3D Transformations and Perspective

CS 4620 Lecture 12

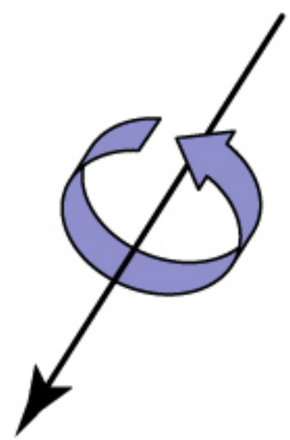
Announcements

- Demos on Monday
  - If you can't make it, send mail to cs4620-staff-l@cornell.edu
  - Post to piazza
- A3 out tonight
  - Written and code due on Thu before break
  - Grading will be after break
  - Start early
    - Hierarchies, transformations
- 4621 class today. PPA1 out tonight

General Rotation Matrices

- A rotation in 2D is around a point
- A rotation in 3D is around an axis
  - so 3D rotation is w.r.t a line, not just a point

2D



3D

Specifying Rotations

- Many ways to specify rotation
  - Indirectly through frame transformations
  - Directly through
    - Euler angles: 3 angles about 3 axes
    - (Axis, angle) rotation: based on Euler's theorem
    - Quaternions

04 08 49 39	Neil Armstrong (CDR)	Houston. Tranquility is going to put the track modes in P00 now.

. . .

04 08 59 27	CapCom	Columbia, Houston. Over.
04 08 59 34	Michael Collins (CMP)	Columbia. Go.
04 08 59 35	CapCom	Columbia, Houston. We noticed you are maneuvering very close to gimbal lock. I suggest you move back away. Over.
04 08 59 43	Michael Collins (CMP)	Yes. I am going around it, doing this CMC AUTO maneuvers to the PAD values of roll 270, pitch 101, yaw 45.
04 08 59 52	CapCom	Roger, Columbia.
04 09 00 30	Michael Collins (CMP)	How about sending me a fourth gimbal for Christmas.
04 09 00 40	CapCom	Columbia, Houston. You were unreadable. Say again please.
04 09 00 46	Michael Collins (CMP)	Disregard.

Building general rotations

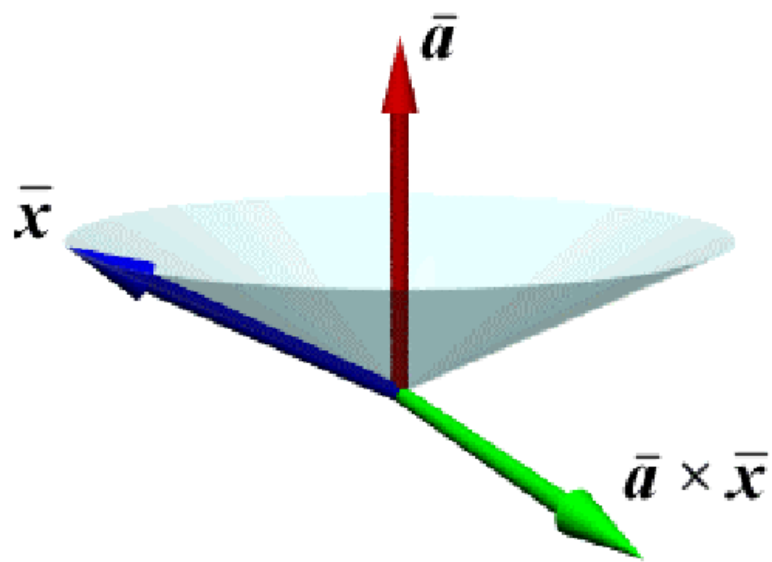
- Construct frame and change coordinates
  - choose *p*, *u*, *v*, *w* to be orthonormal frame with *p* and *u* matching the rotation axis
  - apply similarity transform $T = F\, R_x(\theta)\, F^{-1}$
  - interpretation: move to *x* axis, rotate, move back
  - interpretation: rewrite *u*-axis rotation in new coordinates
  - (each is equally valid)

$$\begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta) & -sin(\theta) \\ 0 & sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$$

  - (note above is linear transform; add affine coordinate)

Derivation of General Rotation Matrix

- Axis angle rotation

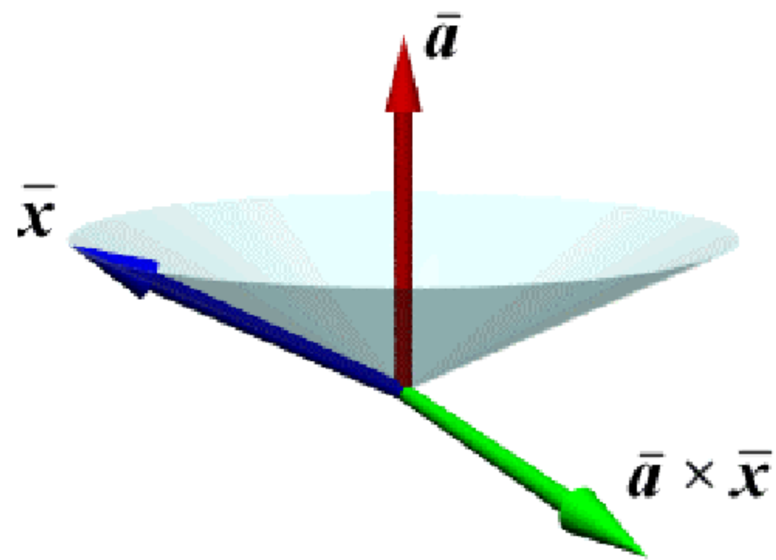


Axis-angle ONB

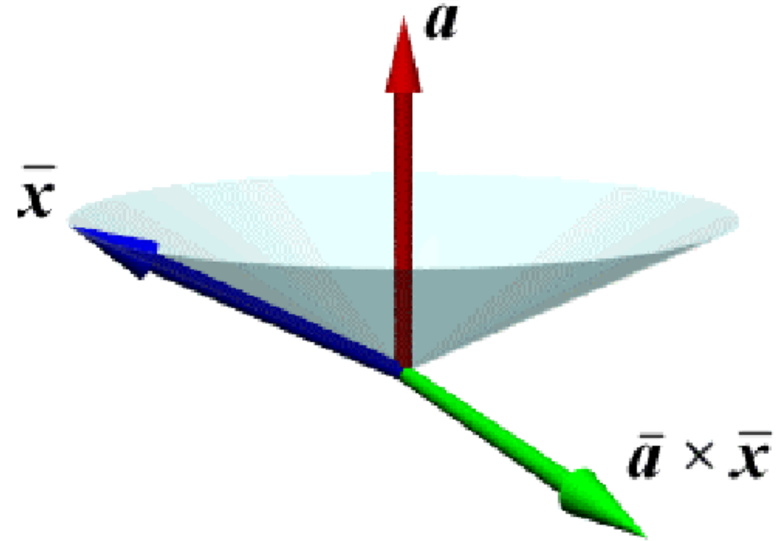
$$\vec{x}_{\parallel} = (\vec{a}.\vec{x})\vec{a}$$

$$\vec{x}_{\perp} = (\vec{x} - \vec{x}_{\parallel}) = (\vec{x} - (\vec{a}.\vec{x})\vec{a})$$

$$\vec{a} \times \vec{x}_{\perp} = \vec{a} \times (\vec{x} - \vec{x}_{\parallel}) = \vec{a} \times (\vec{x} - (\vec{a}.\vec{x})\vec{a}) = \vec{a} \times \vec{x}$$



Axis-angle rotation



$$x_{rotated} = \vec{x}_{||} + \vec{v}$$

$$x_{rotated} = \alpha\ \vec{a} + \beta\ \vec{x}_{\perp} + \gamma\ \vec{a} \times \vec{x}$$

$$\vec{v} = \cos\theta\ \vec{x}_{\perp} + \sin\theta\ \vec{a} \times \vec{x}$$

$$x_{rotated} = \vec{x}_{||} + \cos\theta\ \vec{x}_{\perp} + \sin\theta\ \vec{a} \times \vec{x}$$

$$x_{rotated} = (\vec{a}.\vec{x})\vec{a} + \cos\theta\ (x - (\vec{a}.\vec{x})\vec{a}) + \sin\theta\ \vec{a} \times \vec{x}$$

$$x_{rotated} = (\vec{a}.\vec{x})(1 - \cos\theta)\vec{a} + \cos\theta\ \vec{x} + \sin\theta\ \vec{a} \times \vec{x}$$

Rotation Matrix for Axis-Angle

$$x_{rotated} = (\vec{a}.\vec{x})(1-\cos\theta)\vec{a} + \cos\theta\ \vec{x} + \sin\theta\ \vec{a}\times\vec{x}$$

$$x_{rotated} = (Sym(\vec{a})(1-\cos\theta) + I\cos\theta\ + Skew(\vec{a})\sin\theta\ )\vec{x}$$

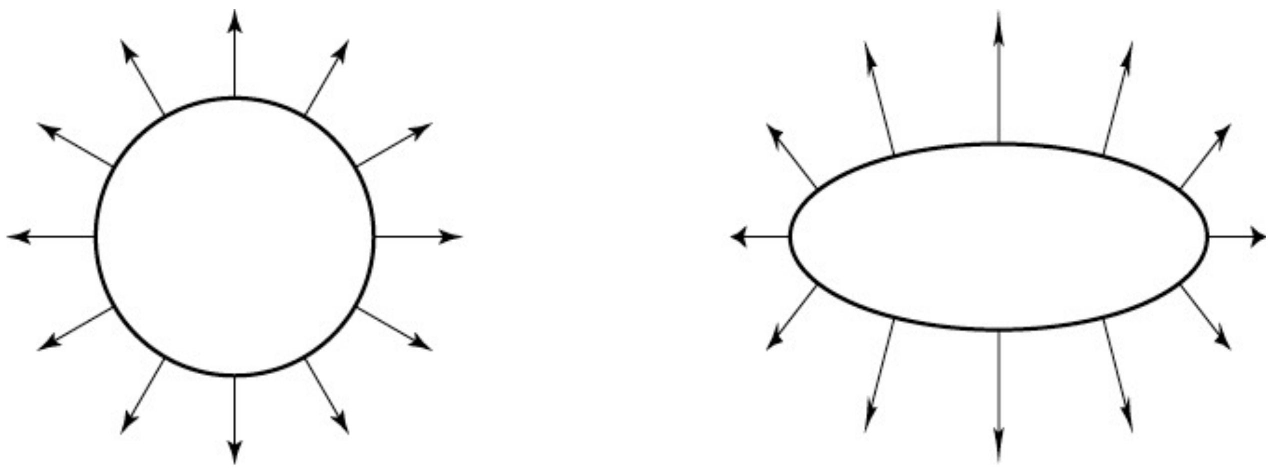
$$Sym(\vec{a}) = \begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix} \begin{bmatrix} a_x & a_y & a_z & 0 \end{bmatrix} = \begin{bmatrix} a_x^2 & a_x a_y & a_x a_z & 0 \\ a_x a_y & a_y^2 & a_y a_z & 0 \\ a_x a_z & a_y a_z & a_z^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Skew(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y & 0 \\ a_z & 0 & -a_x & 0 \\ -a_y & a_x & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Skew(\vec{a})\vec{x} = \vec{a}\times\vec{x}$$

Transforming normal vectors

- Transforming surface normals
  - differences of points (and therefore tangents) transform OK
  - normals do not. Instead, use inverse transpose matrix

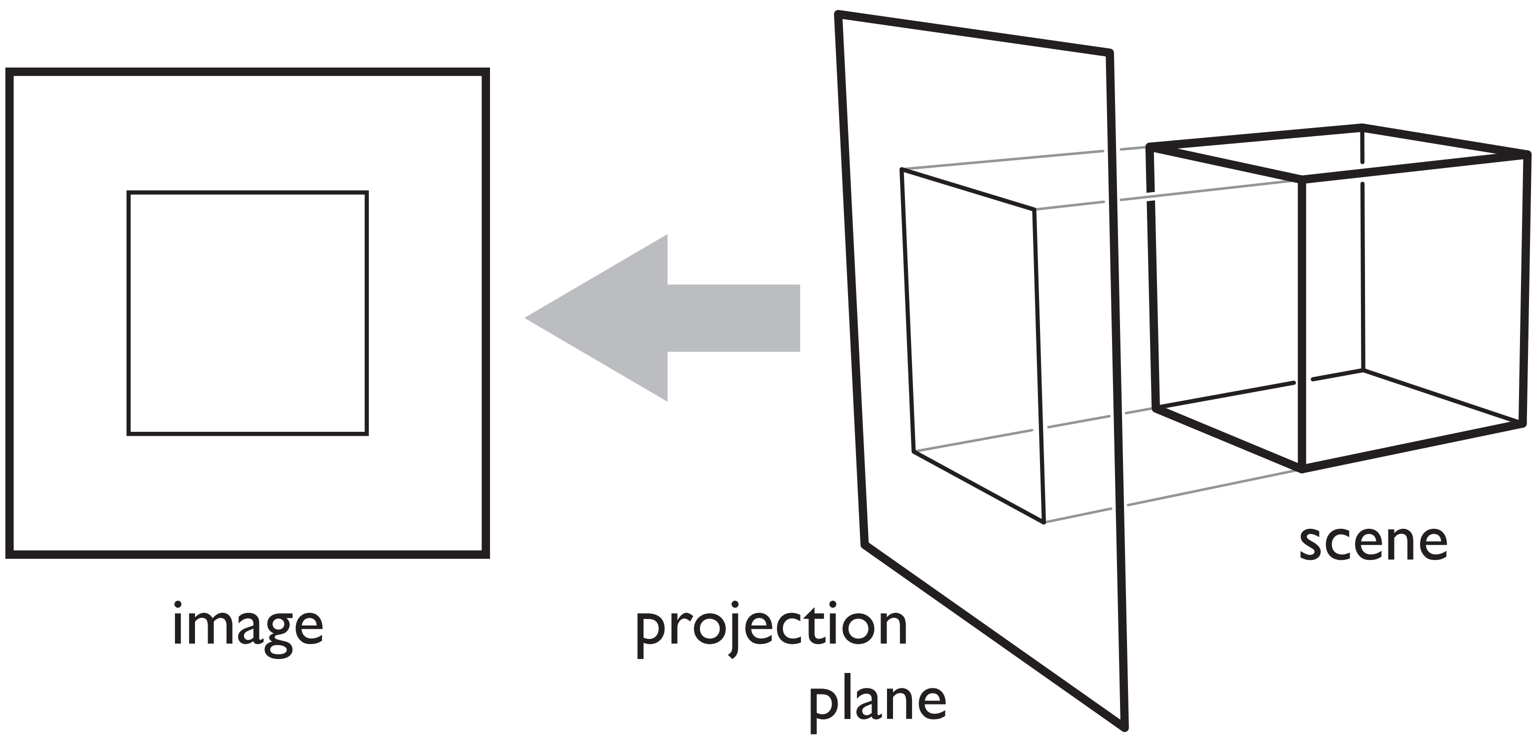


have: $\mathbf{t} \cdot \mathbf{n} = \mathbf{t}^T \mathbf{n} = 0$
want: $M\mathbf{t} \cdot X\mathbf{n} = \mathbf{t}^T M^T X \mathbf{n} = 0$
so set $X = (M^T)^{-1}$
then: $M\mathbf{t} \cdot X\mathbf{n} = \mathbf{t}^T M^T (M^T)^{-1} \mathbf{n} = \mathbf{t}^T \mathbf{n} = 0$

Perspective

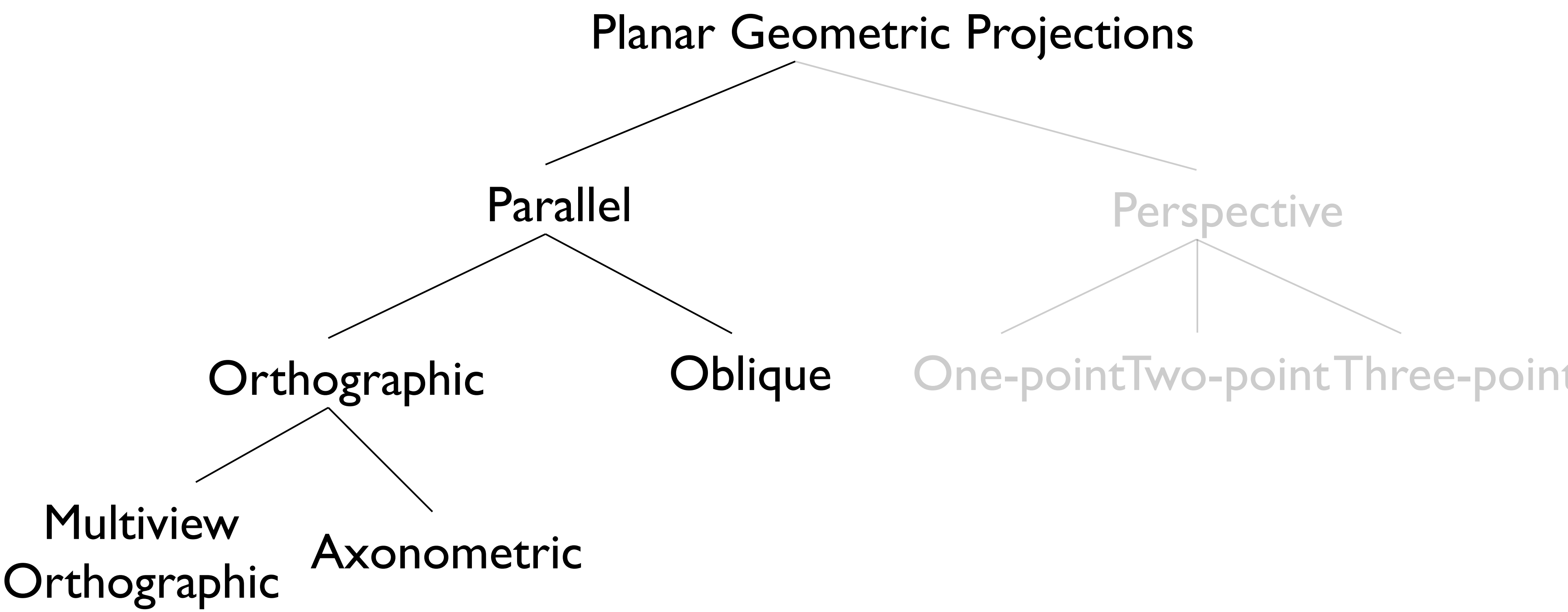
Parallel projection

- To render an image of a 3D scene, we *project* it onto a plane
- Simplest kind of projection is *parallel projection*



Classical projections—parallel

- Emphasis on cube-like objects
  - traditional in mechanical and architectural drawing



[after Carlbom & Paciorek 78]

Orthographic

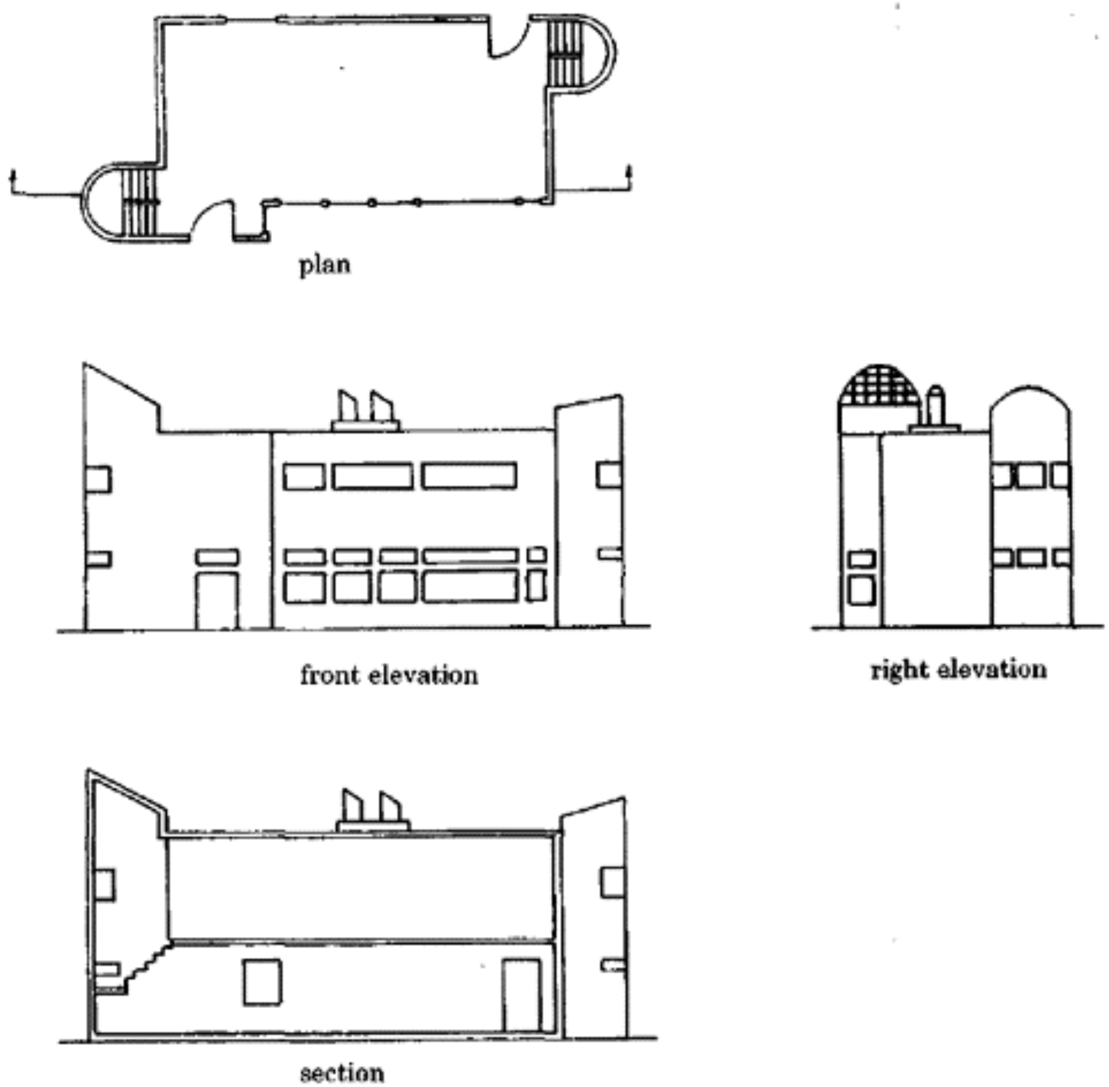


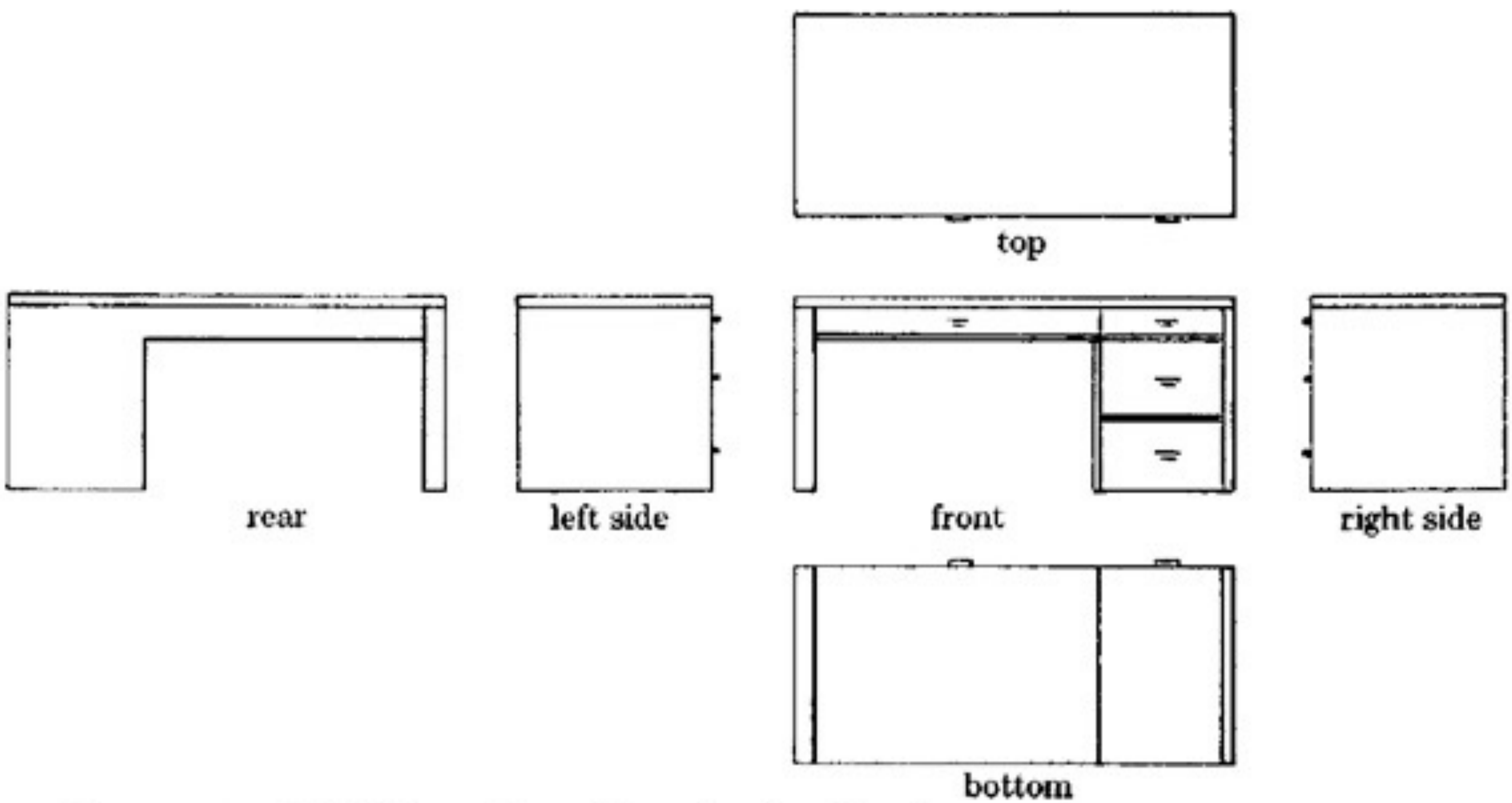
FIGURE 2-1. Multiview orthographic projection: plan, elevations, and section of a building.

[Carlbom & Paciorek 78]

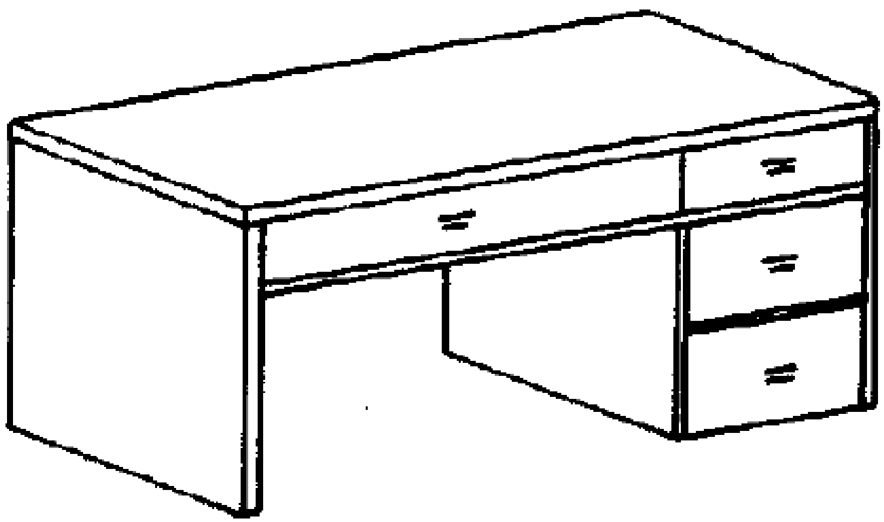
Orthographic

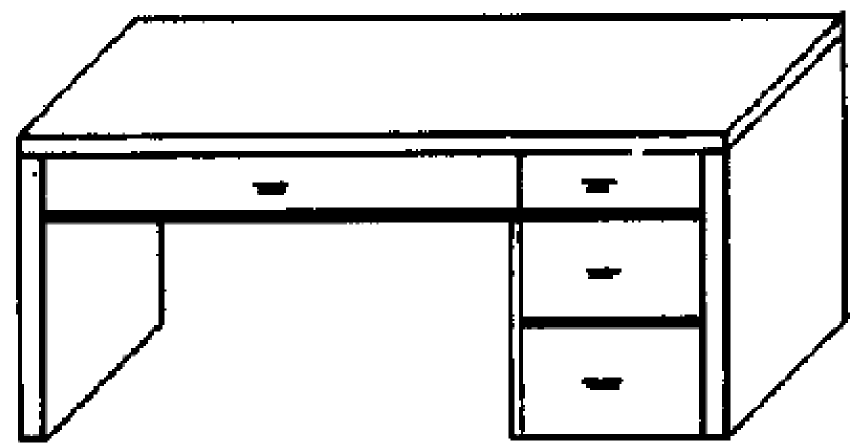
- projection plane parallel to a coordinate plane
- projection direction perpendicular to projection plane

[Carlbom & Paciorek 78]



Off-axis parallel





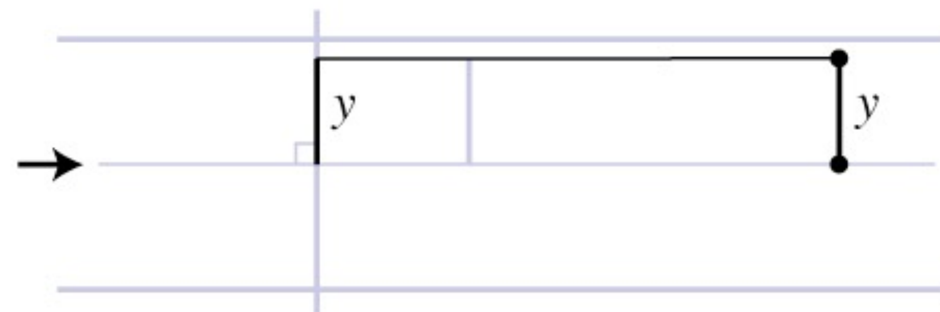
[Carlbom & Paciorek 78]

axonometric: projection plane perpendicular to projection direction but not parallel to coordinate planes

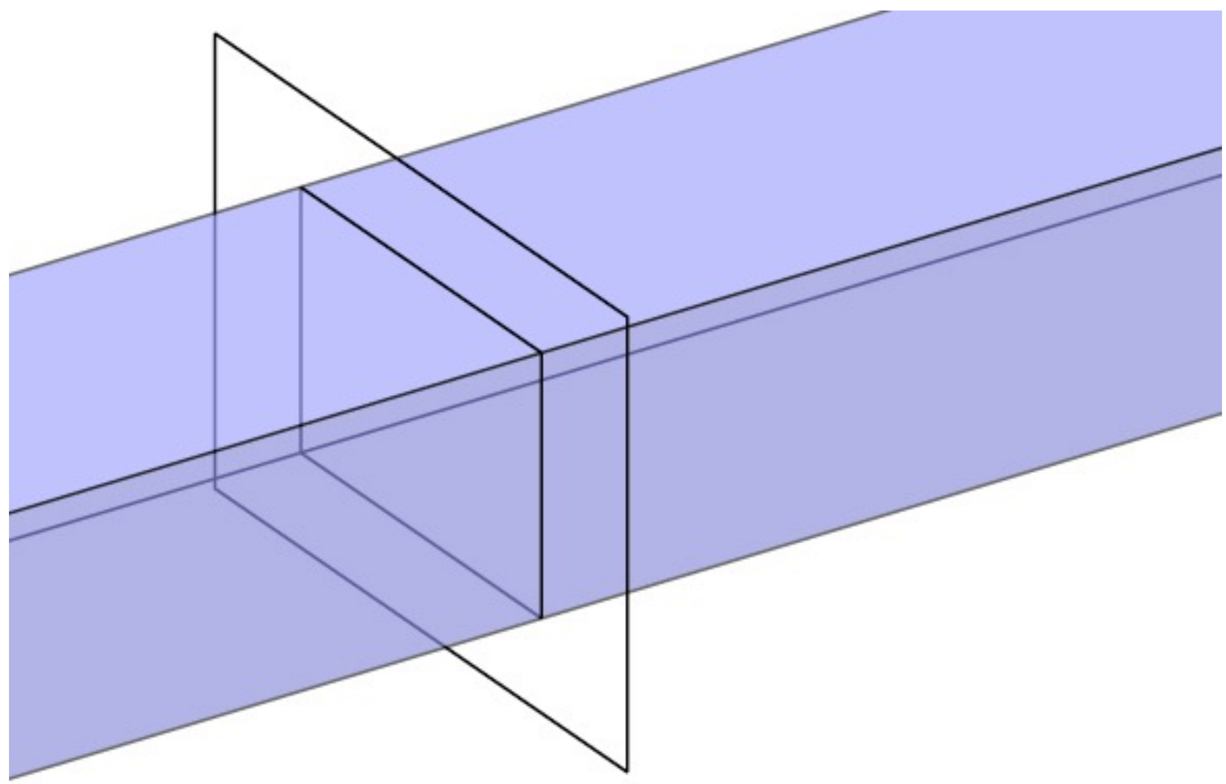
oblique: projection plane parallel to a coordinate plane but not perpendicular to projection direction

"Orthographic" projection

- In graphics usually we lump axonometric with orthographic
  - projection plane perpendicular to projection direction
  - image height determines size of objects in image

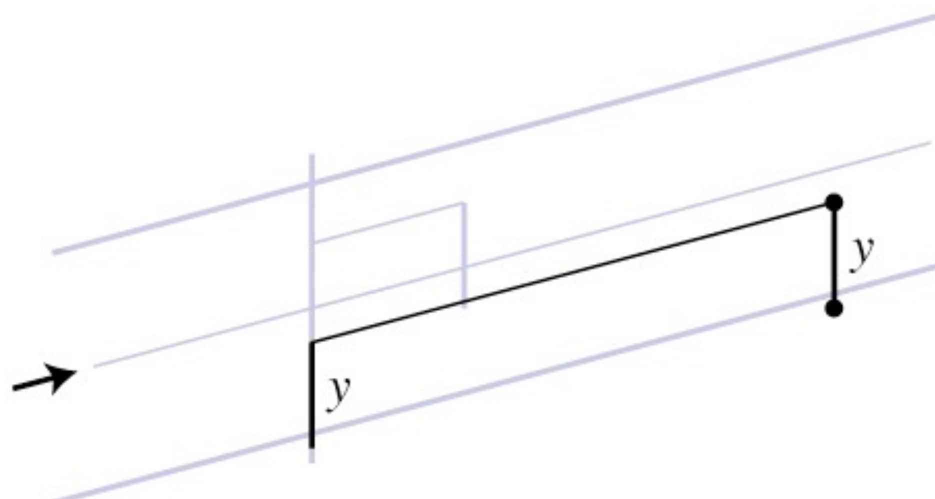


View volume: orthographic



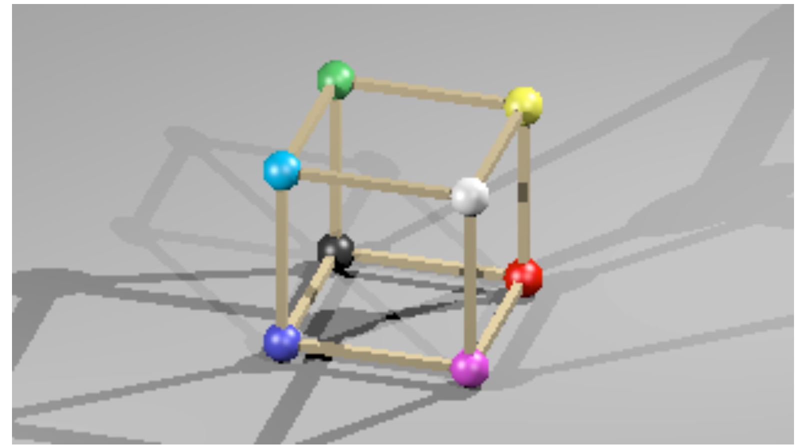
Oblique projection

- View direction no longer coincides with projection plane normal (one more parameter)
  - objects at different distances still same size
  - objects are shifted in the image depending on their depth

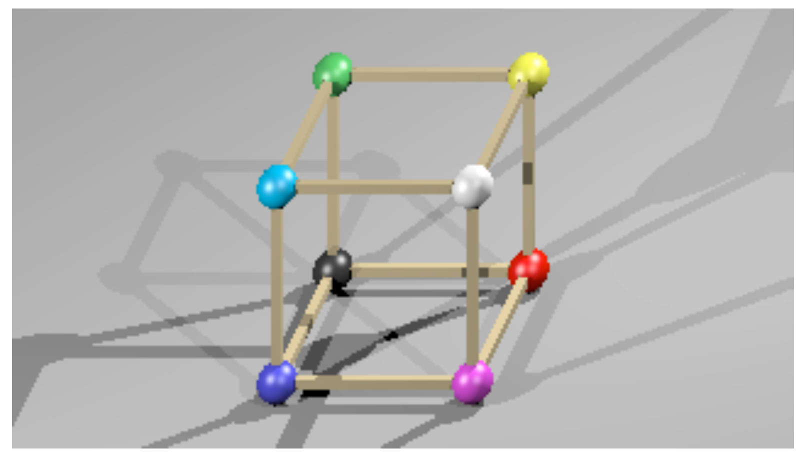


Specifying views in a ray tracer

```xml
<camera type="ParallelCamera">
  <viewPoint>2.0 4.0 7.0</viewPoint>
  <viewDir>–2.0 –4.0 –7.0</viewDir>
  <viewUp>0.0 1.0 0.0</viewUp>
  <viewWidth>8.0</viewWidth>
  <viewHeight>4.5</viewHeight>
</camera>
```



```xml
<camera type="ParallelCamera">
  <viewPoint>2.0 4.0 7.0</viewPoint>
  <viewDir>–2.0 –4.0 –7.0</viewDir>
  <projNormal>0.0 0.0 1.0</projNormal>
  <viewUp>0.0 1.0 0.0</viewUp>
  <viewWidth>8.0</viewWidth>
  <viewHeight>4.5</viewHeight>
</camera>
```



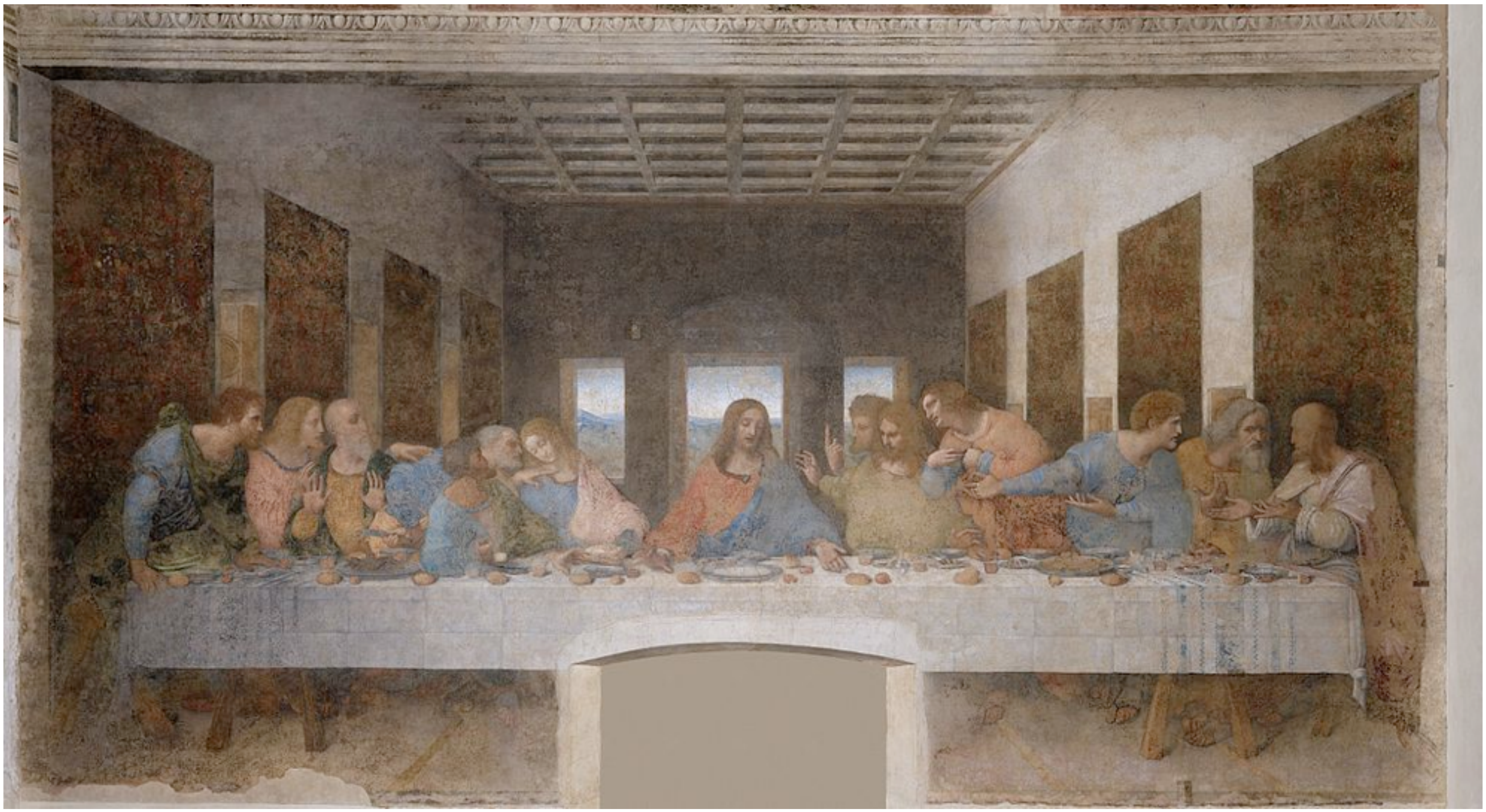
History of projection

- Ancient times: Greeks wrote about laws of perspective
- Renaissance: perspective is adopted by artists

Duccio c. 1308

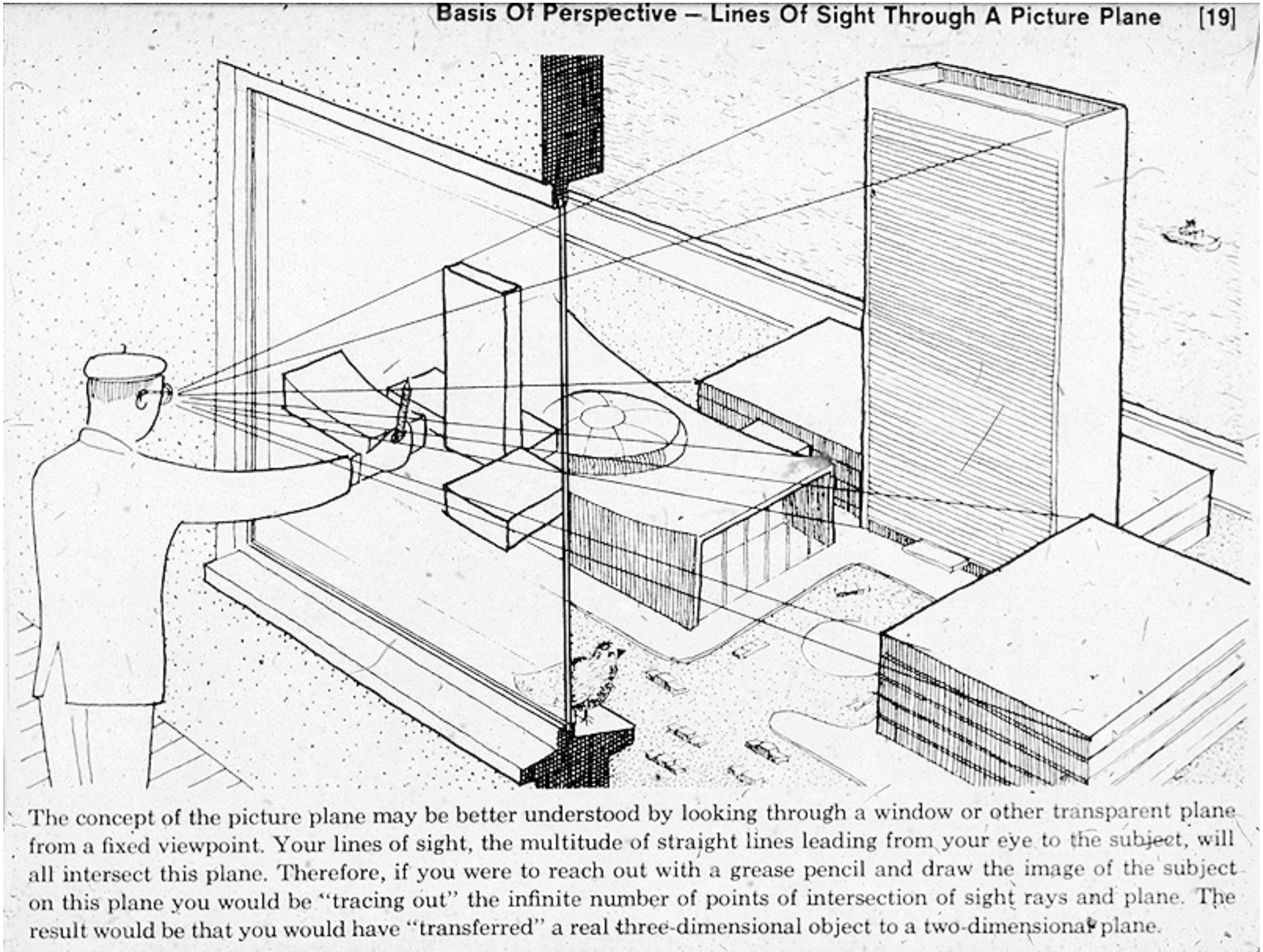
History of projection

- Later Renaissance: perspective formalized precisely



da Vinci c. 1498

Plane projection in drawing



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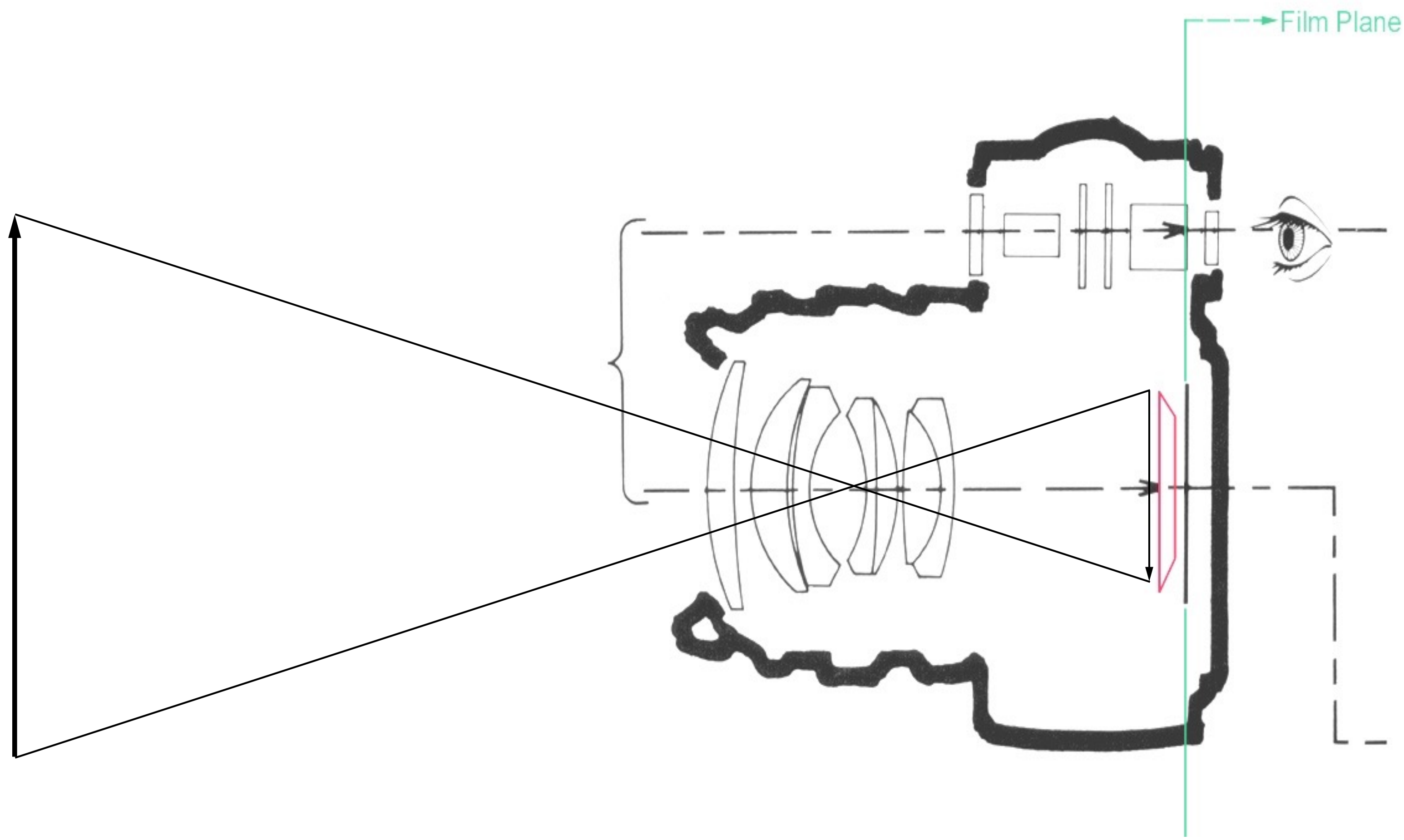
Plane projection in photography

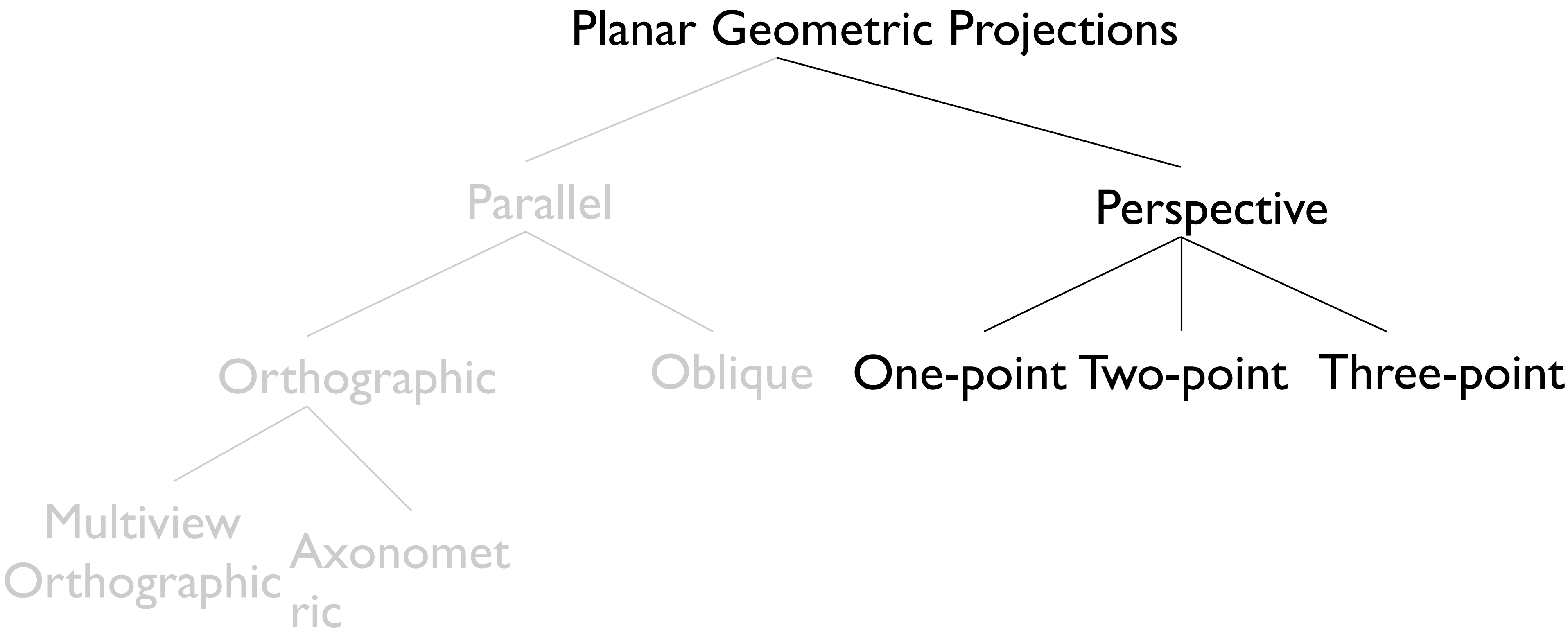
- This is another model for what we are doing
  - applies more directly in realistic rendering



[Source unknown]

Classical projections—perspective

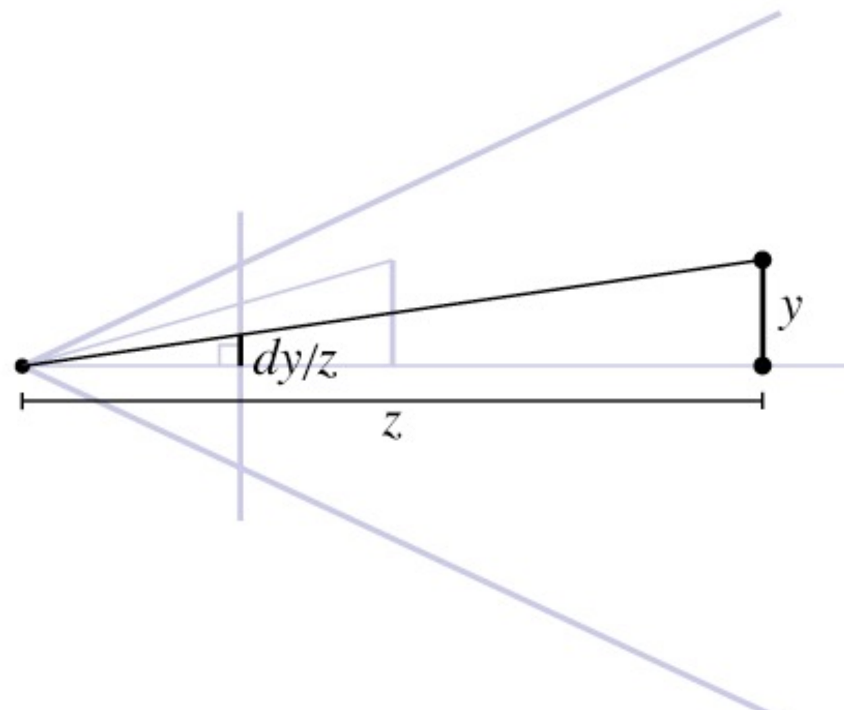
- Emphasis on cube-like objects
  - traditional in mechanical and architectural drawing



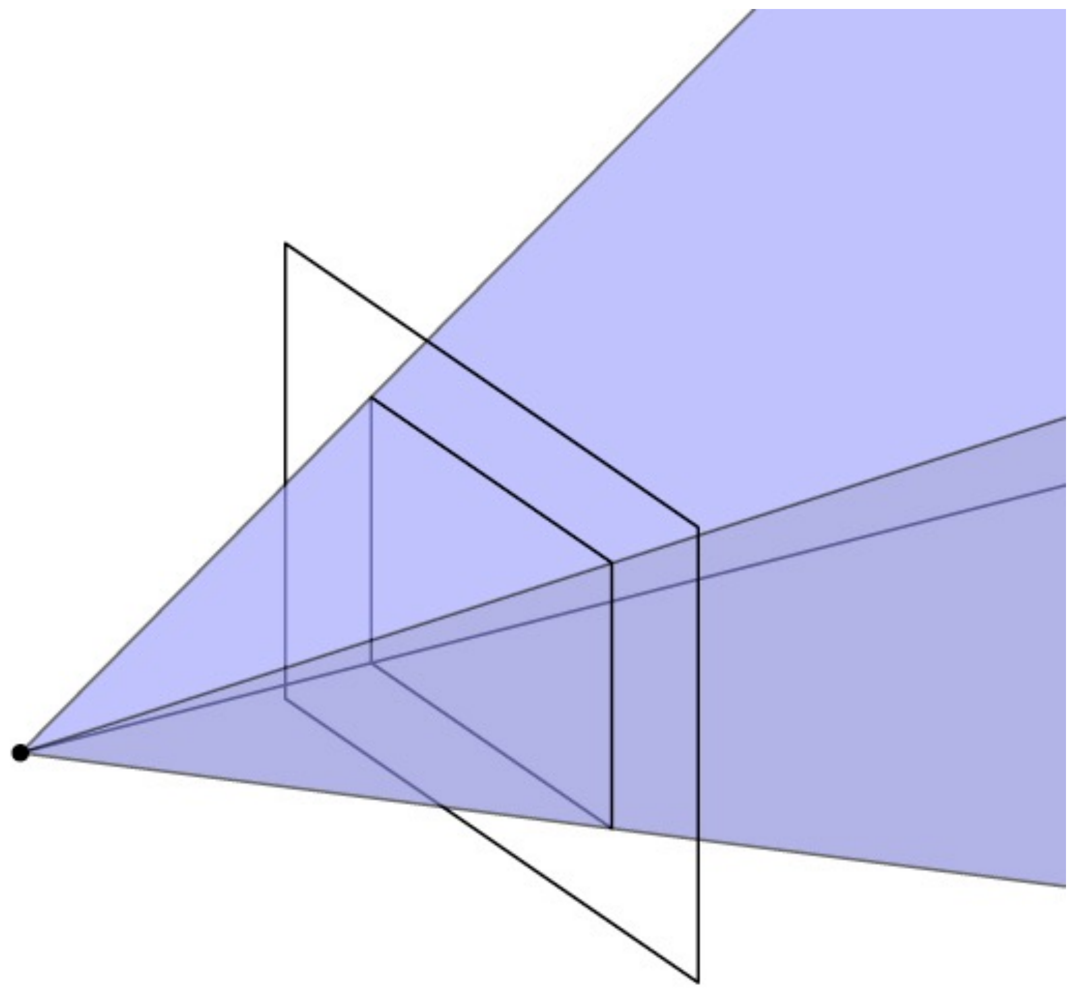
[after Carlbom & Paciorek 78]

Perspective projection (normal)

- Perspective is projection by lines through a point;
- "normal" = plane perpendicular to view direction
  - magnification determined by:
    - image height
    - object depth
    - image plane distance
  - f.o.v. $\alpha = 2 \operatorname{atan}(h/(2d))$
  - $y' = d\,y / z$
  - "normal" case corresponds to common types of cameras

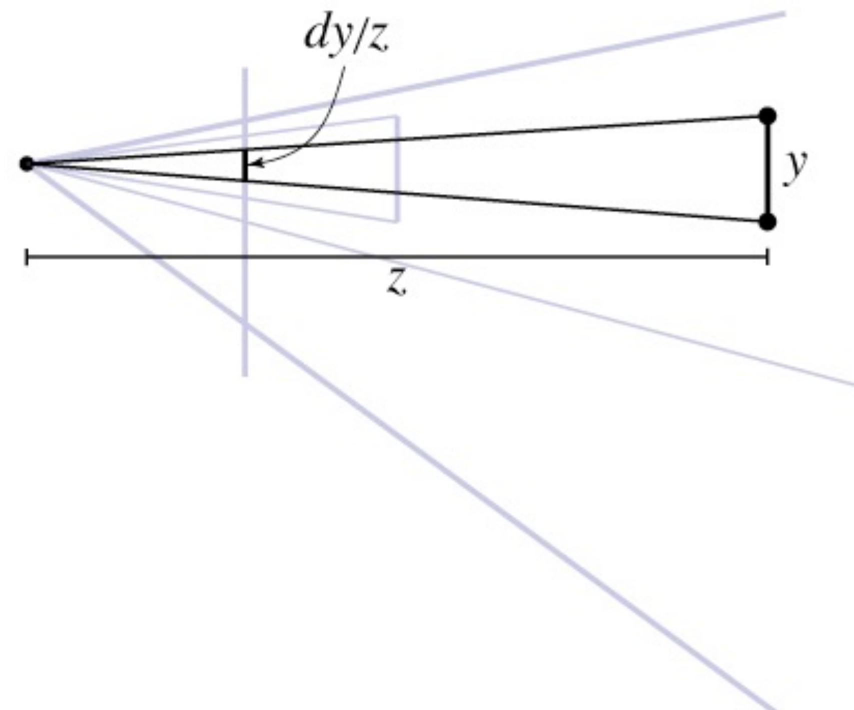


View volume: perspective



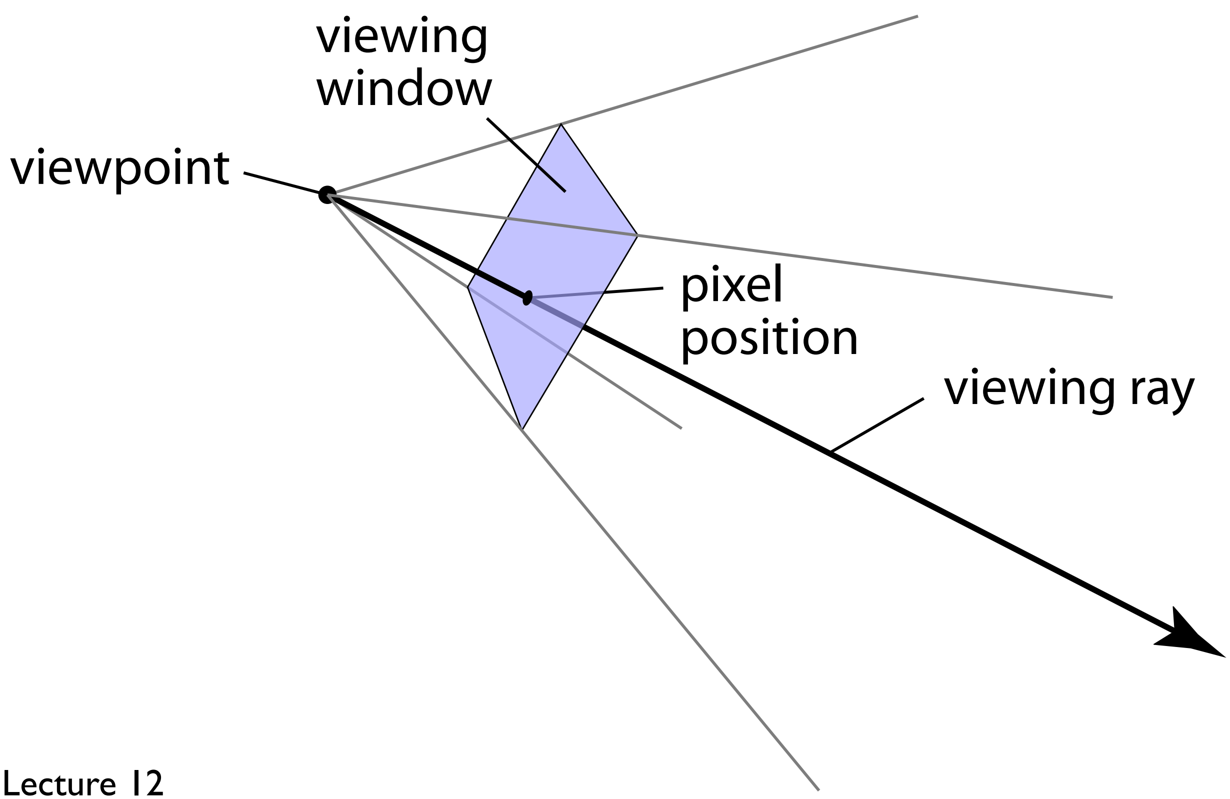
Shifted perspective projection

- Perspective but with projection plane not perpendicular to view direction
  - additional parameter: projection plane normal
  - exactly equivalent to cropping out an off-center rectangle from a larger "normal" perspective
  - corresponds to *view camera* in photography



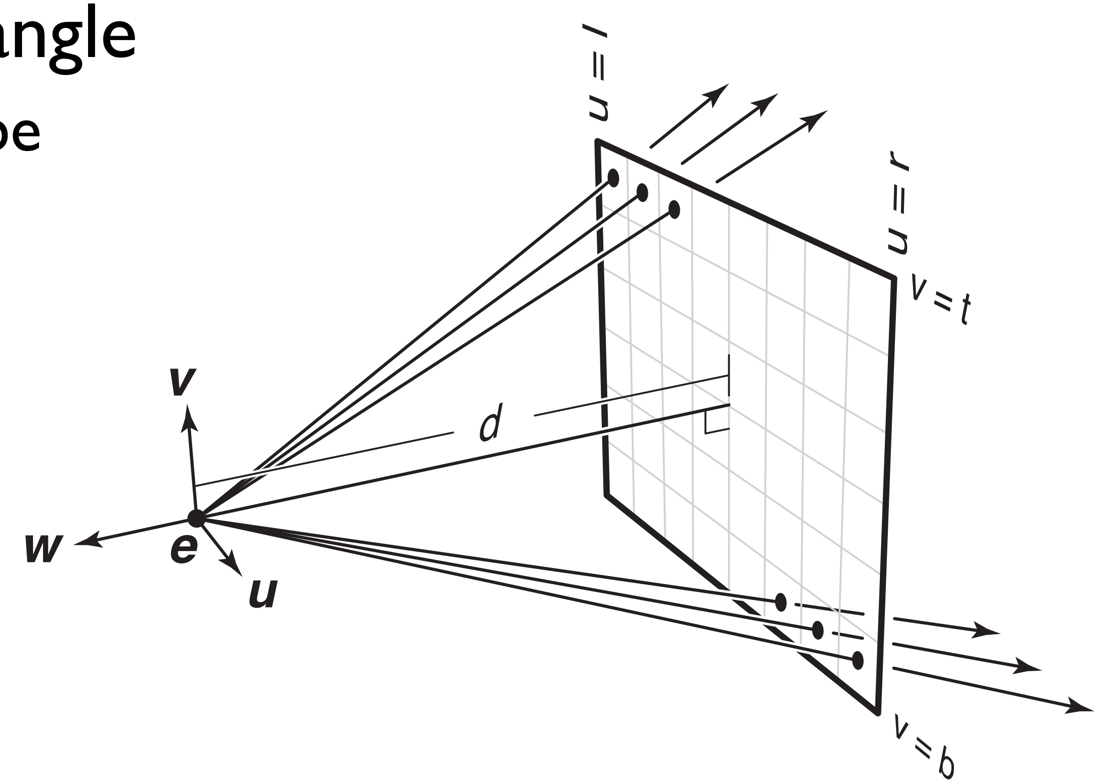
Generating eye rays—perspective

- Use window analogy directly
- Ray origin (constant): viewpoint
- Ray direction (varying): toward pixel position on viewing window



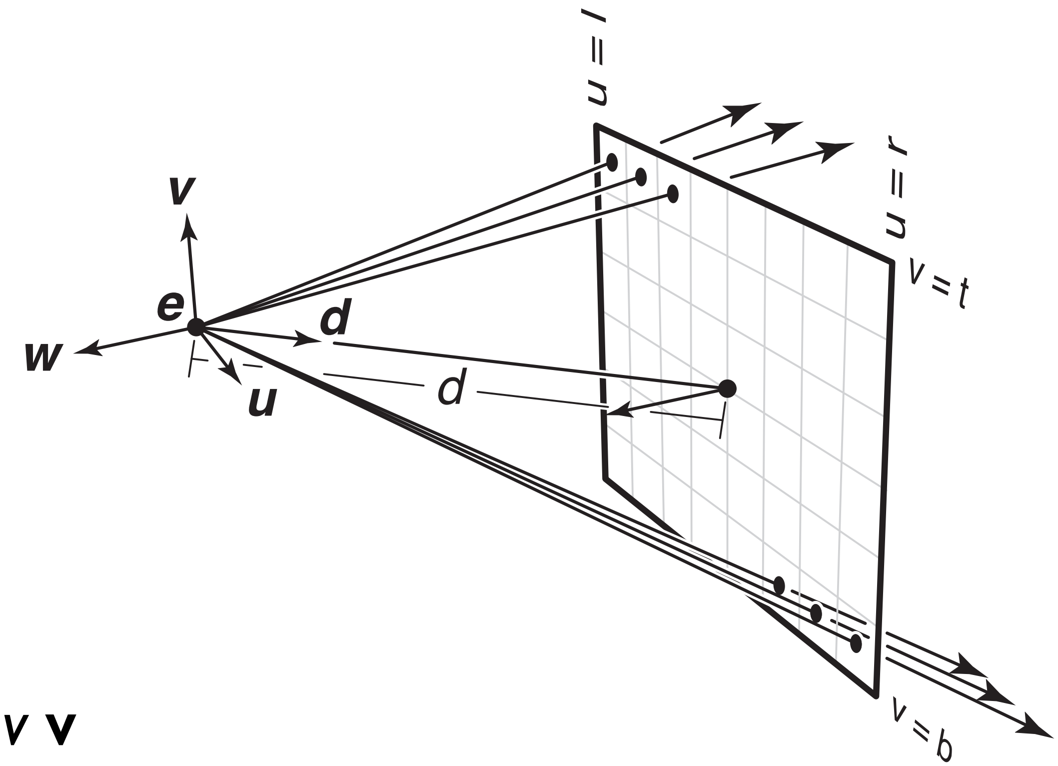
Generating eye rays—perspective

- Positioning the view rectangle
  - establish three vectors to be *camera basis:* **u**, **v**, **w**
  - view rectangle is parallel to **u**–**v** plane, at $w = -d$, specified by l, r, t, b
- Generating rays
  - for (u, v) in $[l, r] \times [b, t]$
  - ray.origin = **e**
  - ray.direction = $-d\,\mathbf{w} + u\,\mathbf{u} + v\,\mathbf{v}$



Oblique perspective views

- Positioning the view rectangle
  - establish three vectors to be *camera basis:* **u**, **v**, **w**
  - view rectangle is the same, but shifted so that the center is in the direction **d** from **e**
- Generating rays
  - for (u, v) in [l, r] × [b, t]
  - ray.origin = **e**
  - ray.direction = d **d** + u **u** + v **v**



Field of view (or f.o.v.)

- The angle between the rays corresponding to opposite edges of a perspective image
  – simpler to compute for "normal" perspective
  – have to decide to measure vert., horiz., or diag.
- In cameras, determined by focal length
  – confusing because of many image sizes
  – for 35mm format (36mm by 24mm image)
    - 18mm = 67° v.f.o.v. — super-wide angle
    - 28mm = 46° v.f.o.v. — wide angle
    - 50mm = 27° v.f.o.v. — "normal"
    - 100mm = 14° v.f.o.v. — narrow angle ("telephoto")

Field of view

- Determines "strength" of perspective effects

close viewpoint
wide angle
prominent foreshortening

far viewpoint
narrow angle
little foreshortening

[Ansel Adams]

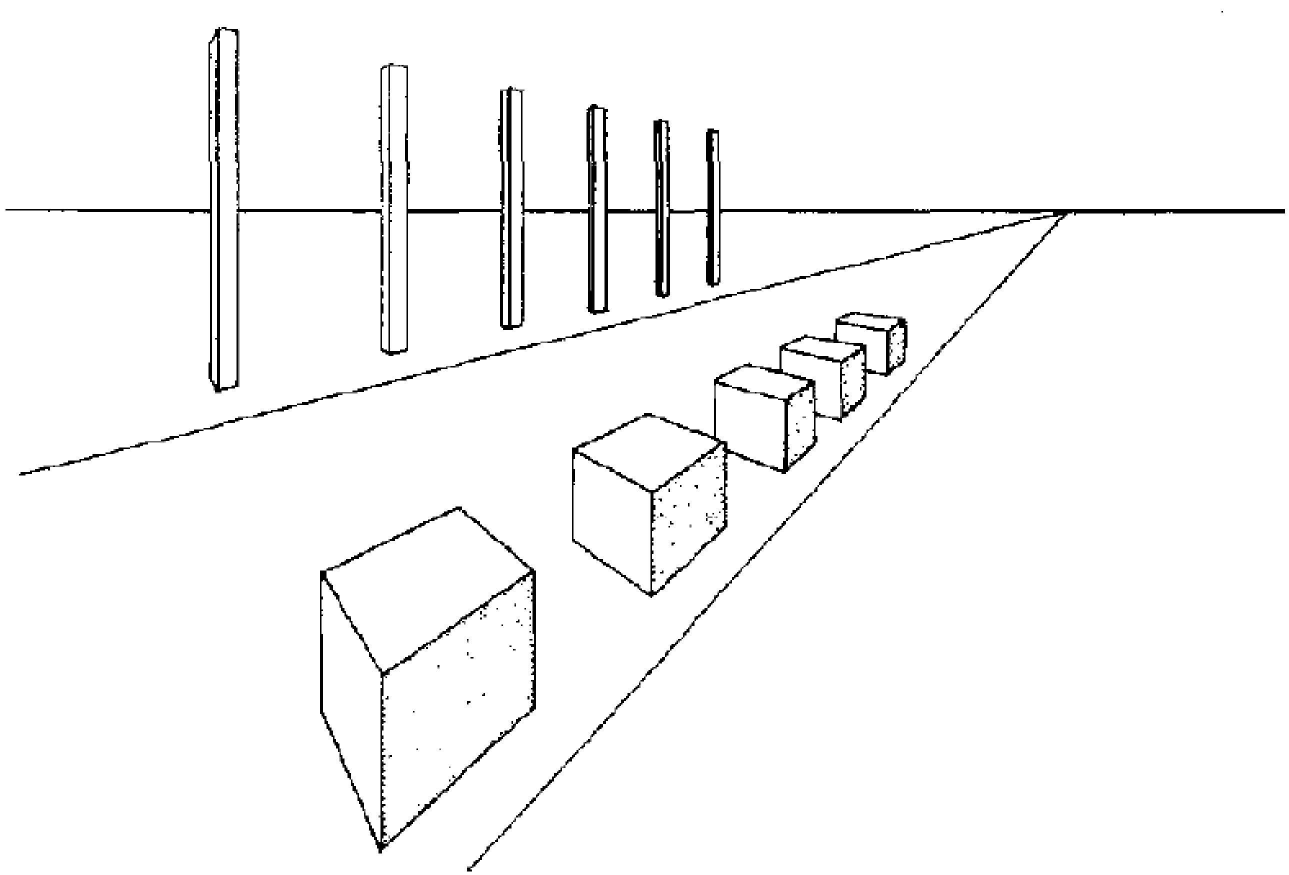
Choice of field of view

- In photography, wide angle lenses are specialty tools
  - "hard to work with"
  - easy to create weird-looking perspective effects
- In graphics, you can type in whatever f.o.v. you want
  - and people often type in big numbers!

[Ken Perlin]

Perspective distortions

- Lengths, length ratios



[Carlbom & Paciorek 78]

Pipeline of transformations

- Standard sequence of transforms