Subdivision Surfaces

CS 4620 Lecture 31

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Administration

- A5 due on Friday
- Dreamworks visiting Thu/Fri
- Rest of class
  - Surfaces, Animation, Rendering

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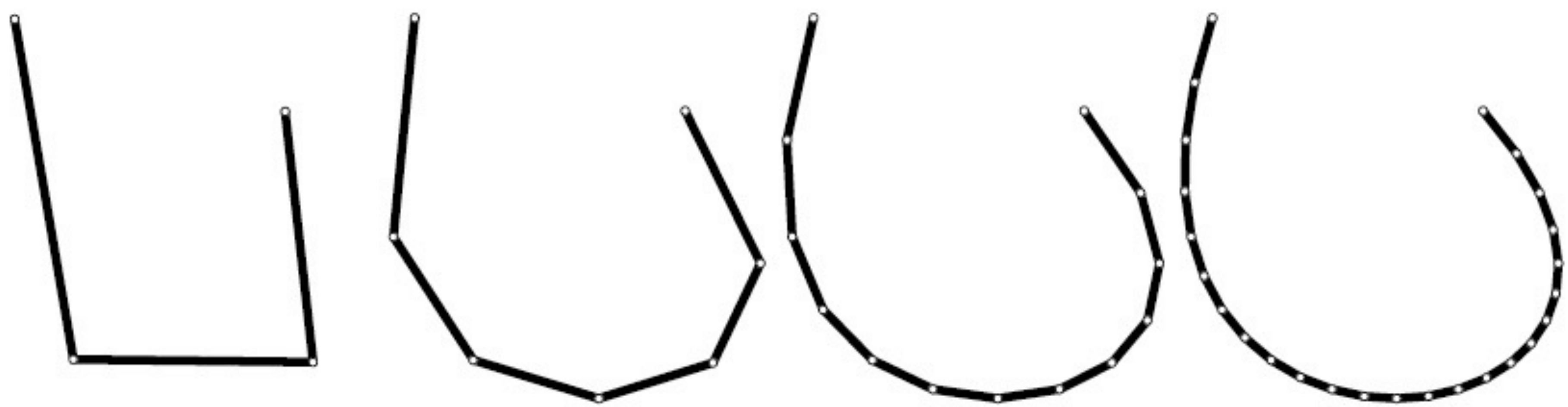
Subdivision curves

- Key idea: let go of the polynomials as the definition of the curve, and let the refinement rule define the curve
- Curve is defined as the *limit of a refinement process*
  – properties of curve depend on the rules
  – some rules make polynomial curves, some don't
  – complexity shifts from implementations to proofs



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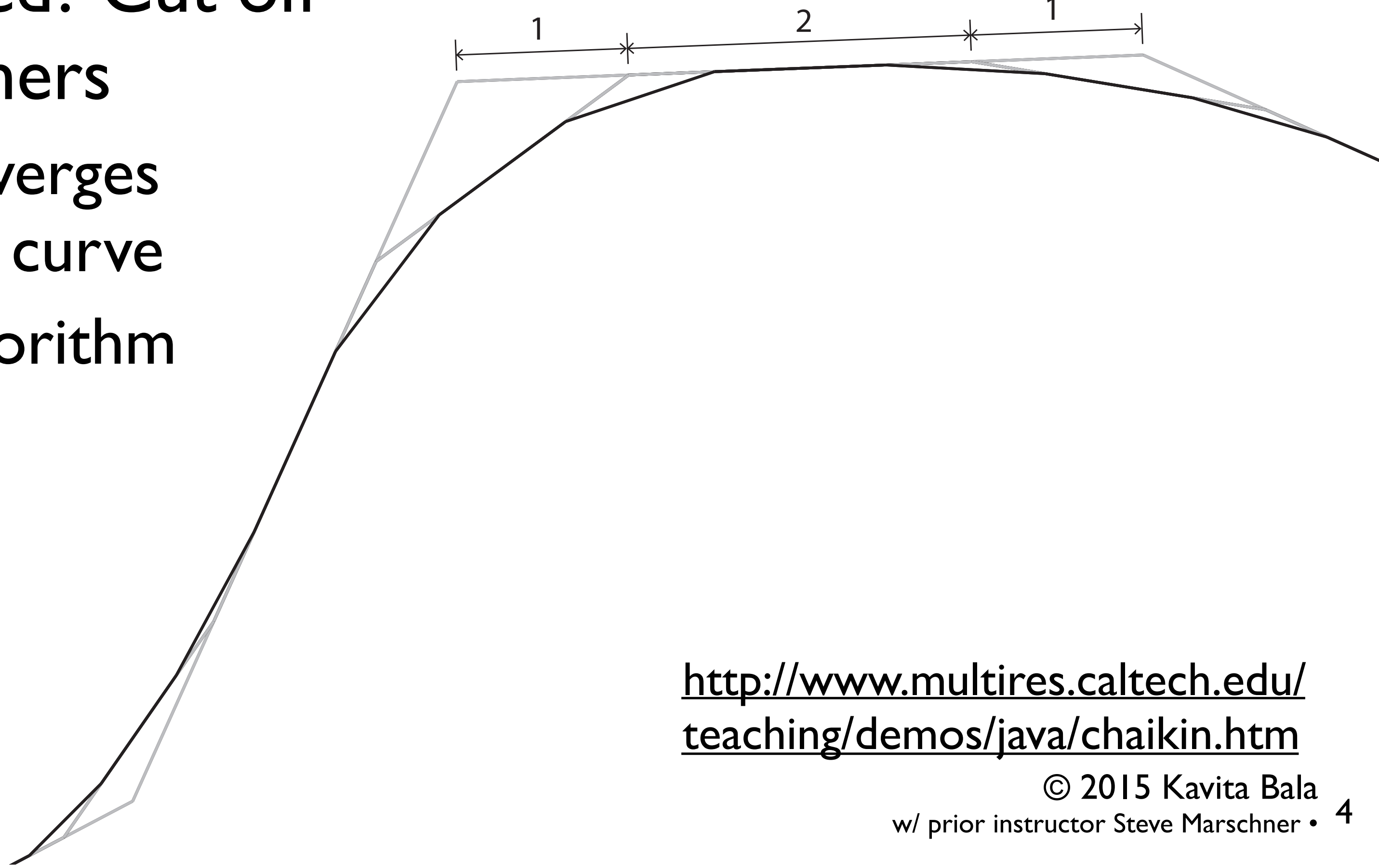
Introduction: corner cutting

- Piecewise linear curve too jagged for you? Lop off the corners!
  - results in a curve with twice as many corners
- Still too jagged? Cut off the new corners
  - process converges to a smooth curve
  - Chaikin's algorithm



http://www.multires.caltech.edu/teaching/demos/java/chaikin.htm

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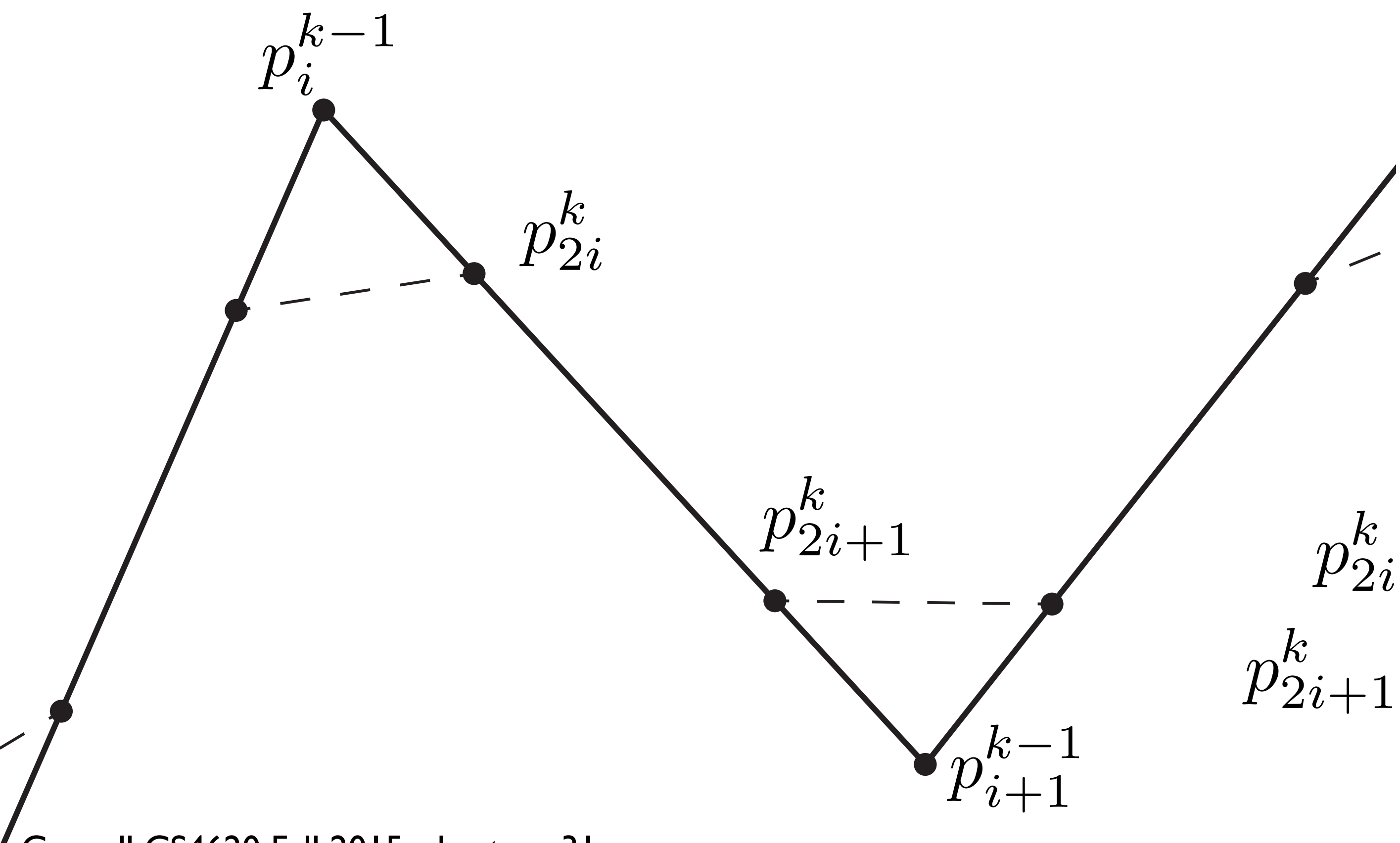
Corner cutting in equations

- New points are linear combinations of old ones
- Different treatment for odd-numbered and even-numbered points.



$$p_{2i}^k = (3p_i^{k-1} + p_{i+1}^{k-1})/4$$

$$p_{2i+1}^k = (p_i^{k-1} + 3p_{i+1}^{k-1})/4$$

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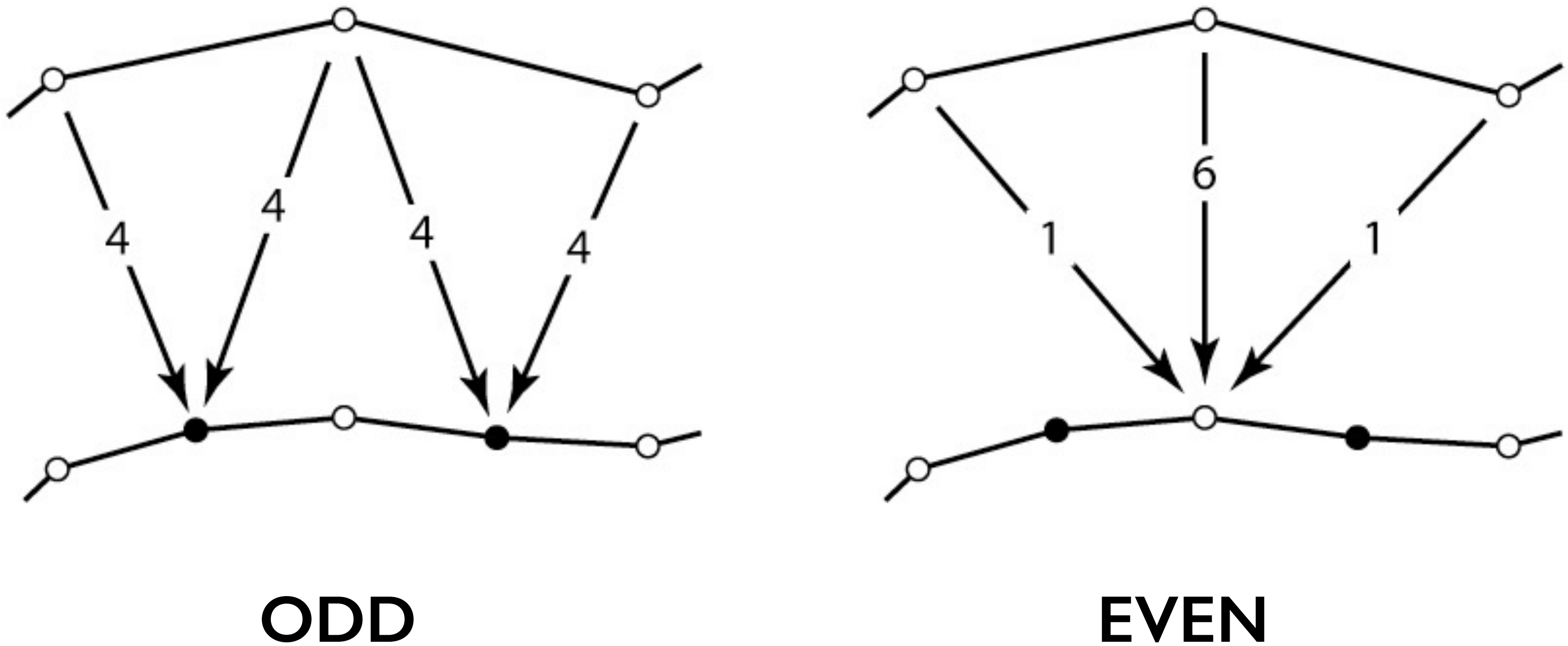
Subdivision for B-splines

- Control vertices of refined spline are linear combinations of the c.v.s of the coarse spline



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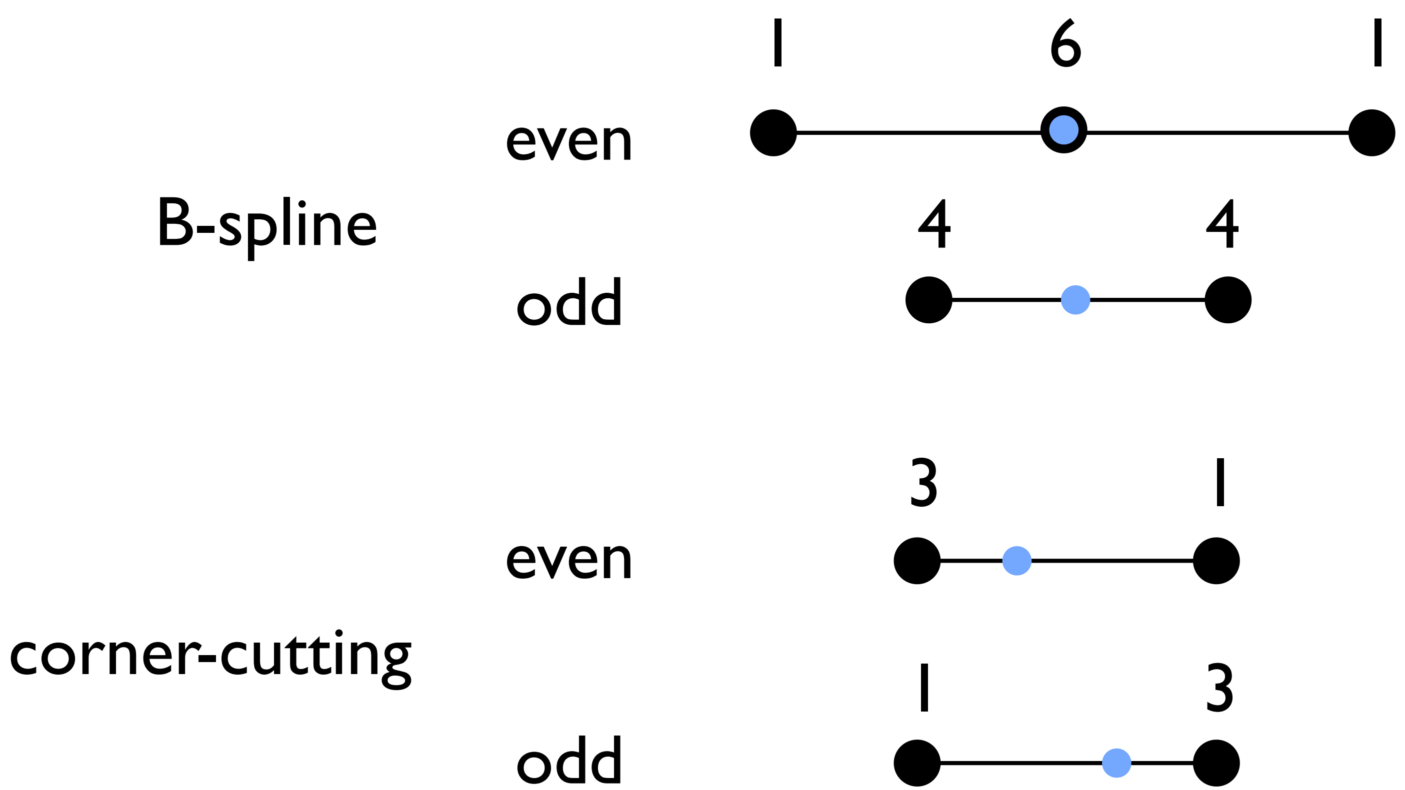
Drawing a picture of the rule

- Conventionally illustrate subdivision rules as a "mask" that you match against the neighborhood
  – often implied denominator = sum of weights



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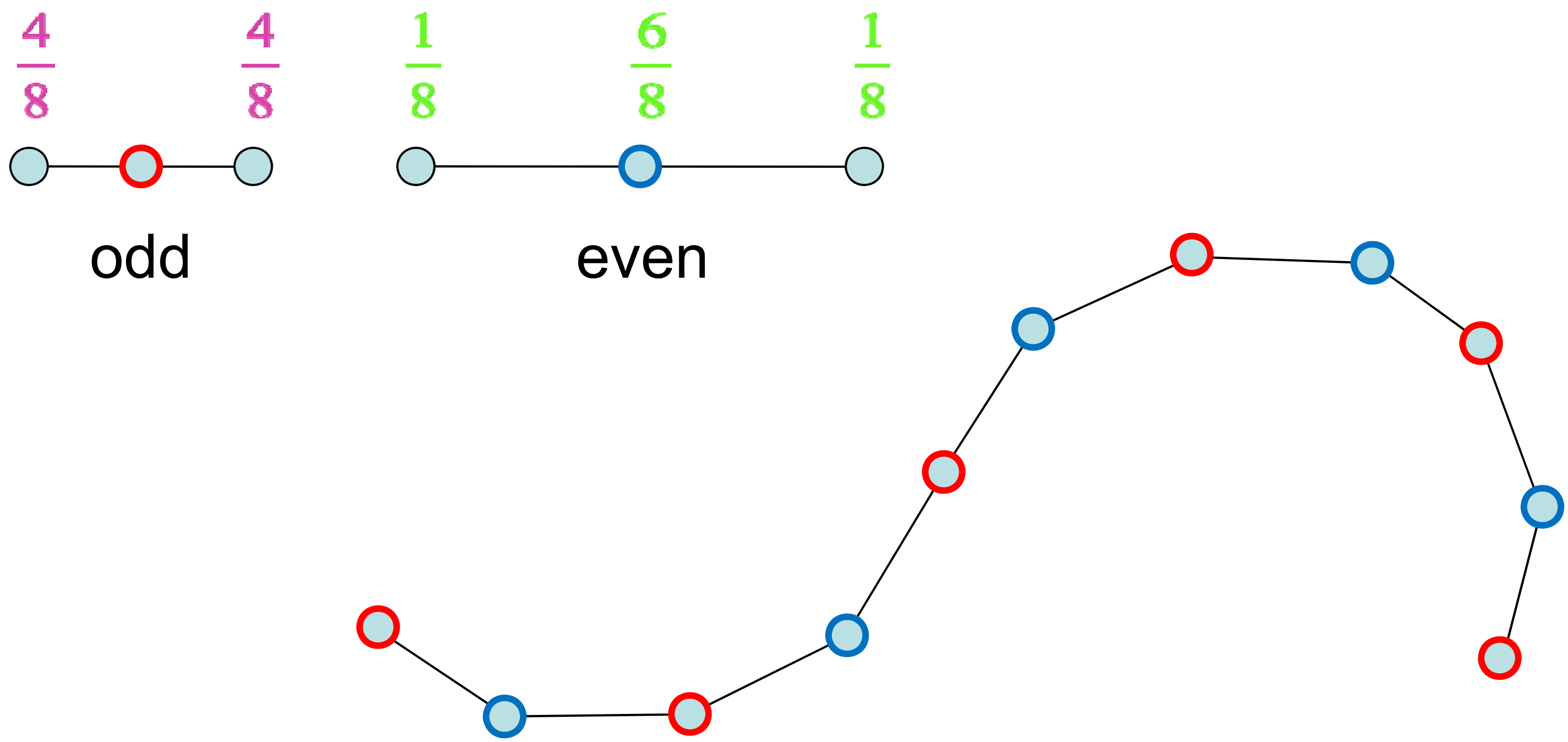
Cubic B-Spline



[Stanford CS468 Fall 2010 slides]

Playing with the rules

- Once a curve is *defined* using subdivision we can customize its behavior by making exceptions to the rules.
- Example: handle endpoints differently
- Resulting curve *is* a uniform B-spline in the middle, but near the exceptional points it is something different.
  - it might not be a polynomial
  - but it is still linear, still has basis functions
  - the three coordinates of a surface point are still separate

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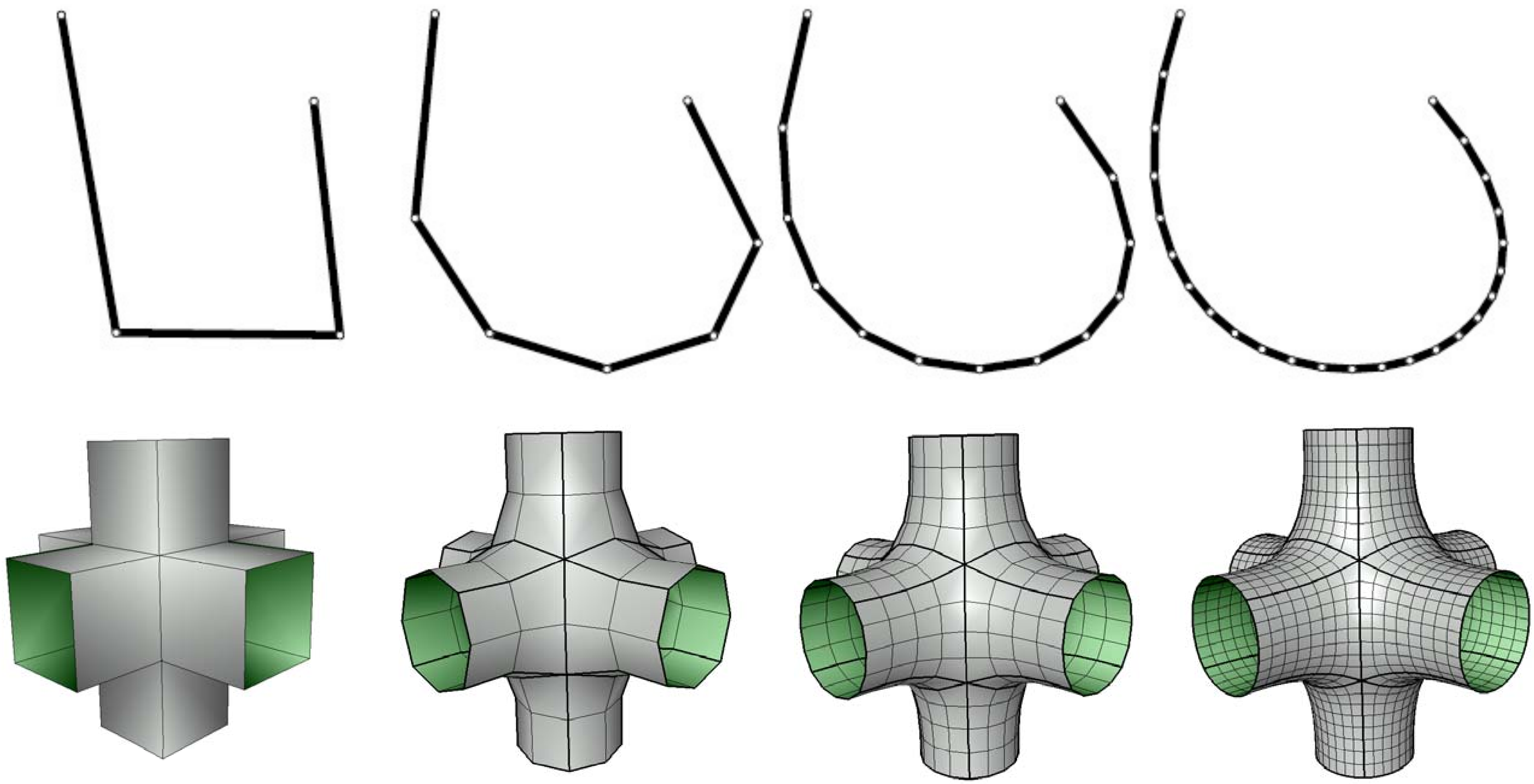
From curves to surfaces



[Schröder & Zorin SIGGRAPH 2000 course 23]

[Stanford CS468 Fall 2010 slides]

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Subdivision surfaces

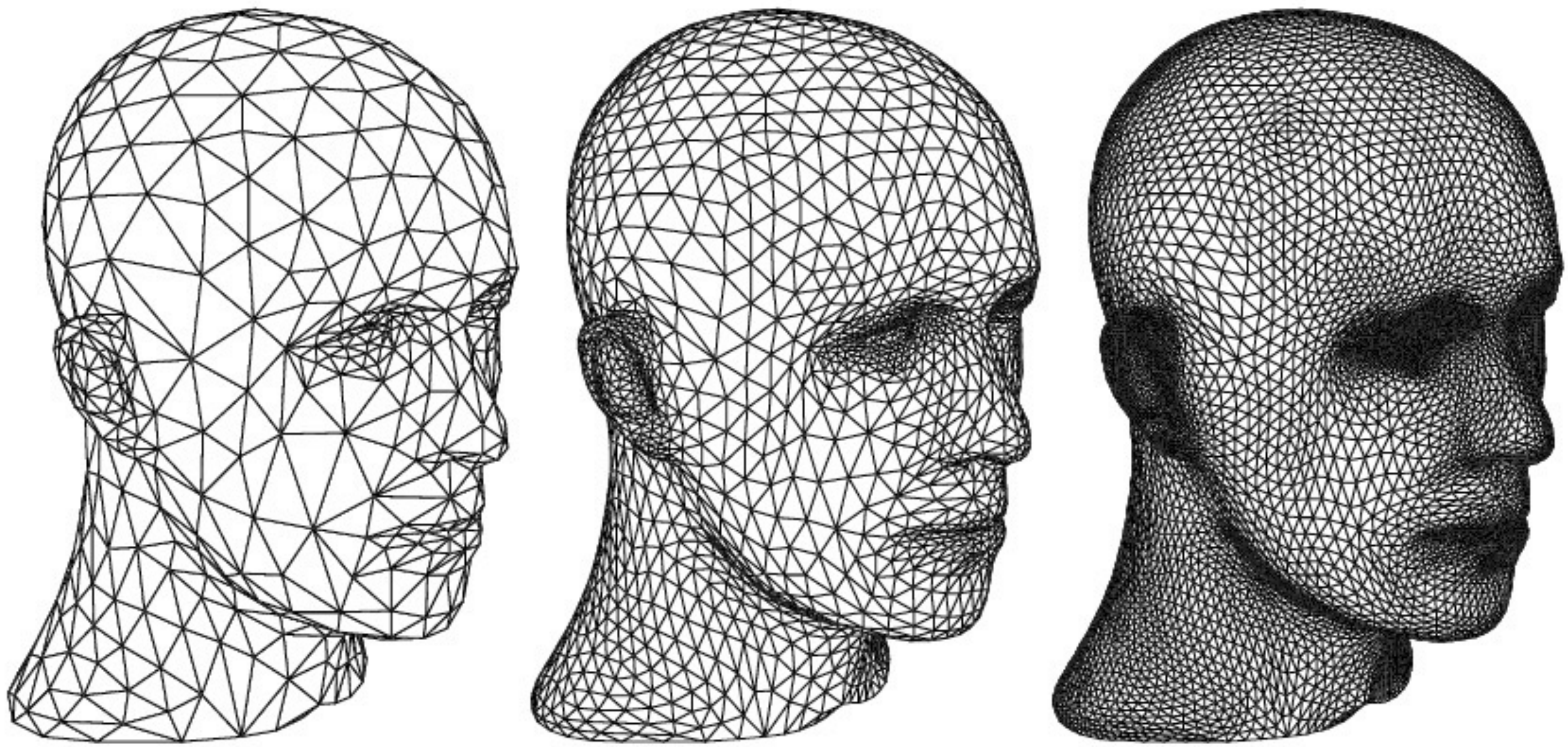

Figure 2.2: *Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.*

[Schröder & Zorin SIGGRAPH 2000 course 23]

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Generalizing from curves to surfaces

- Two parts to subdivision process
- Subdividing the mesh (computing new topology)
  - For curves: replace every segment with two segments
  - For surfaces: replace every face with some new faces
- Positioning the vertices (computing new geometry)
  - For curves: two rules (one for *odd* vertices, one for *even*)
    - New vertex's position is a weighted average of positions of old vertices that are nearby along the sequence
  - For surfaces: two kinds of rules (still called odd and even)
    - New vertex's position is a weighted average of positions of old vertices that are nearby in the mesh

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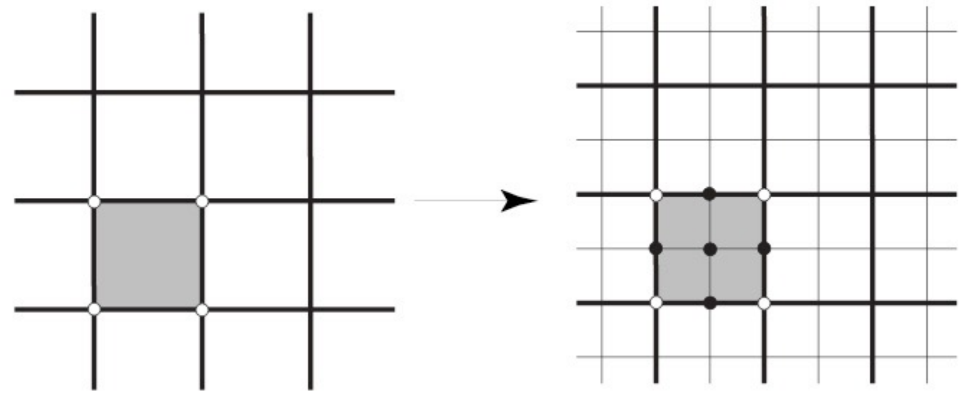
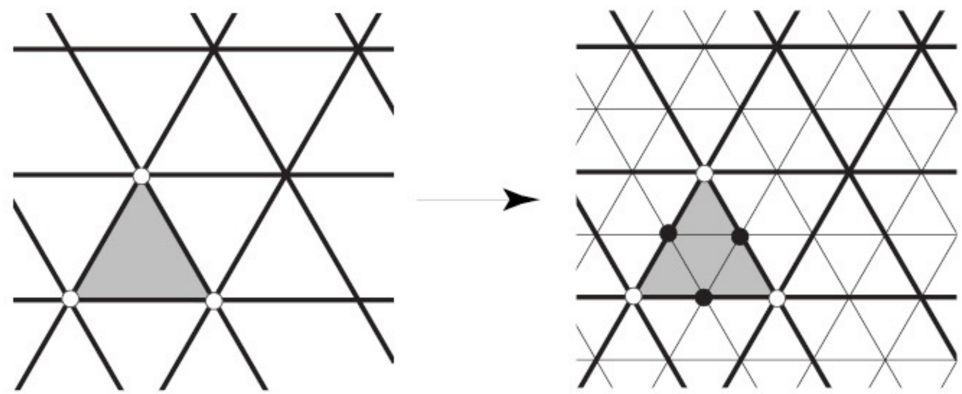
Subdivision of meshes

- Quadrilaterals
  – Catmull-Clark 1978
- Triangles
  – Loop 1987



Face split for quads



Face split for triangles

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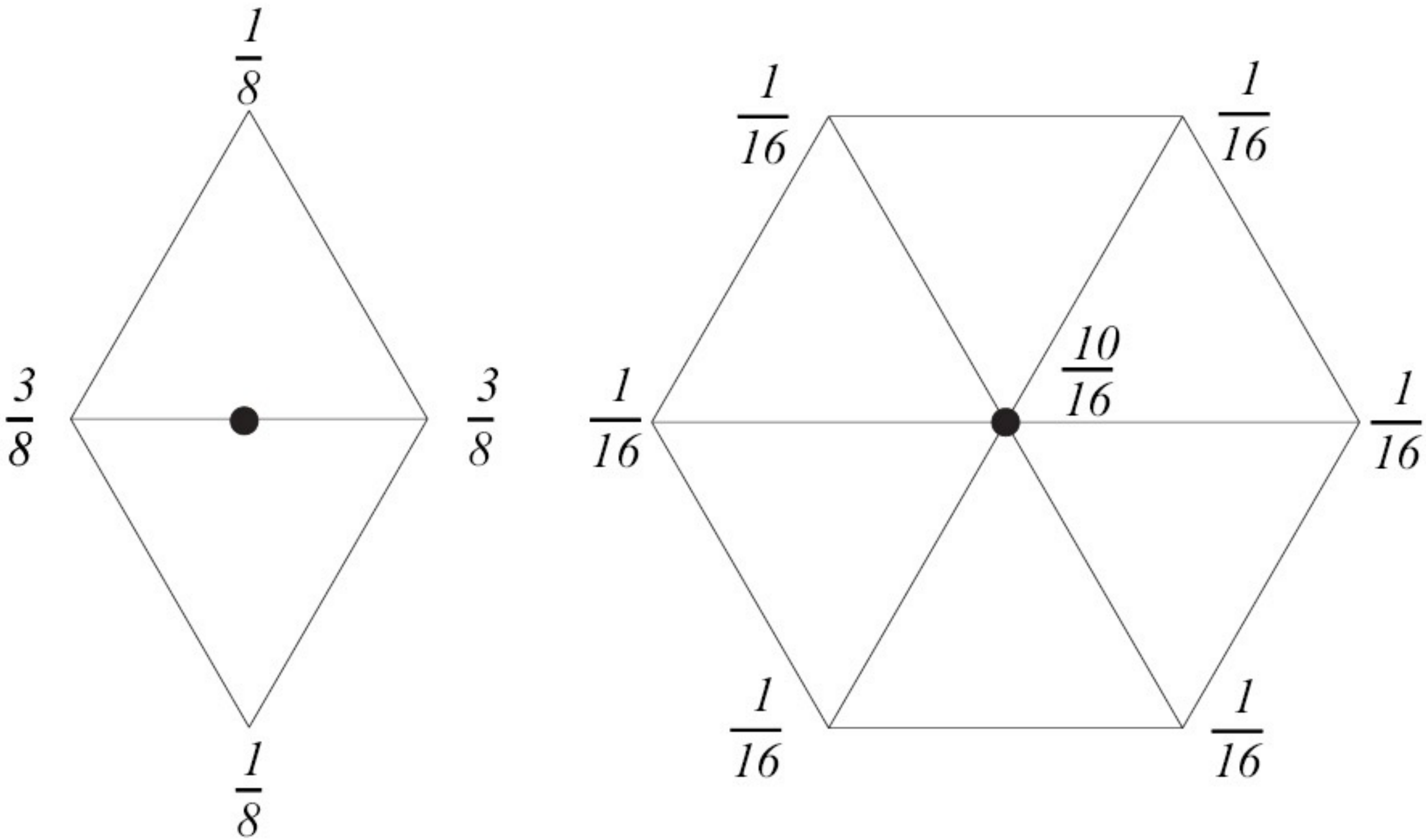
Loop regular rules



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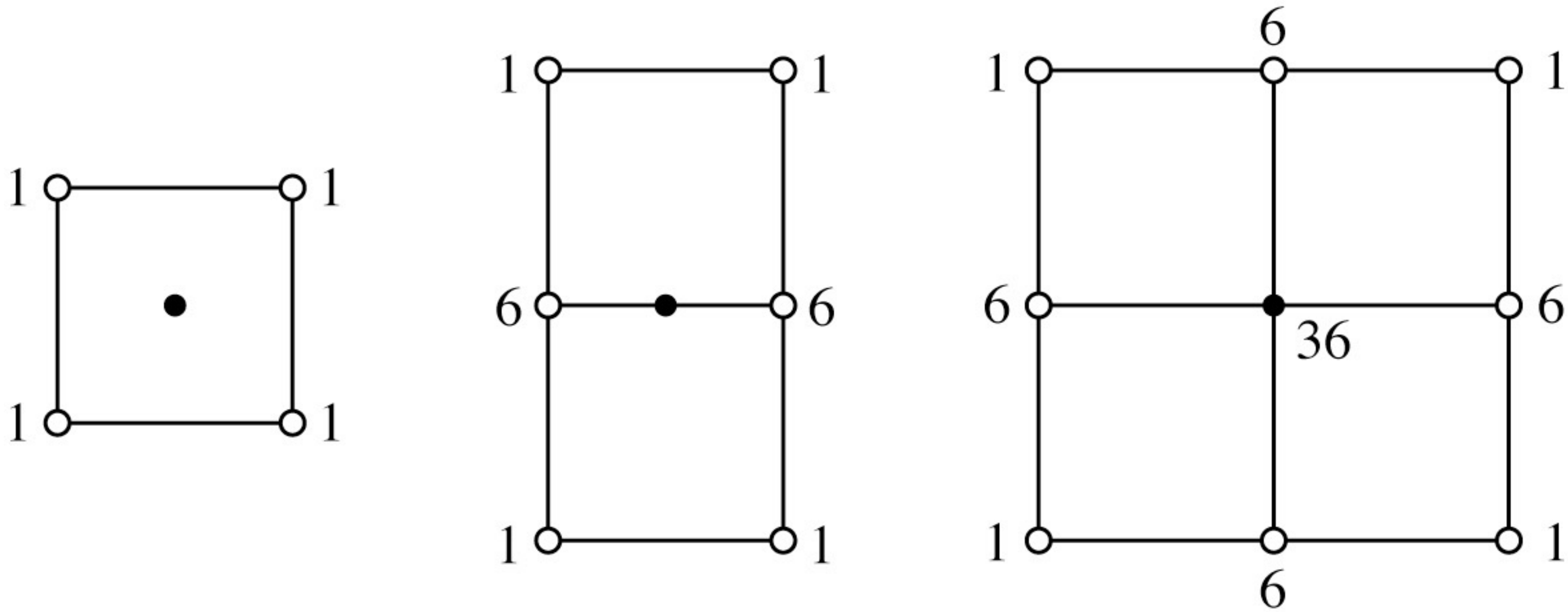
Catmull-Clark regular rules



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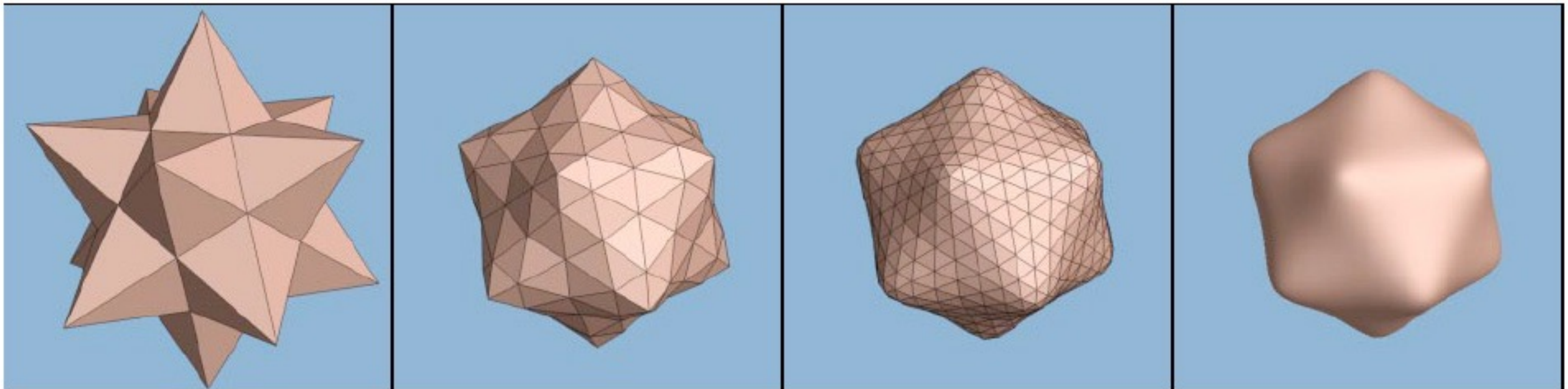
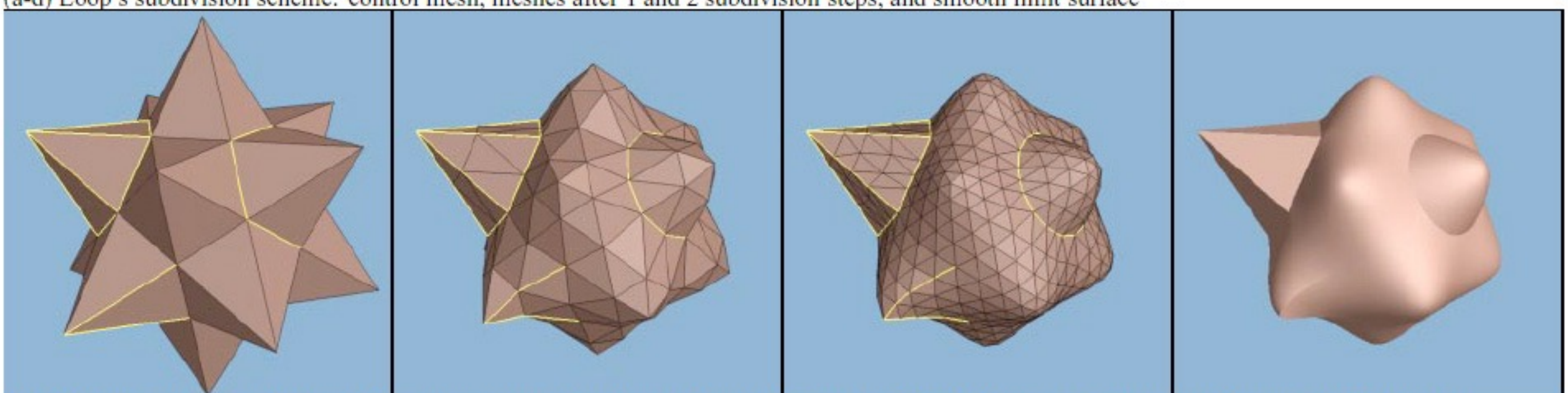
Loop with creases



(a-d) Loop's subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface



(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface

[Hugues Hoppe]

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Geri's Game

- Pixar short film to test subdivision in production
  - Catmull-Clark (quad mesh) surfaces
  - complex geometry
  - extensive use of creases
  - subdivision surfaces to support cloth dynamics

[DeRose et al. SIGGRAPH 1998]

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Representing geometry

- Volumes
  - CSG (Constructive Solid Geometry)
    - apply boolean operations on solids
    - simple to define
    - simple to compute in some cases
      - [e.g. ray tracing, implicit surfaces]

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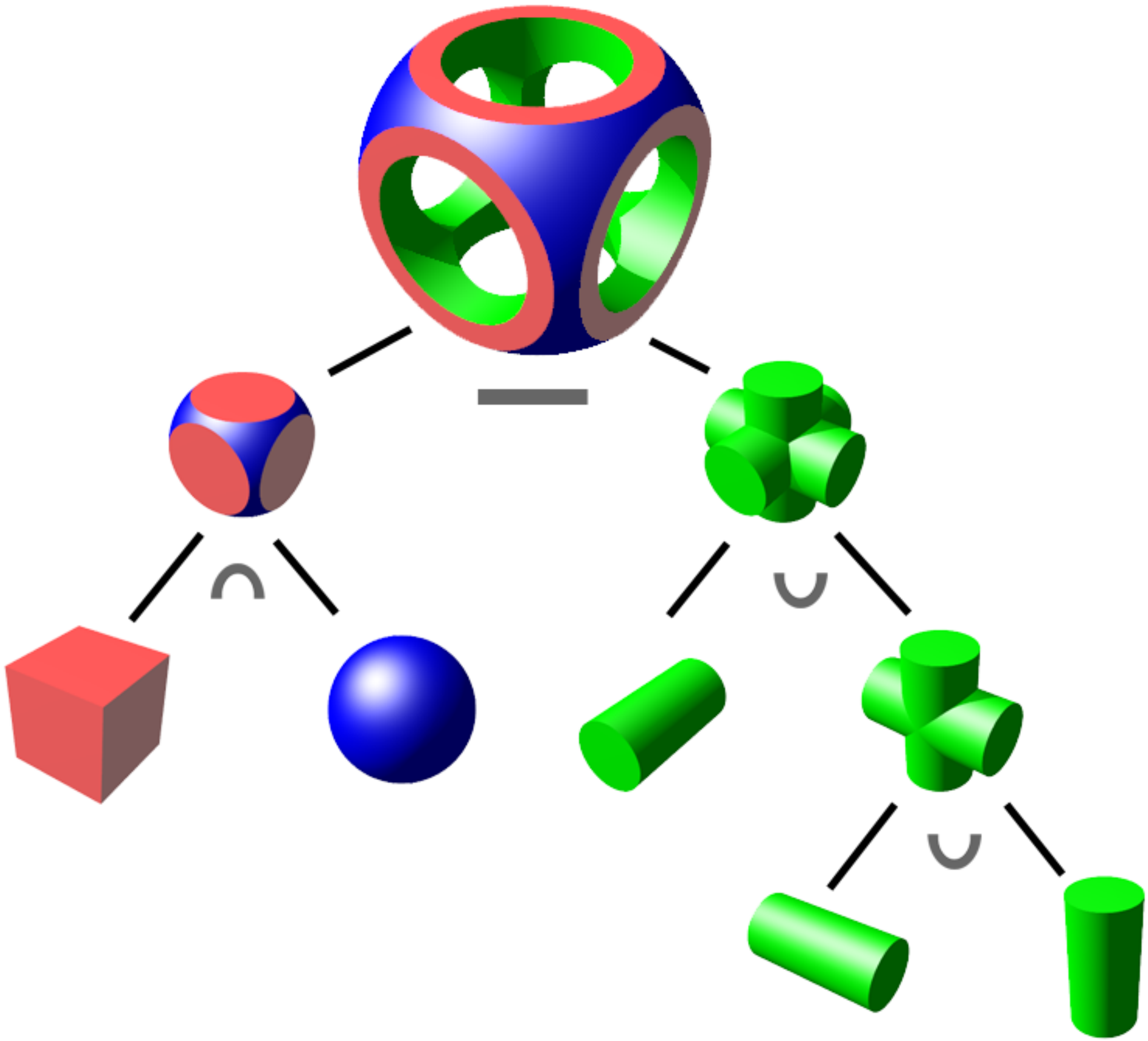
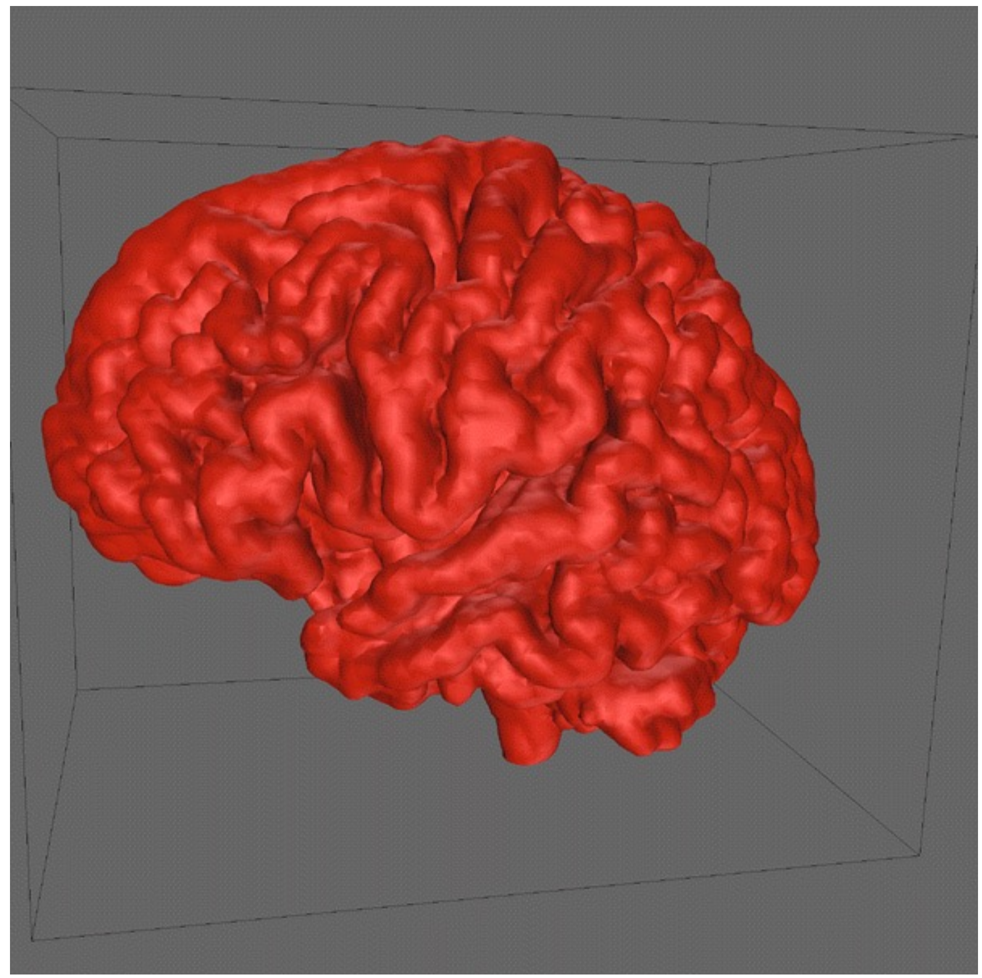


Specific surface representations

- Isosurface of volume data
  - implicit representation
  - function defined by regular samples on a 3D grid
    - (like an image but in 3D)
  - example uses:
    - medical imaging
    - numerical simulation



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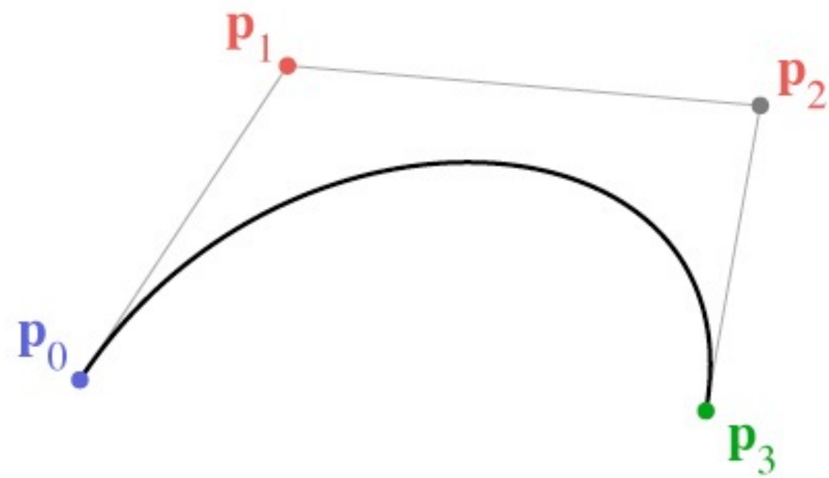
Modeling



- Curves
- Surfaces
- Volumes

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Matrix form of spline

$$\mathbf{f}(t) = \mathbf{a}t^3 + \mathbf{b}t^2 + \mathbf{c}t + \mathbf{d}$$

$$\begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix}$$

$$\mathbf{f}(t) = b_0(t)\mathbf{p}_0 + b_1(t)\mathbf{p}_1 + b_2(t)\mathbf{p}_2 + b_3(t)\mathbf{p}_3$$

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How splines depend on their controls

- Each coordinate is separate
  - the function $x(t)$ is determined solely by the x coordinates of the control points
  - this means 1D, 2D, 3D, … curves are all really the same
- Spline curves are **linear** functions of their controls
  - moving a control point two inches to the right moves $x(t)$ twice as far as moving it by one inch

– $x(t)$, for fixed t, is a linear combination (weighted sum) of the controls' x coordinates

– $\mathbf{f}(t)$, for fixed t, is a linear combination (weighted sum) of the controls

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