
 - translation sits in the top right
- If we allow S to be a scale along arbitrary axes
- $M = TRS$ where
 - T is a translation
 - R is a rotation
 - S is a symmetric matrix (positive definite if no reflection)
 - Linear algebra name
 - Polar decomposition (at least the $A = RS$ part)

Parameterizing rotations

- Unit quaternions
 - A 4D representation (like 3D unit vectors for 2D sphere)
 - Good choice for interpolating rotations
- These are first examples of motion control
 - Matrix = deformation
 - Angles/quaternion = animation controls

The artistic process of animation

- What are animators trying to do?
- "Principles of Traditional Animation Applied to 3D Computer **Graphics**," SIGGRAPH'87, by John Lasseter
- Widely cited set of principles laid out by Frank Thomas and Ollie Johnston in *The Illusion of Life* (1981)
- The following slides follow Michael Comet's examples:
www.comet-cartoons.com

Animation principles: timing

- Speed of an action is crucial to the impression it makes

examples with same keyframes, different times:



60 fr: looking around



30 fr: "no"



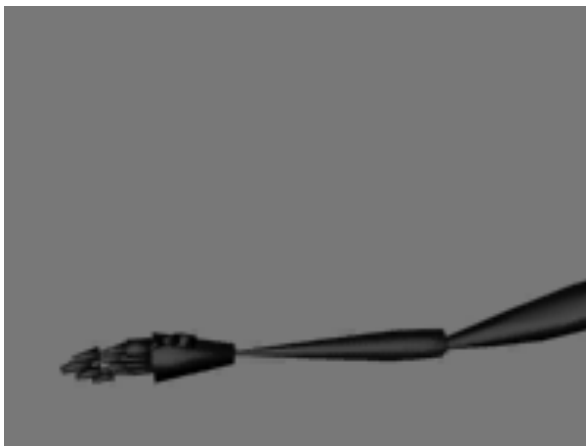
5 fr: just been hit

[Michael B. Comet]

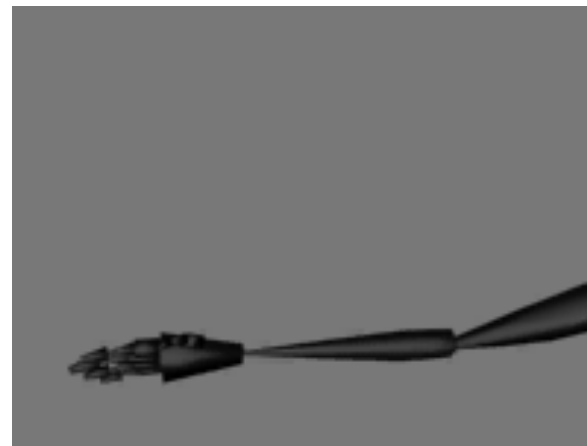


Animation principles: ease in/out

- Real objects do not start and stop suddenly
animation parameters shouldn't either



straight linear interp.



ease in/out

a little goes a long way (just a few frames acceleration or deceleration for “snappy” motions)

Animation principles: moving in arcs

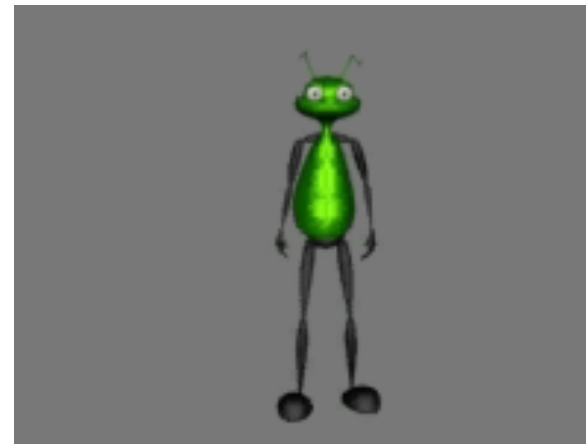
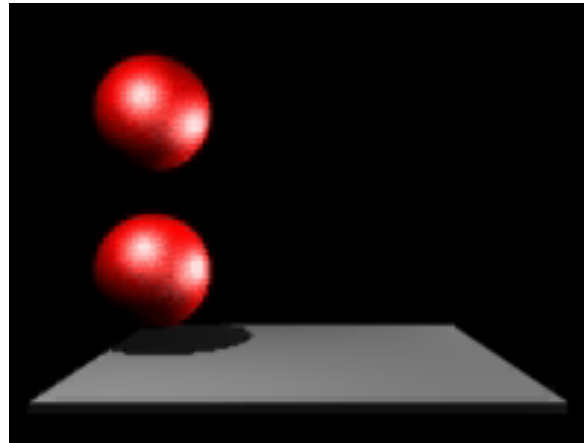
- Real objects also don't move in straight lines
generally curves are more graceful and realistic



[Michael B. Comet]

Animation principles: anticipation

- Most actions are preceded by some kind of “wind-up”



[Michael B. Comet]

[Michael B. Comet]



Animation principles: exaggeration

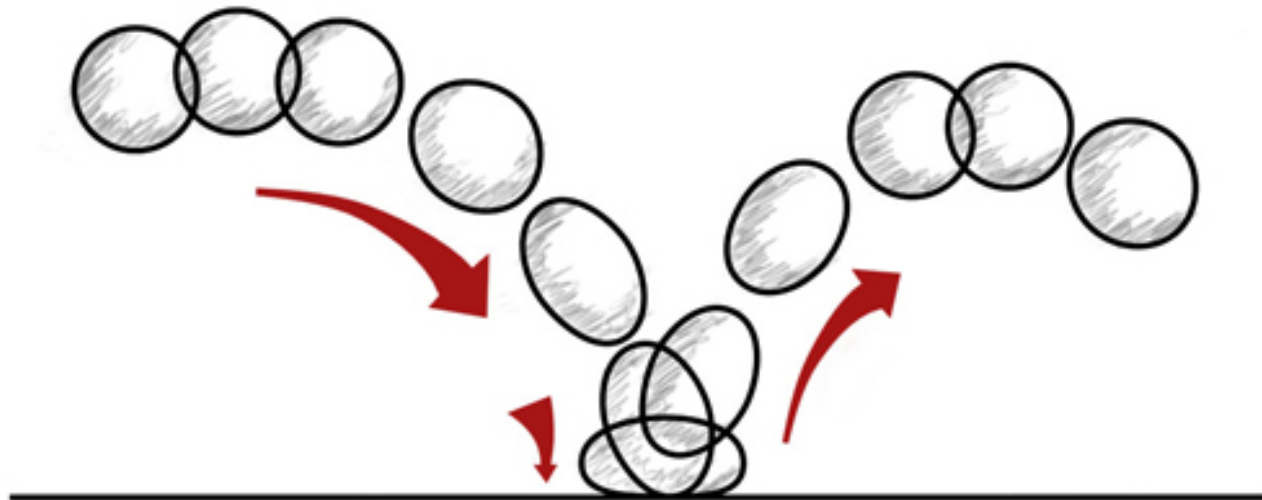
- Animation is not about exactly modeling reality
- Exaggeration is very often used for emphasis



[Michael B. Comet]

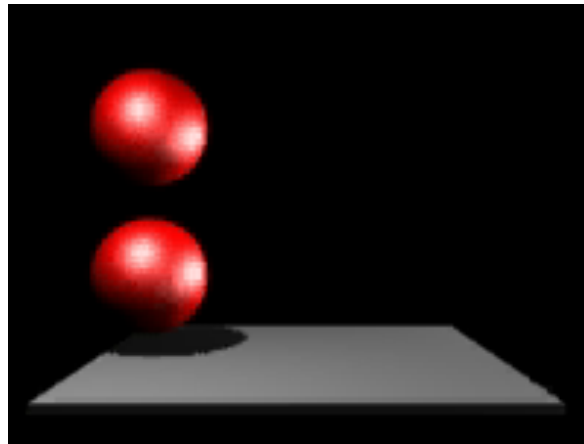
Animation principles: squash & stretch

- Objects do not remain perfectly rigid as they move
- Adding stretch with motion and squash with impact:
models deformation of soft objects
indicates motion by simulating exaggerated “motion blur”



Animation principles: follow through

- We've seen that objects don't start suddenly
- They also don't stop on a dime



[Michael B. Comet]

[Michael B. Comet]



Anim. principles: overlapping action

- Usually many actions are happening at once



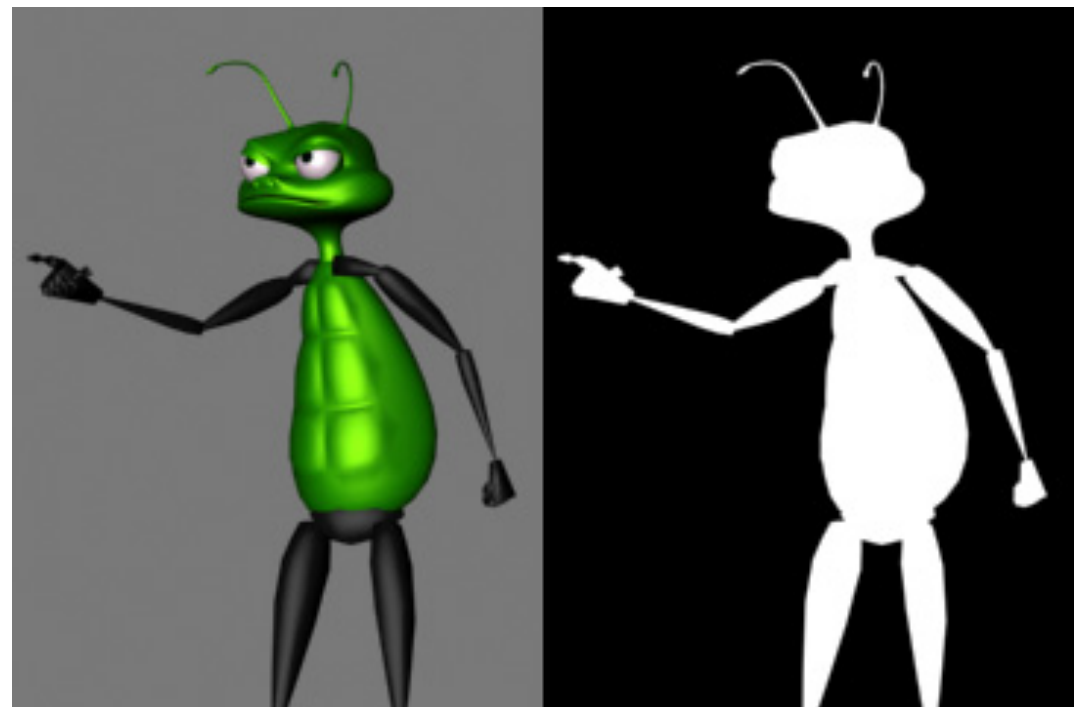
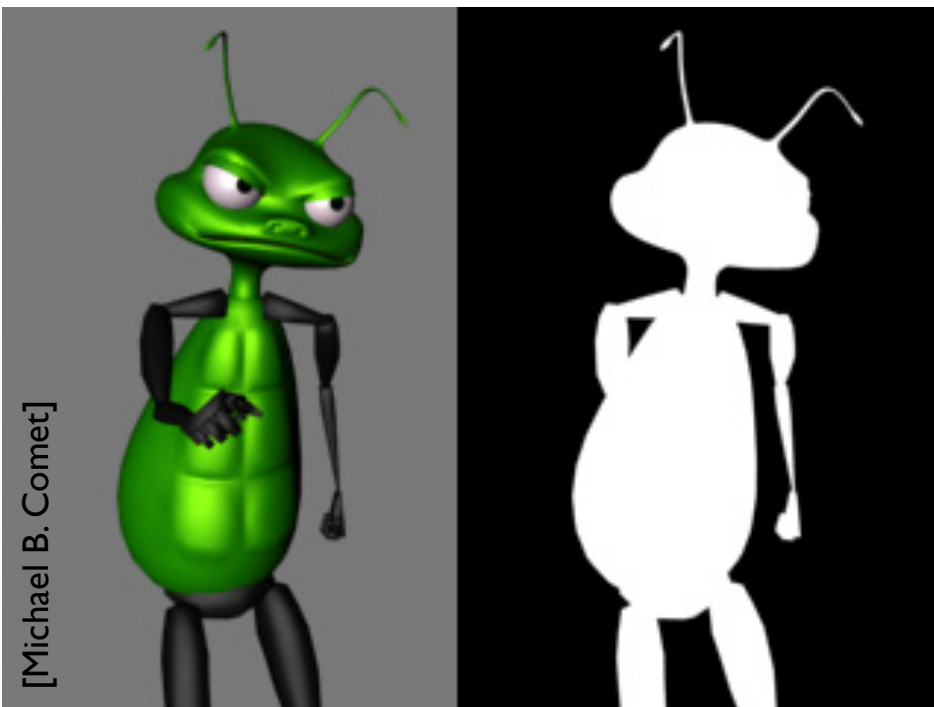
[Michael B. Comet]

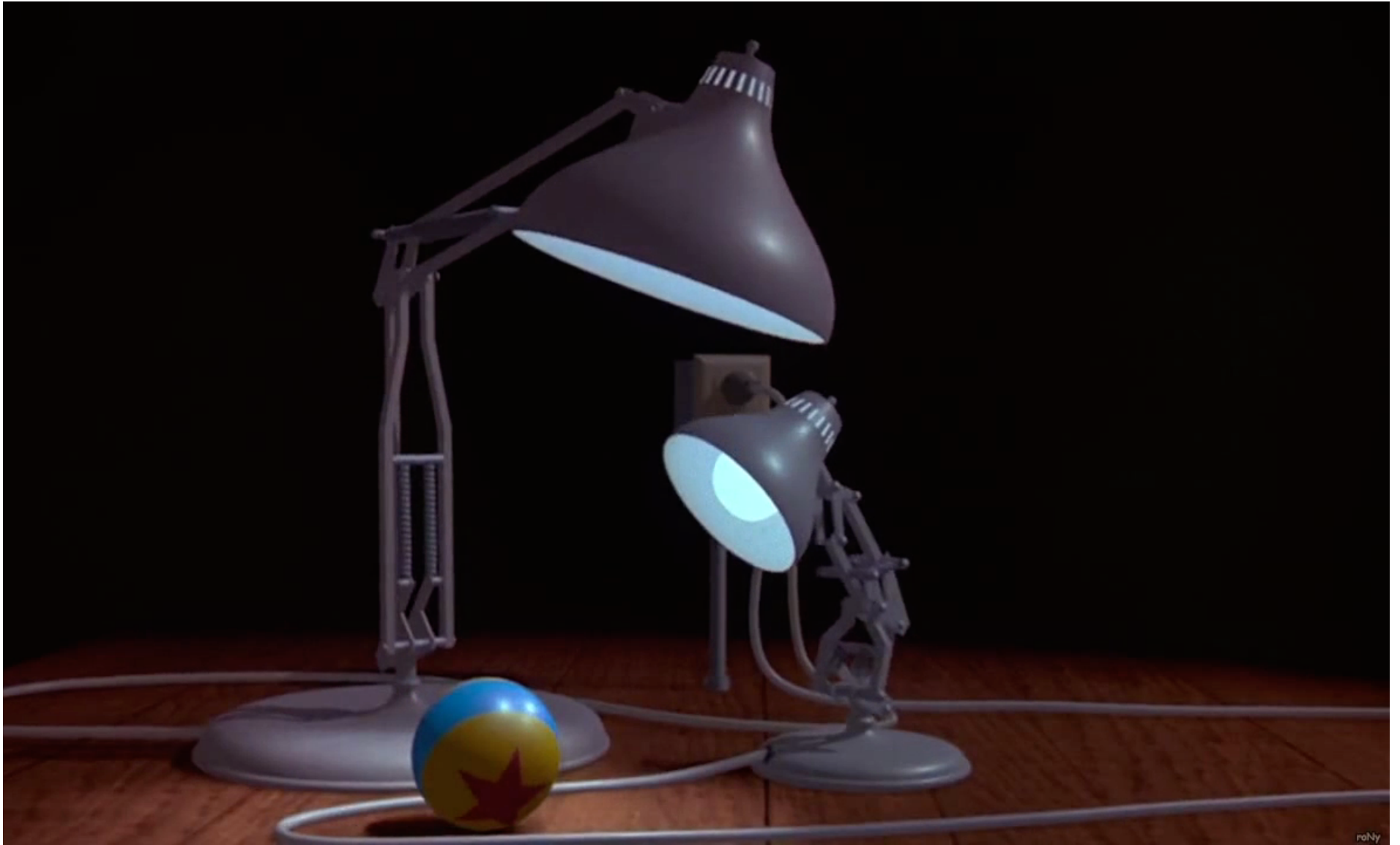


roNy

Animation principles: staging

- Want to produce clear, good-looking 2D images need good camera angles, set design, and character positions





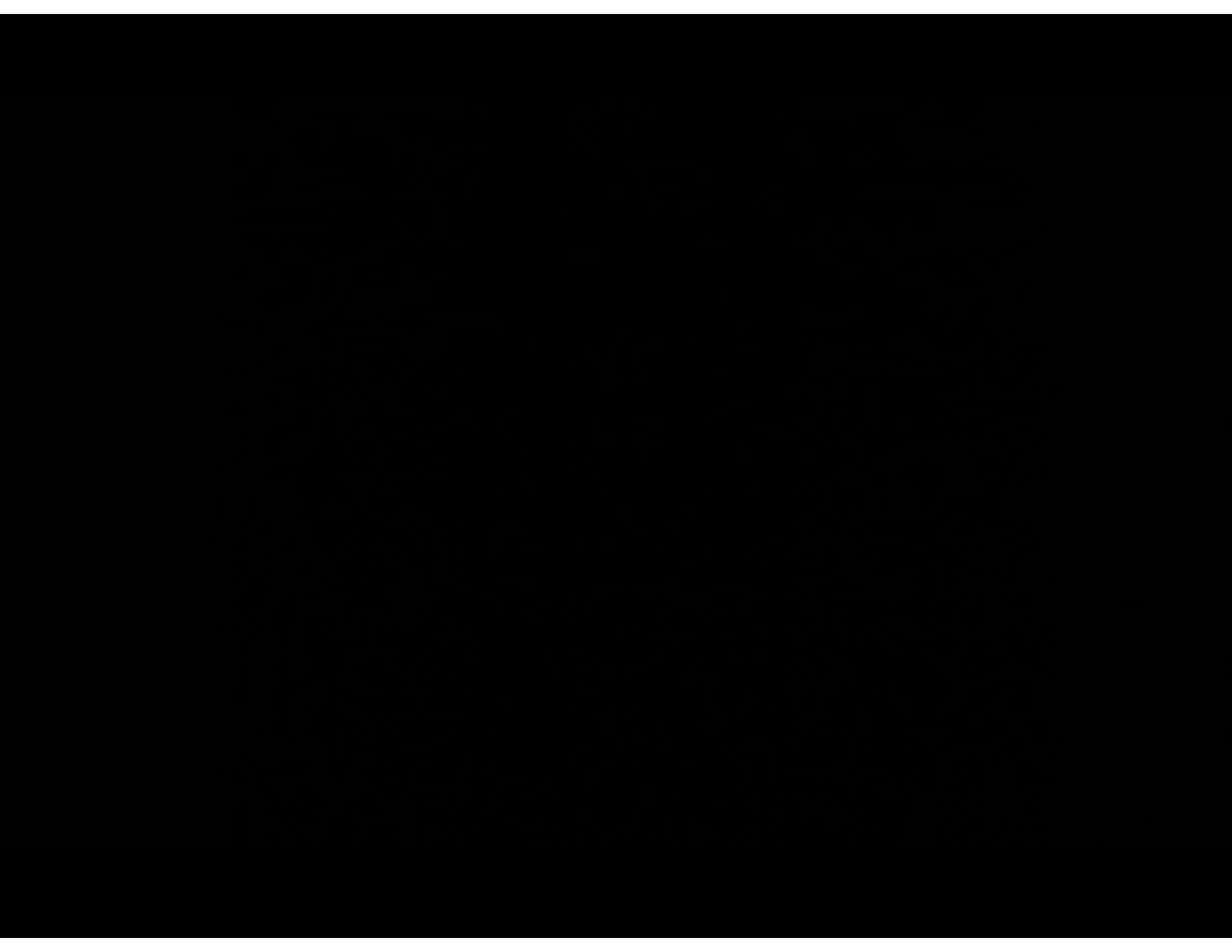
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Principles at work: weight



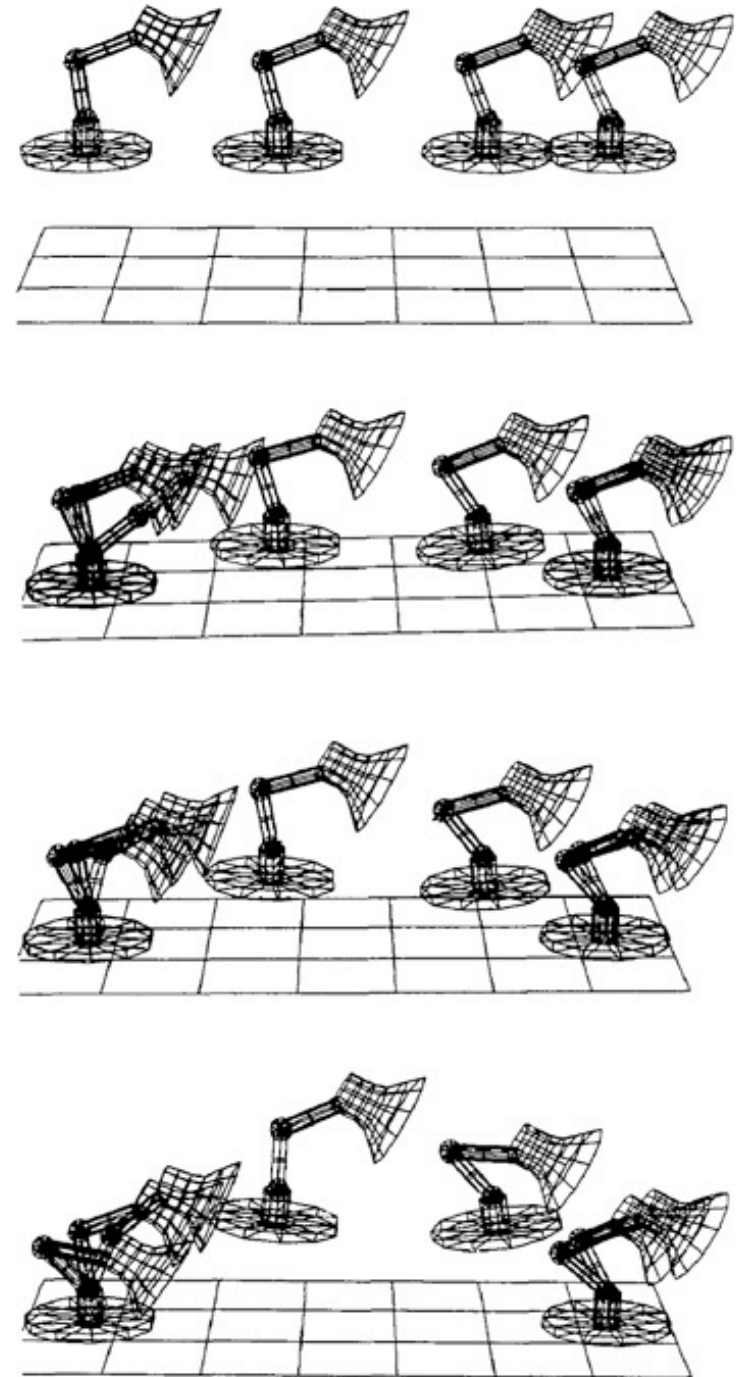
[Michael B. Comet]

Extended example: Luxo, Jr.



Computer-generated motion

- Interesting aside: many principles of character animation follow indirectly from physics
- Anticipation, follow-through, and many other effects can be produced by simply minimizing physical energy
- Seminal paper: “Spacetime Constraints” by Witkin and Kass in SIGGRAPH 1988





Forward Kinematics

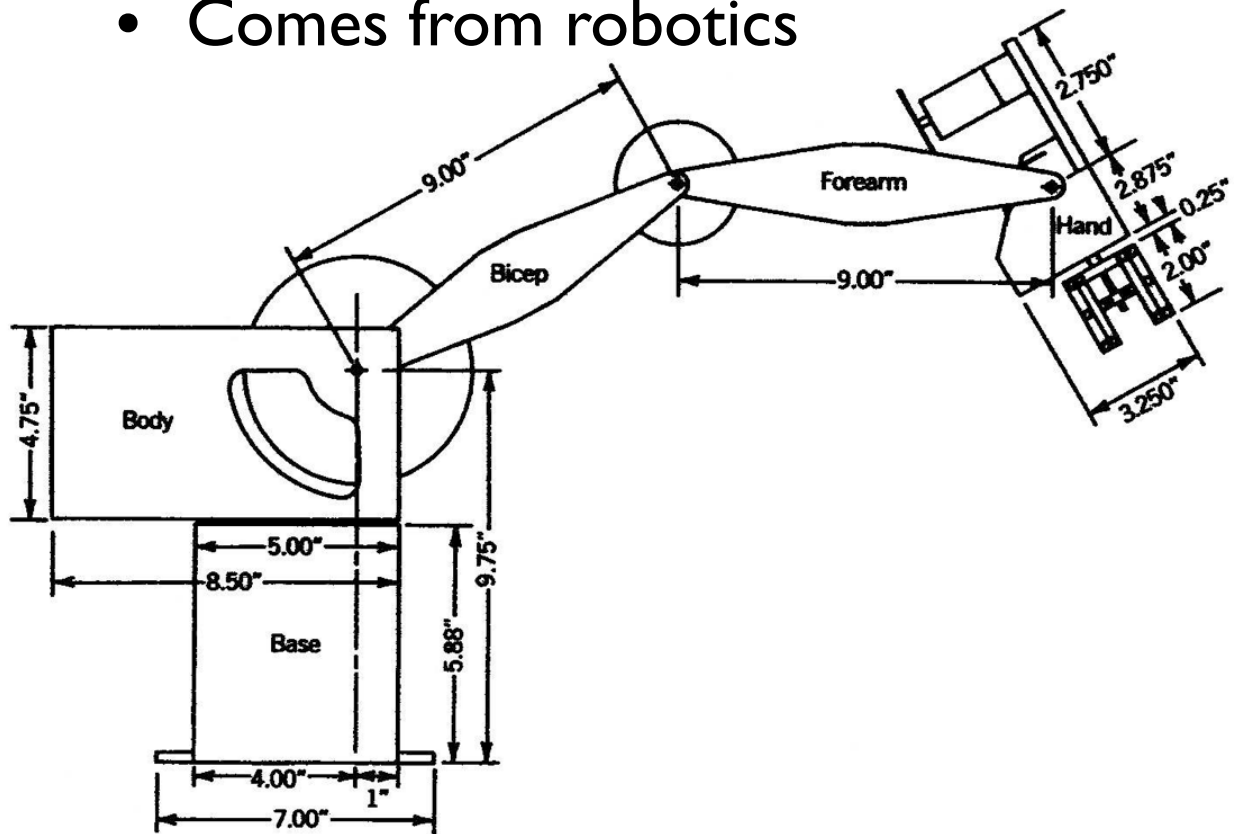


Inverse Kinematics

- Forward kinematics
 - Describe positions of body parts as fn of joint angles
 - Body parts: bones
- Inverse kinematics
 - Constrain locations for bones and solve for joint angles

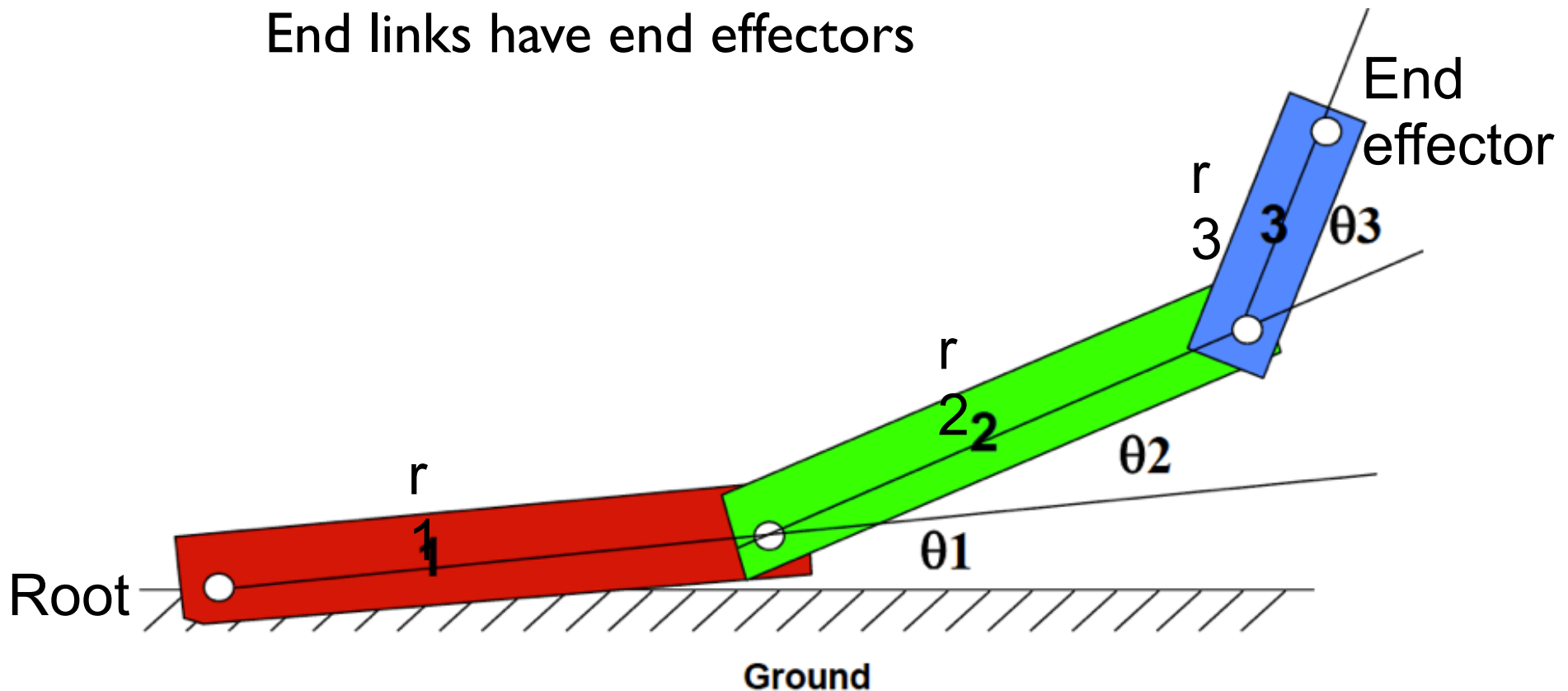
Forward Kinematics

- Articulated body
 - Hierarchical transforms
 - Comes from robotics



Rigid Links and Joint Structure

- Links connected by joints
 - Joints are purely rotational (single DOF)
 - Links form a tree (no loops)
 - End links have end effectors

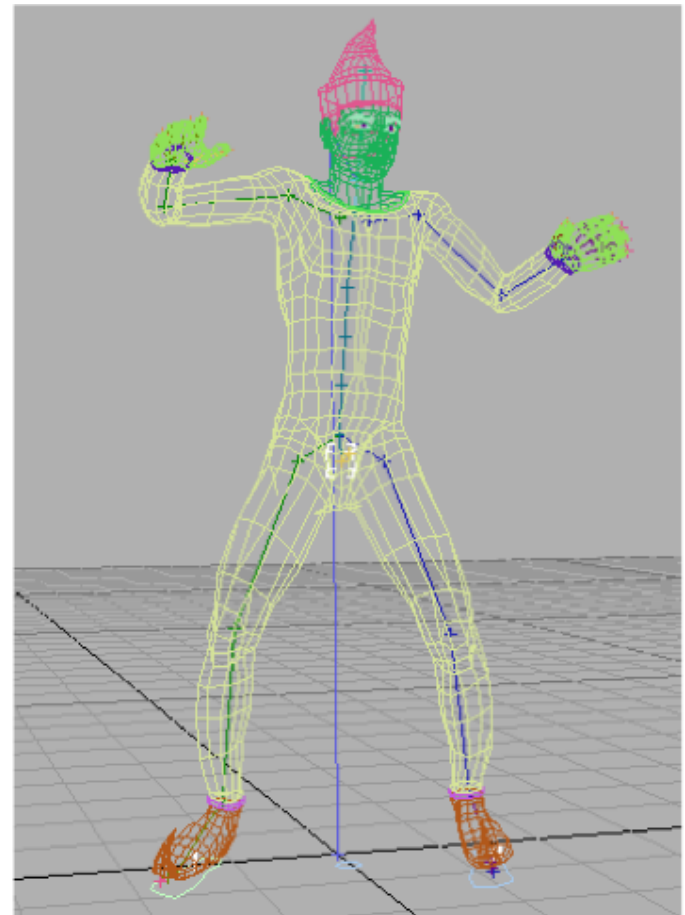
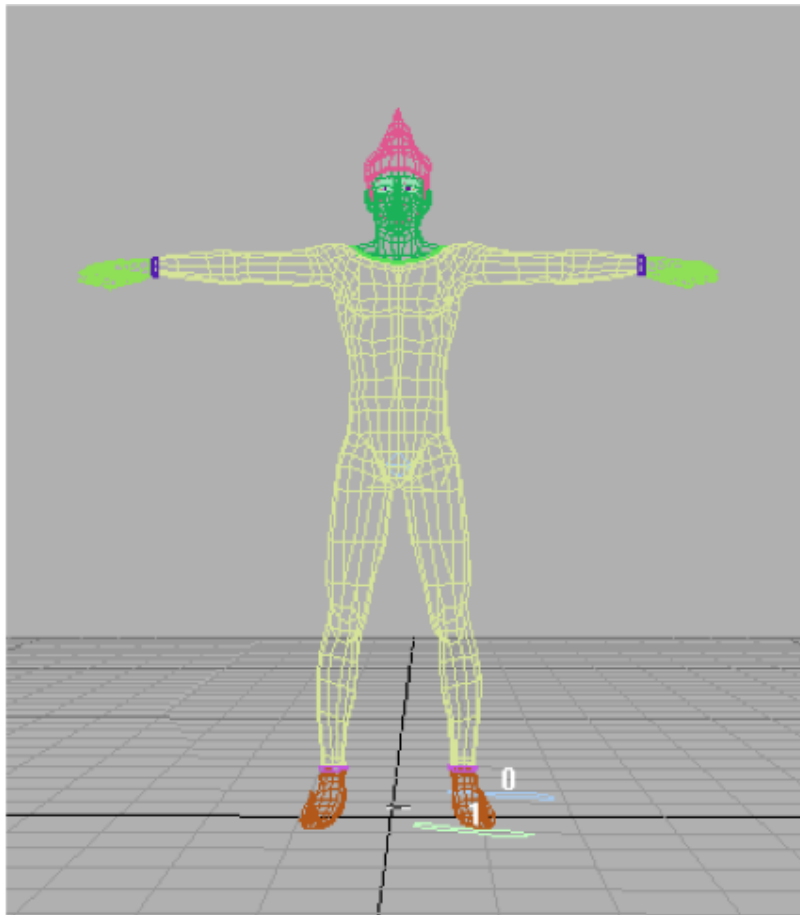


Basic surface deformation methods

- Mesh skinning: deform a mesh based on an underlying skeleton
- Blend shapes: make a mesh by combining several meshes
- Both use simple linear algebra
 - Easy to implement—first thing to try
 - Fast to run—used in games
- The simplest tools in the offline animation toolbox

Mesh skinning

- A simple way to deform a surface to follow a skeleton



[Sébastien Dominé | NVIDIA]

Skinning

- Embed a skeleton into a character mesh
- Animate “bones”
 - Change joint angles over time
 - Key framing, etc.
- Bind skin vertices to bones
 - Animate skeleton
 - Skin will move with it

Mesh skinning math: setup

- Surface has control points p_i
 - Triangle vertices, spline control points, subdiv base vertices
- Each bone has a transformation matrix M_j
 - Normally a rigid motion
- Every point–bone pair has a weight w_{ij}
 - In practice only nonzero for small # of nearby bones
 - The weights are provided by the user



Colored tris
attached to one
bone

Black to $>$ one bone

James & Twigg, Skinning Mesh Animations, 2005, used with permission from ACM, Inc.

Mesh skinning math

- Deformed position of a point is a weighted sum of the positions determined by each bone's transform alone
weighted by that vertex's weight for that bone
 w_{ij} : How much should vertex i move with bone j

$$\mathbf{p}'_i = \sum_j w_{ij} M_j \mathbf{p}_i$$

Mesh skinning

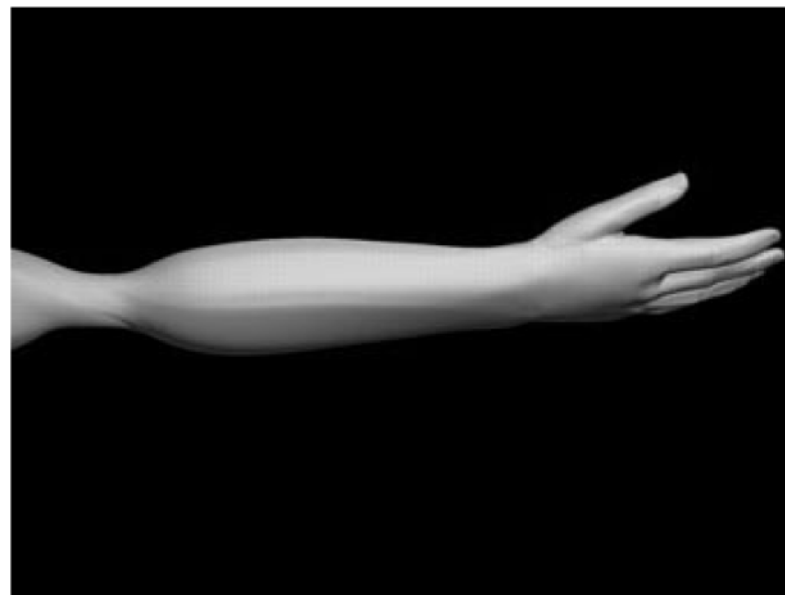
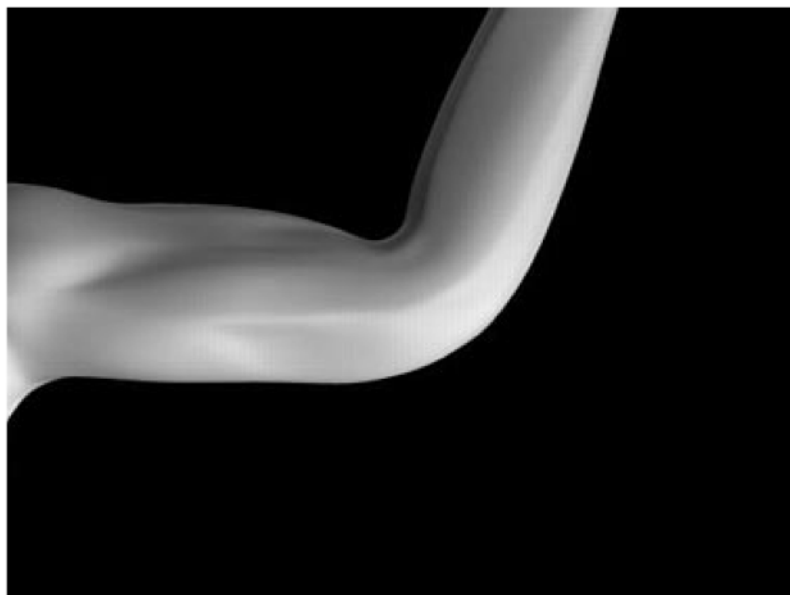
- Simple and fast to compute
 - Can even compute in the vertex stage of a graphics pipeline
- Used heavily in games
- One piece of the toolbox for offline animation
 - Many other deformers also available

Mesh skinning: classic problems

- Surface collapses on the inside of bends and in the presence of strong twists

Average of two rotations is not a rotation!

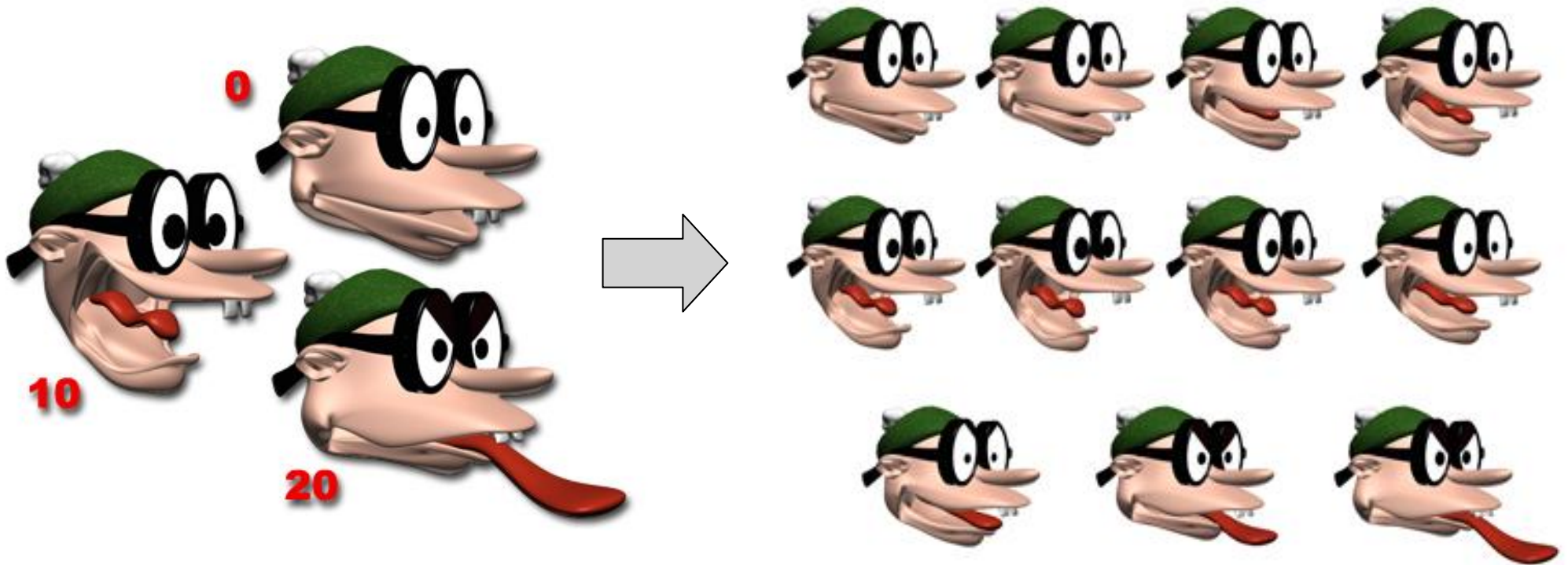
Add more bones to keep adjacent bones from being too different, or change the blending rules.



[Lewis et al. SIGGRAPH 2000]

Blend shapes

- Another very simple surface control scheme
- Based on interpolating among several key poses
Aka. blend shapes or morph targets



[3D Studio Max example]

Blend shapes math

- Simple setup

User provides key shapes—that is, a position for every control point in every shape: \mathbf{p}_{ij} for point i , shape j

Per frame: user provides a weight w_j for each key shape

- Must sum to 1.0

- Computation of deformed shape

$$\mathbf{p}'_i = \sum_j w_j \mathbf{p}_{ij}$$

- Works well for relatively small motions

Often used for facial animation

Runs in real time; popular for games

Animation

- Key frame
- Motion capture
- Physics-based