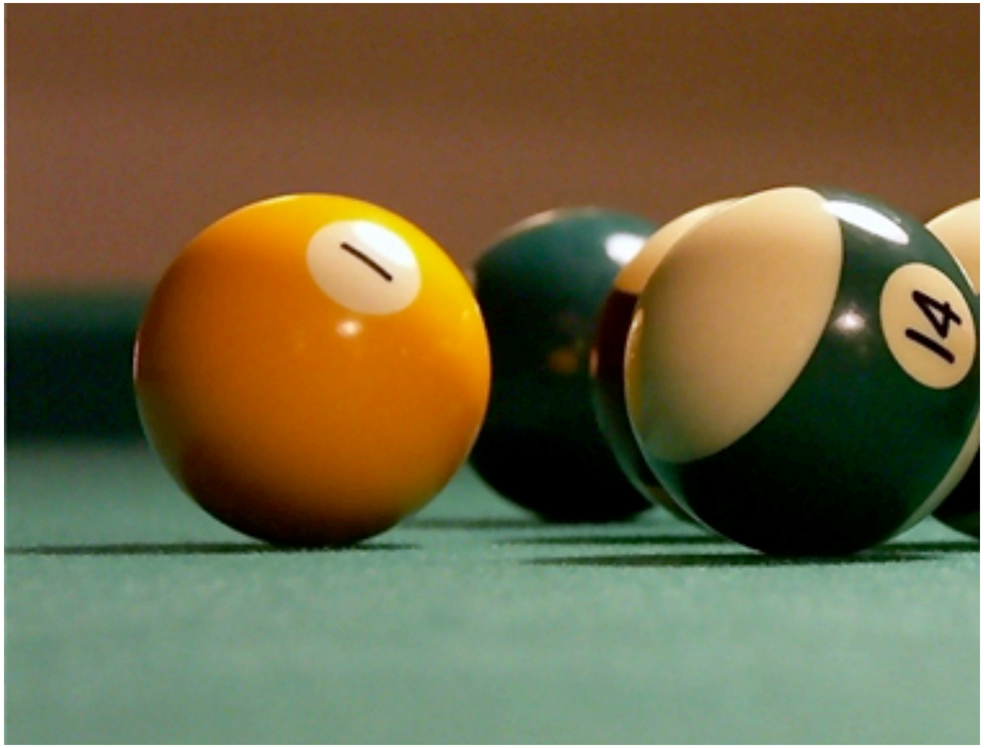
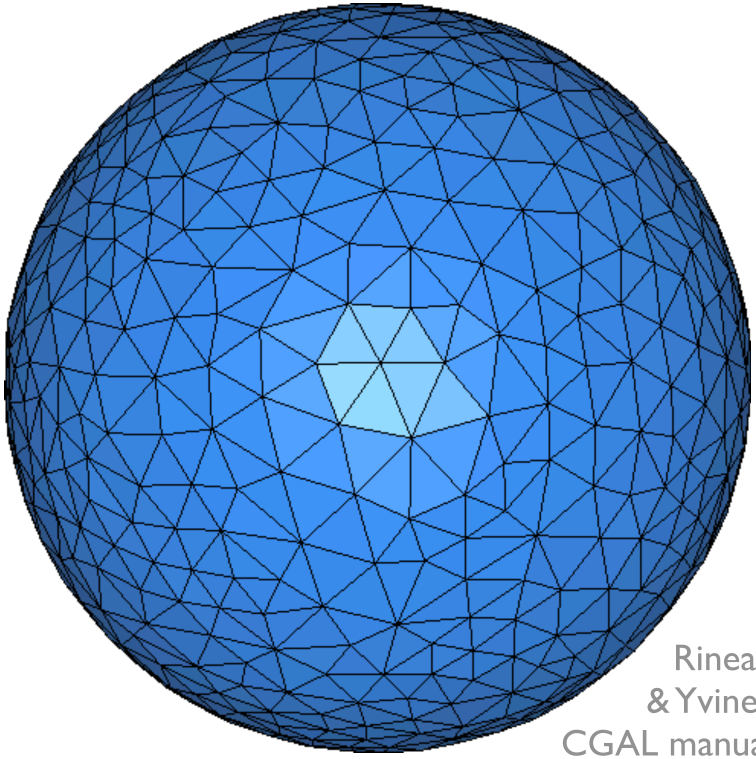
Triangle meshes I

CS 4620 Lecture 2



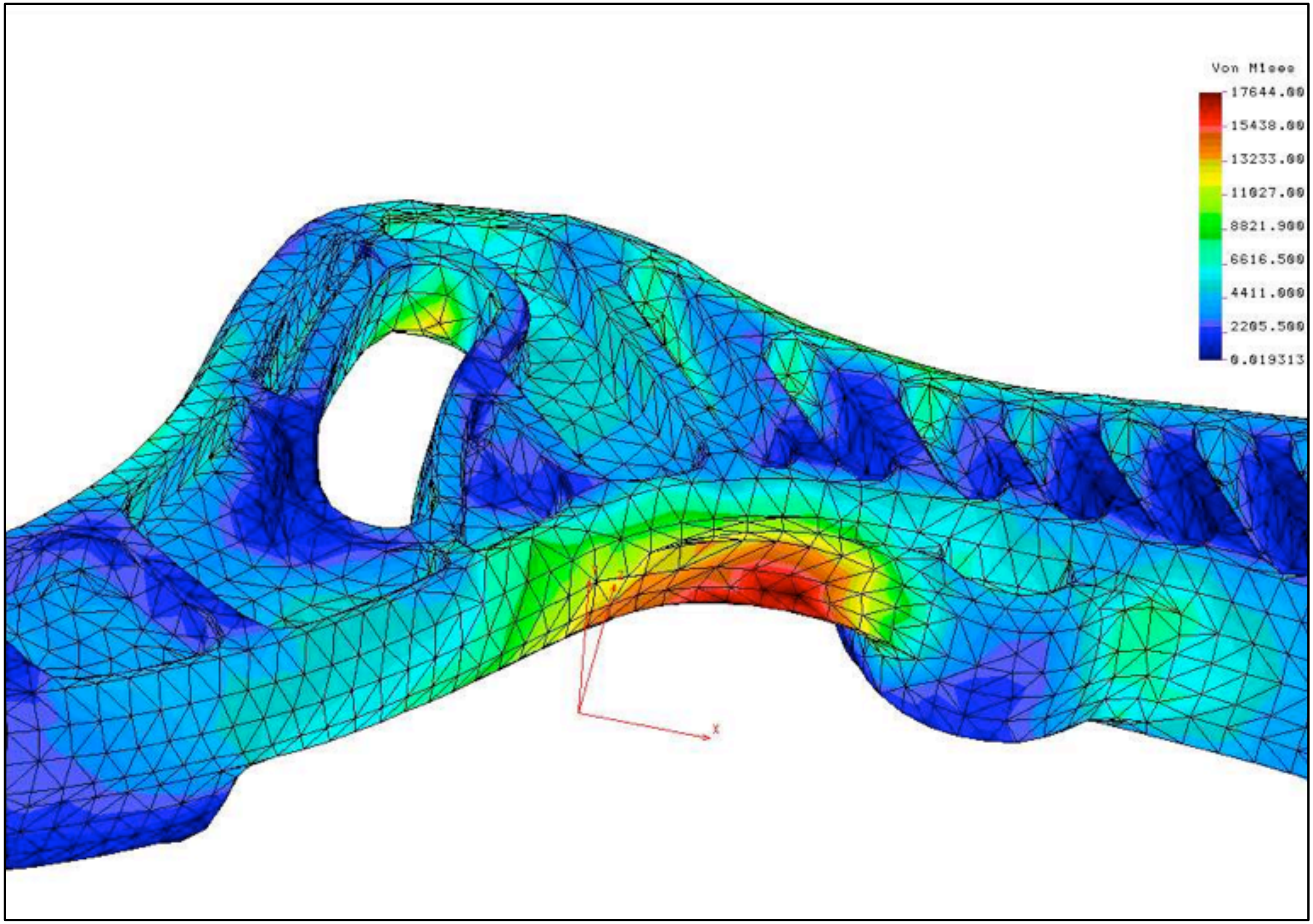
Andrzej Barabasz

spheres



Rineau & Yvinec CGAL manual

approximate sphere

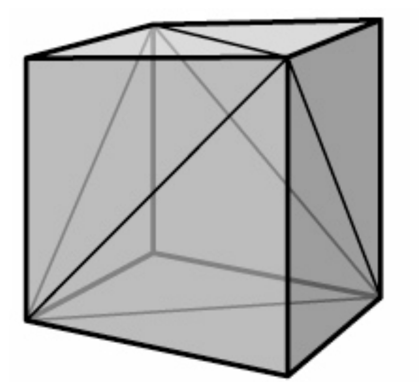


PATRIOT Engineering

finite element analysis

Ottawa Convention Center

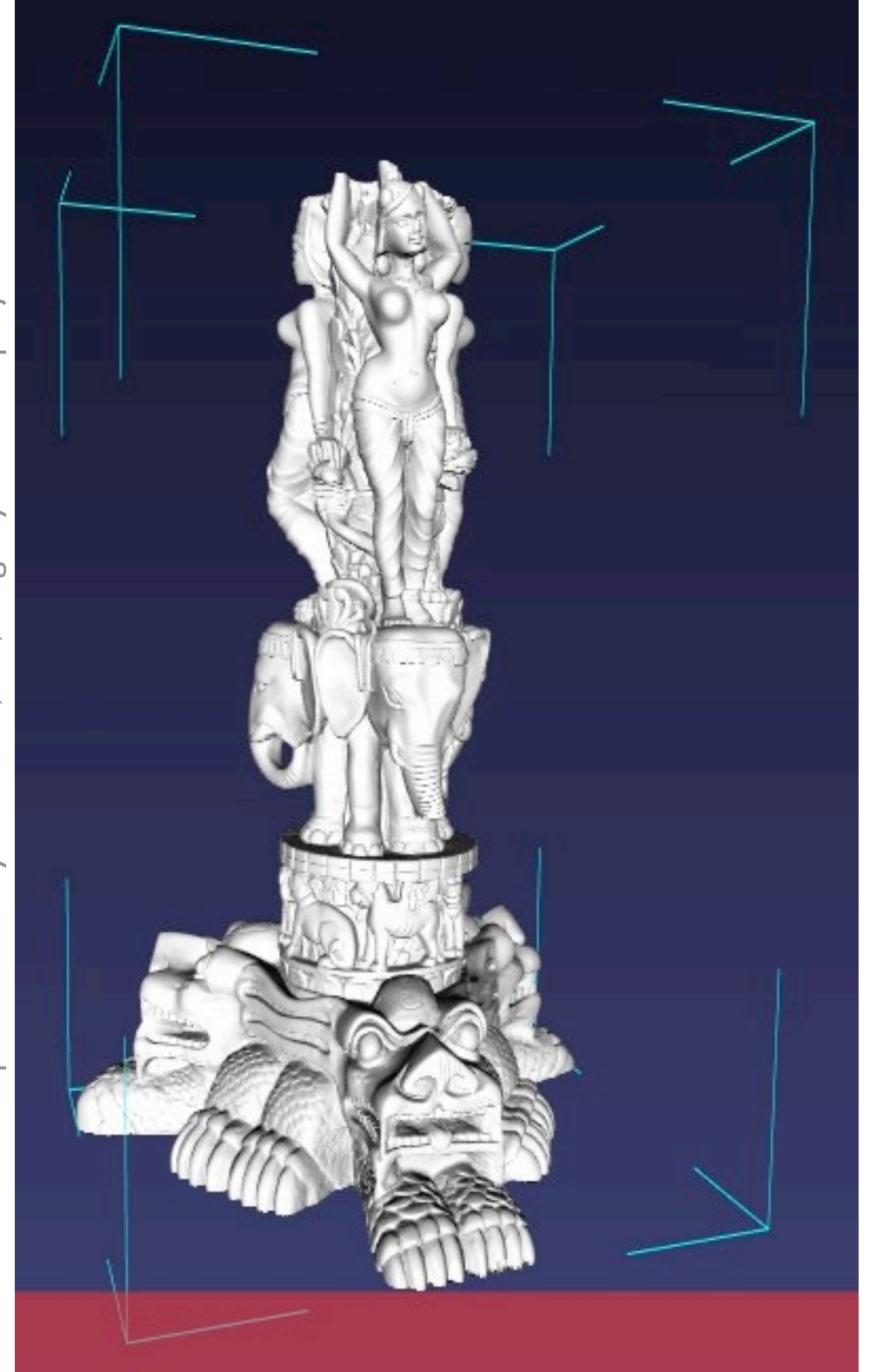
A small triangle mesh



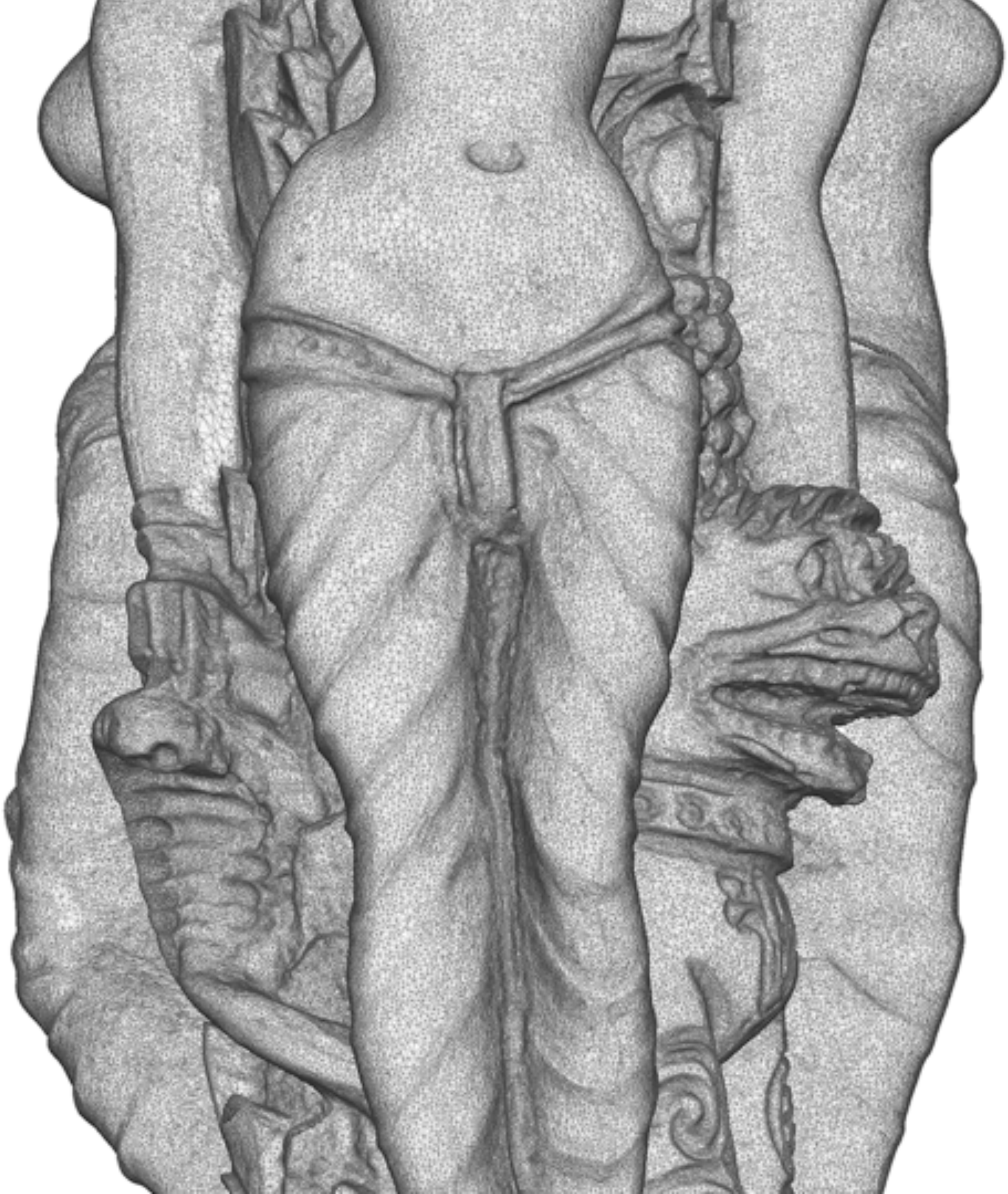
12 triangles, 8 vertices

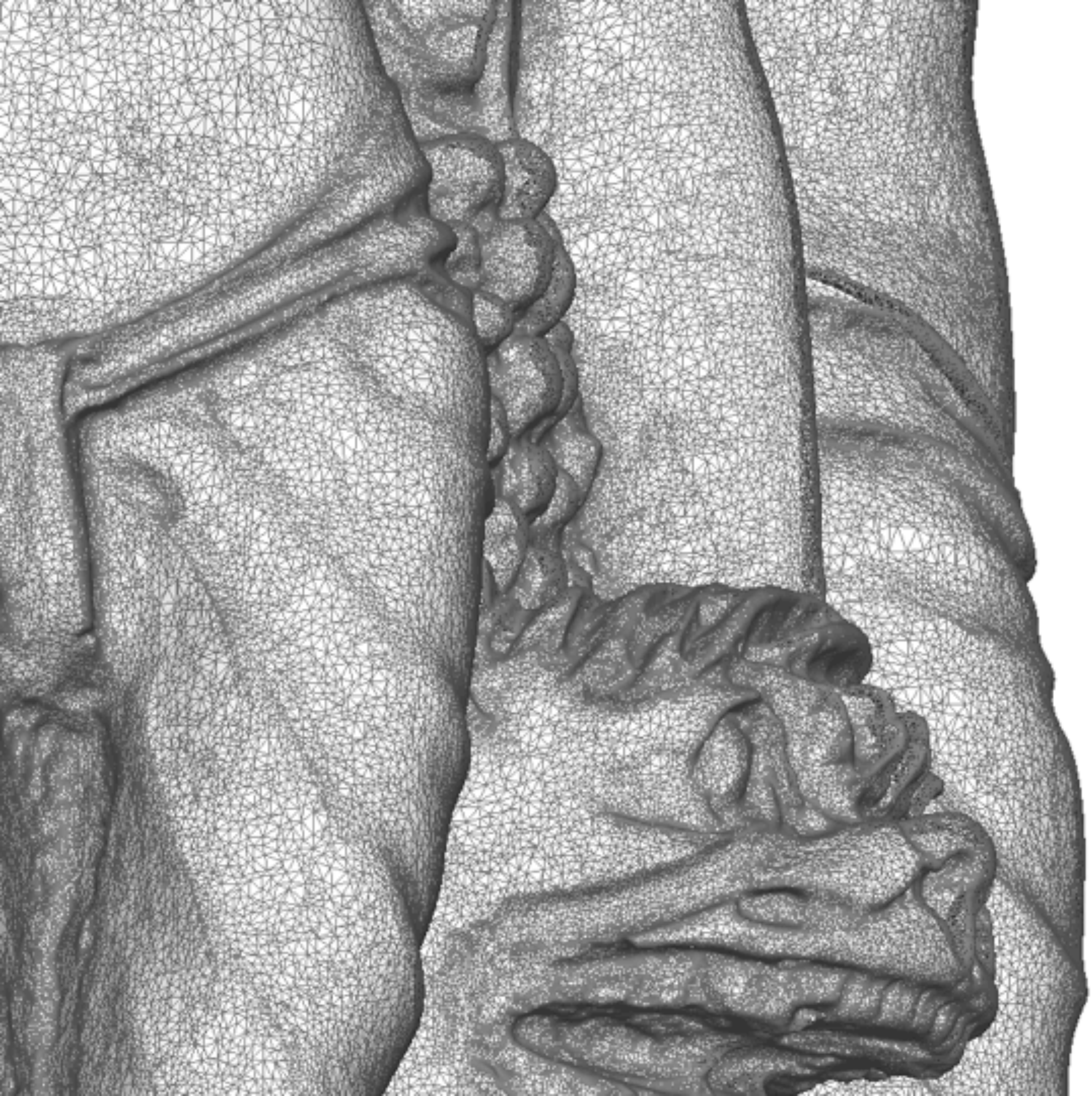
A large mesh

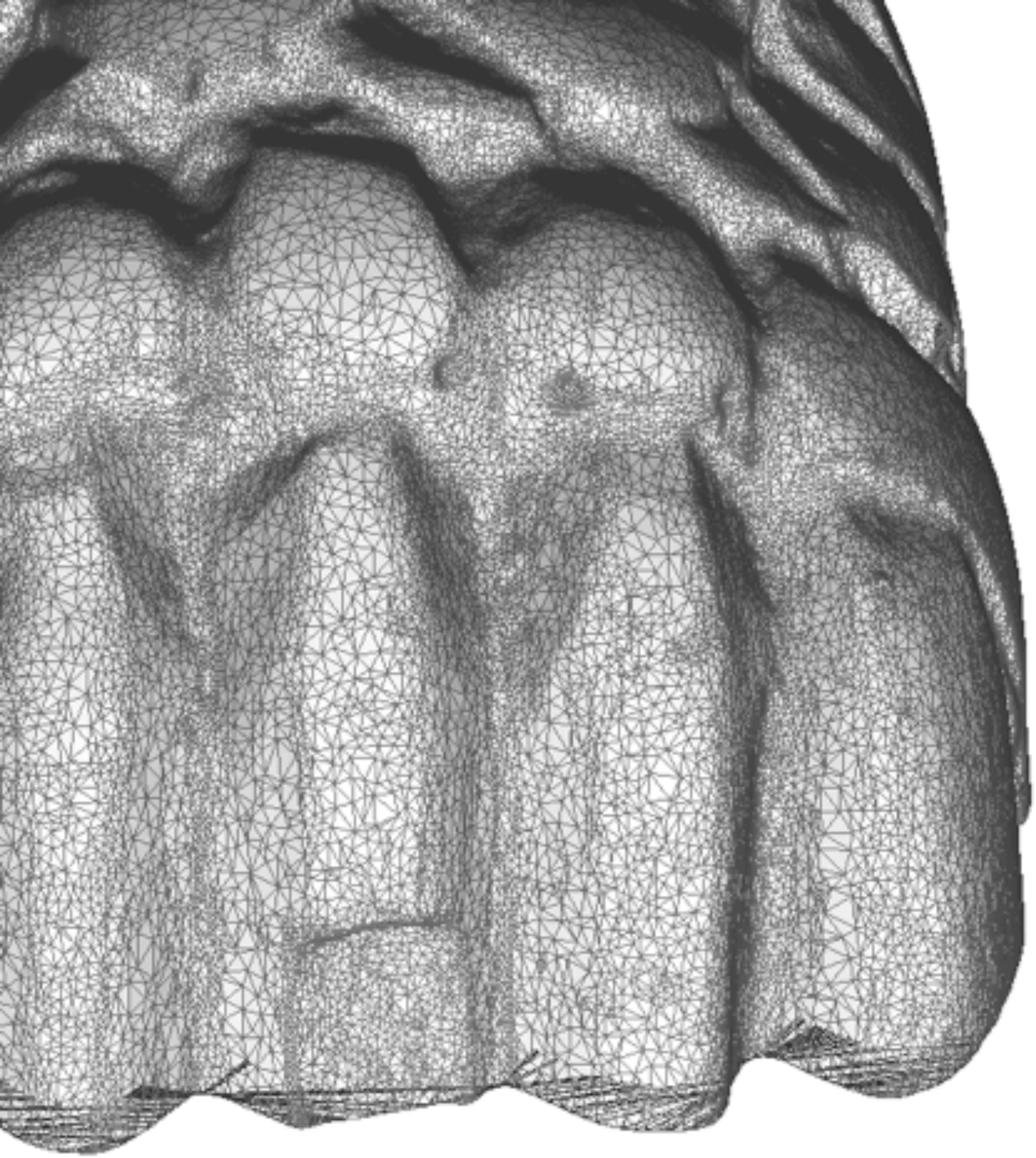
10 million triangles
from a high-resolution
3D scan



Traditional Thai sculpture—scan by XYZRGB, inc., image by MeshLab project








about a trillion triangles
from automatically processed
satellite and aerial photography
Google earth

Triangles

- Defined by three *vertices*
- Lives in the plane containing those vertices
- Vector normal to plane is the triangle's normal
- Conventions (for this class, not everyone agrees):
  - vertices are counter-clockwise as seen from the "outside" or "front"
  - surface normal points towards the outside ("outward facing normals")

Triangle meshes

- A bunch of triangles in 3D space that are **connected together** to form a surface
- Geometrically, a mesh is a *piecewise planar* surface
  – almost everywhere, it is planar
  – exceptions are at the edges where triangles join
- Often, it's a piecewise planar **approximation of a smooth surface**
  – in this case the creases between triangles are artifacts—we don't want to see them

Representation of triangle meshes

- Compactness
- Efficiency for rendering
  - enumerate all triangles as triples of 3D points
- Efficiency of queries
  - all vertices of a triangle
  - all triangles around a vertex
  - neighboring triangles of a triangle
  - (need depends on application)
    - finding triangle strips
    - computing subdivision surfaces
    - mesh editing

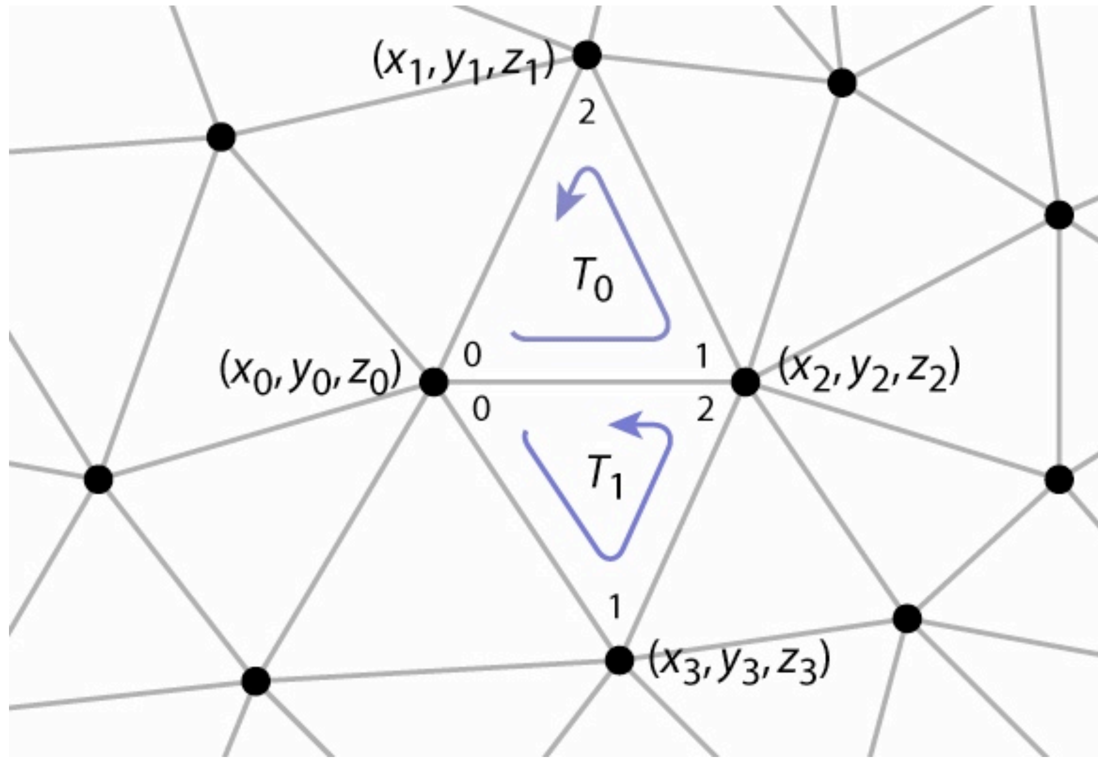
Representations for triangle meshes

- Separate triangles
- Indexed triangle set ⟵ crucial for first assignment
  - shared vertices
- Triangle strips and triangle fans
  - compression schemes for fast transmission
- Triangle-neighbor data structure
  - supports adjacency queries
- Winged-edge data structure
  - supports general polygon meshes

can read about in textbook (will discuss later if time)

Separate triangles

	[0]	[1]	[2]
tris[0]	x_0, y_0, z_0	x_2, y_2, z_2	x_1, y_1, z_1
tris[1]	x_0, y_0, z_0	x_3, y_3, z_3	x_2, y_2, z_2
	⋮	⋮	⋮



Separate triangles

- array of triples of points
  - float[n_T][3][3]: about 72 bytes per vertex
    - 2 triangles per vertex (on average)
    - 3 vertices per triangle
    - 3 coordinates per vertex
    - 4 bytes per coordinate (float)
- various problems
  - wastes space (each vertex stored 6 times)
  - cracks due to roundoff
  - difficulty of finding neighbors at all

Indexed triangle set

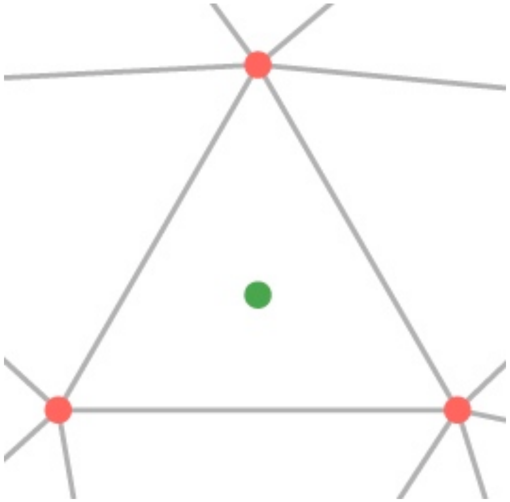
- Store each vertex once
- Each triangle points to its three vertices

```
Triangle {
    Vertex vertex[3];
    }

Vertex {
    float position[3];  // or other data
    }

// ... or ...

Mesh {
    float verts[nv][3];  // vertex positions (or other data)
    int tInd[nt][3];  // vertex indices
    }
```



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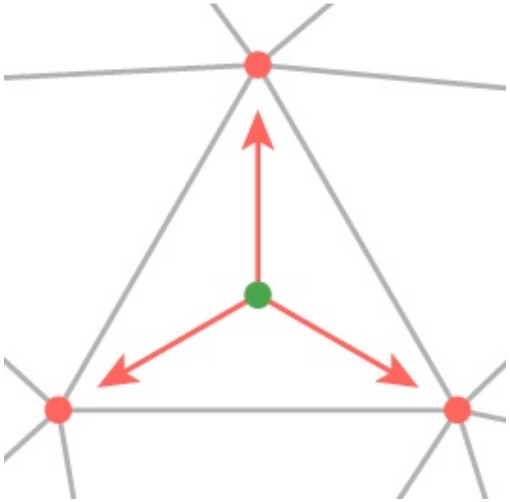
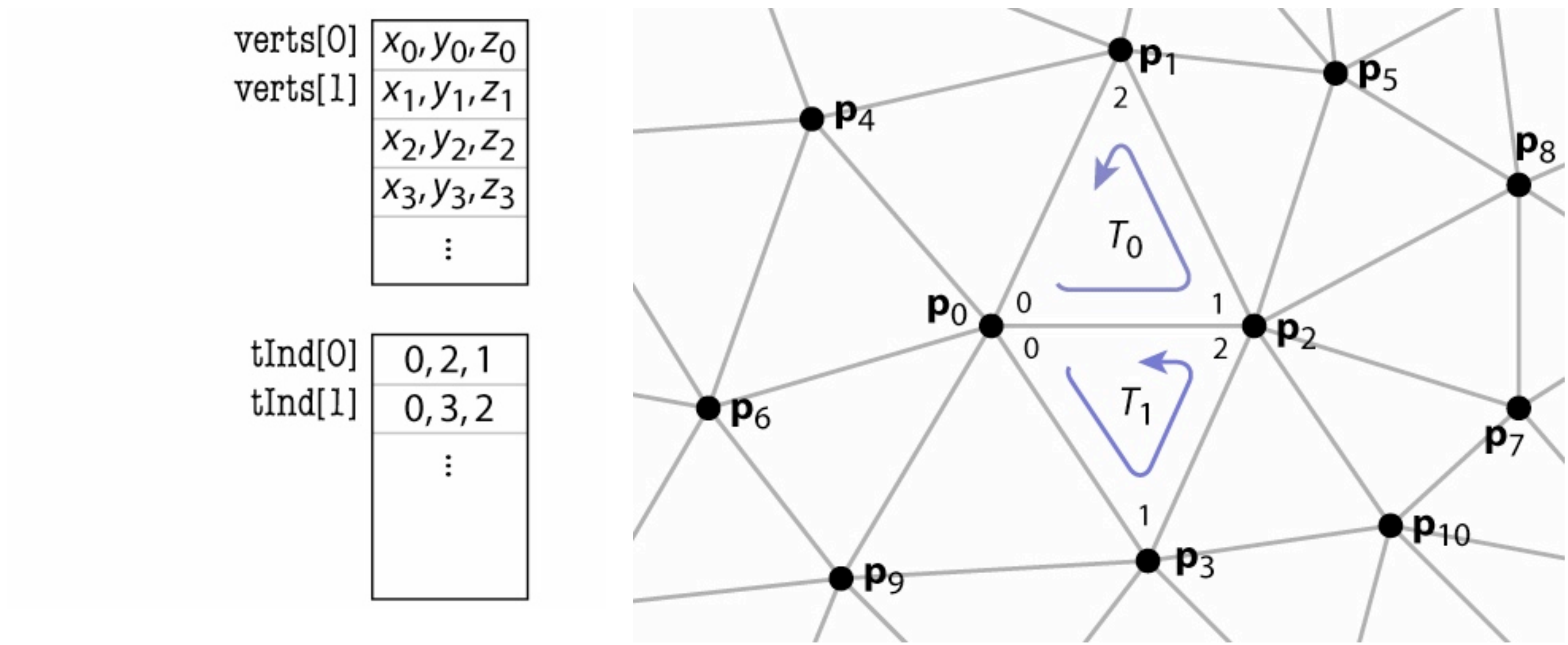
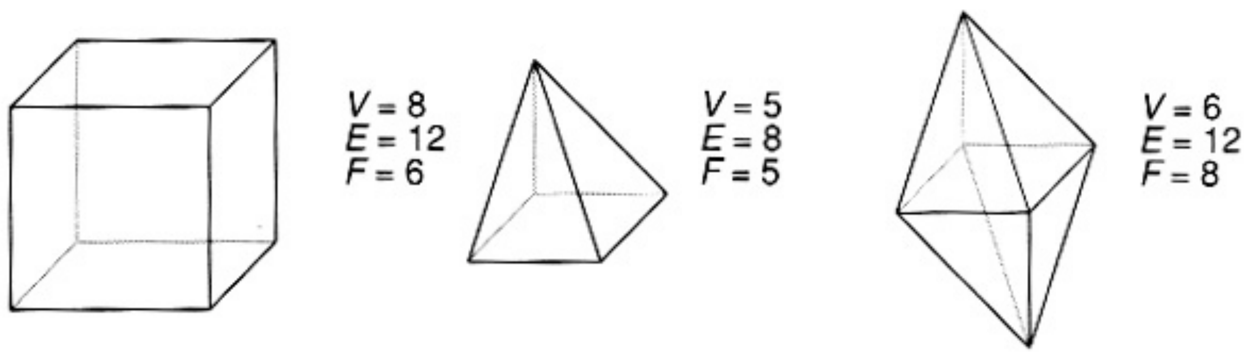


Indexed triangle set



Estimating storage space

- n_T = #tris; n_V = #verts; n_E = #edges
- Euler: $n_V - n_E + n_T = 2$ for a simple closed surface
  - and in general sums to small integer
  - argument for implication that n_T:n_E:n_V is about 2:3:1



[Foley et al.]

Indexed triangle set

- array of vertex positions
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- array of triples of indices (per triangle)
  - int[n_T][3]: about 24 bytes per vertex
    - 2 triangles per vertex (on average)
    - (3 indices x 4 bytes) per triangle
- total storage: 36 bytes per vertex (factor of 2 savings)
- represents topology and geometry separately
- finding neighbors is at least well defined

Data on meshes

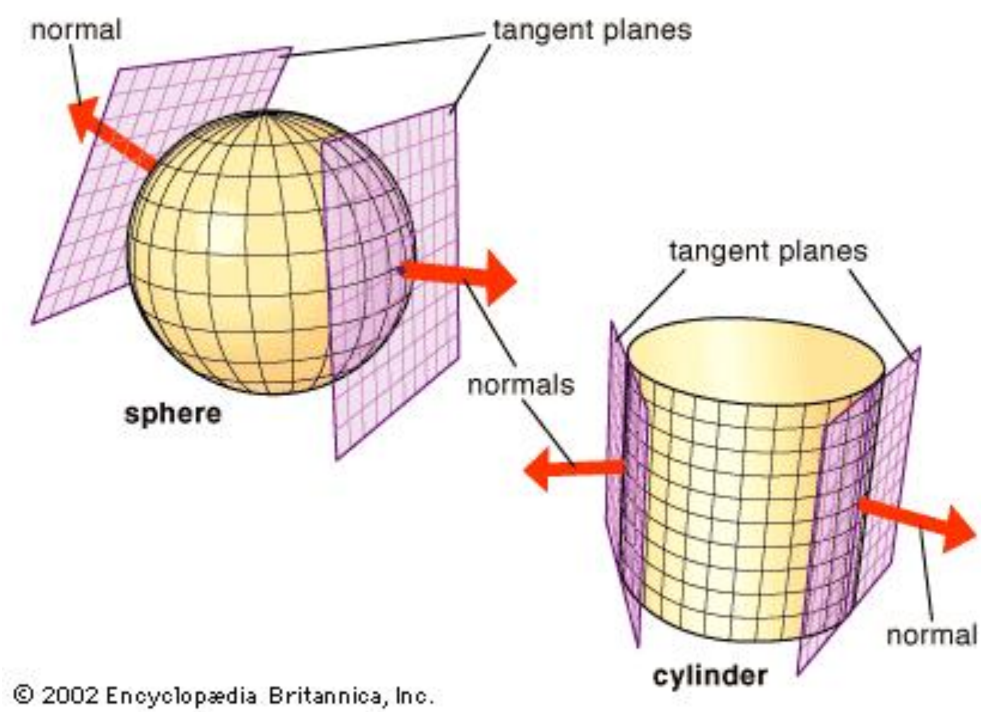
- Often need to store additional information besides just the geometry
- Can store additional data at faces, vertices, or edges
- Examples
  - colors stored on faces, for faceted objects
  - information about sharp creases stored at edges
  - any quantity that varies *continuously* (without sudden changes, or *discontinuities*) gets stored at vertices

Key types of vertex data

- Surface normals
  - when a mesh is approximating a curved surface, store normals at vertices
- Texture coordinates
  - 2D coordinates that tell you how to paste images on the surface
- Positions
  - at some level this is just another piece of data
  - position varies continuously between vertices

Differential geometry 101

- Tangent plane
  - at a point on a smooth surface in 3D, there is a unique plane tangent to the surface, called the *tangent plane*
- Normal vector
  - vector perpendicular to a surface (that is, to the tangent plane)
  - only unique for smooth surfaces (not at corners, edges)



Surface parameterization

- A surface in 3D is a two-dimensional thing
- Sometimes we need 2D coordinates for points on the surface
- Defining these coordinates is *parameterizing* the surface
- Examples:
  - cartesian coordinates on a rectangle (or other planar shape)
  - cylindrical coordinates (θ, y) on a cylinder
  - latitude and longitude on the Earth's surface
  - spherical coordinates (θ, ϕ) on a sphere

Example: unit sphere

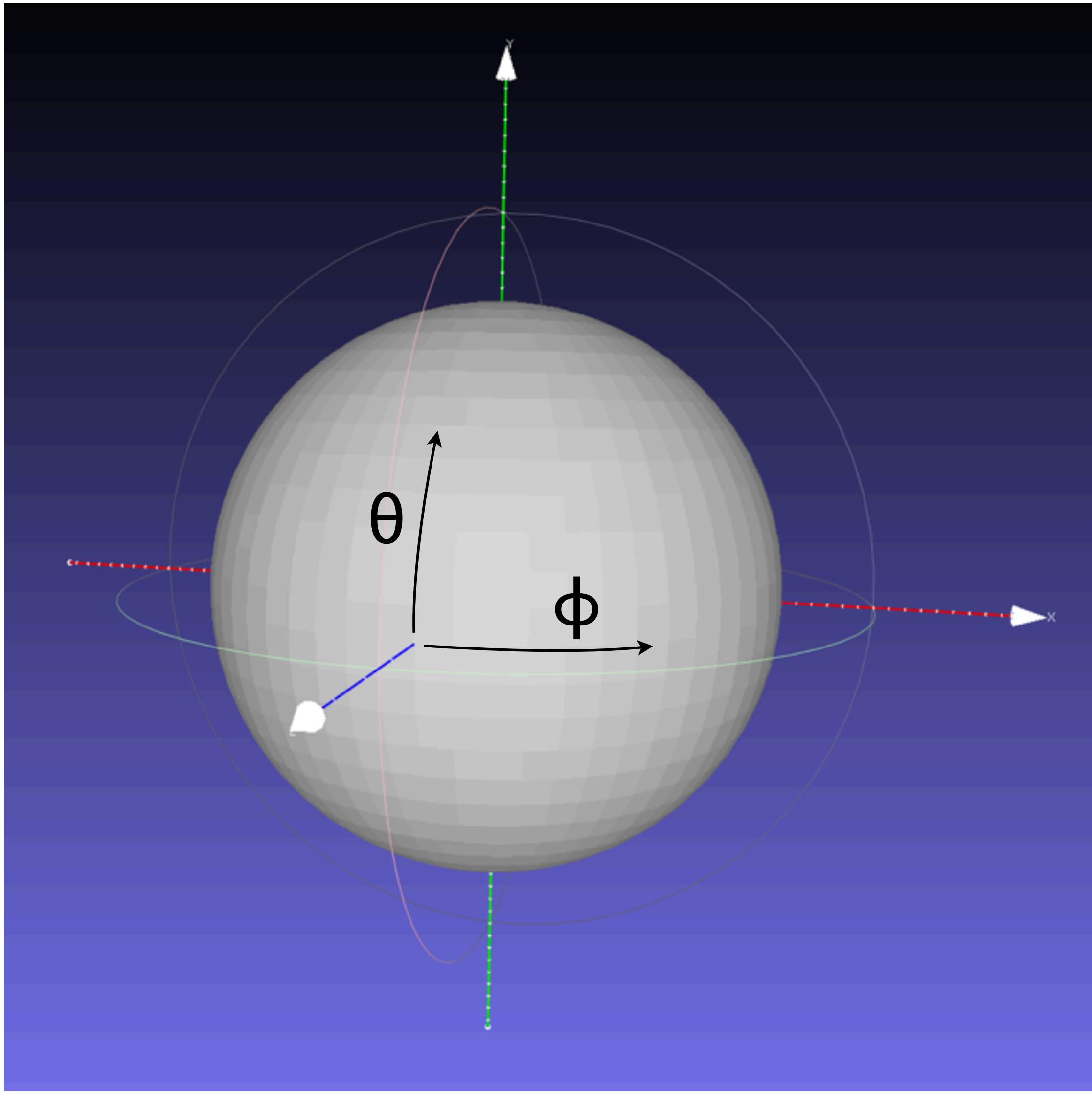
- position:

$$x = \cos\theta \sin\phi$$

$$y = \sin\theta$$

$$z = \cos\theta \cos\phi$$

- normal is position (easy!)

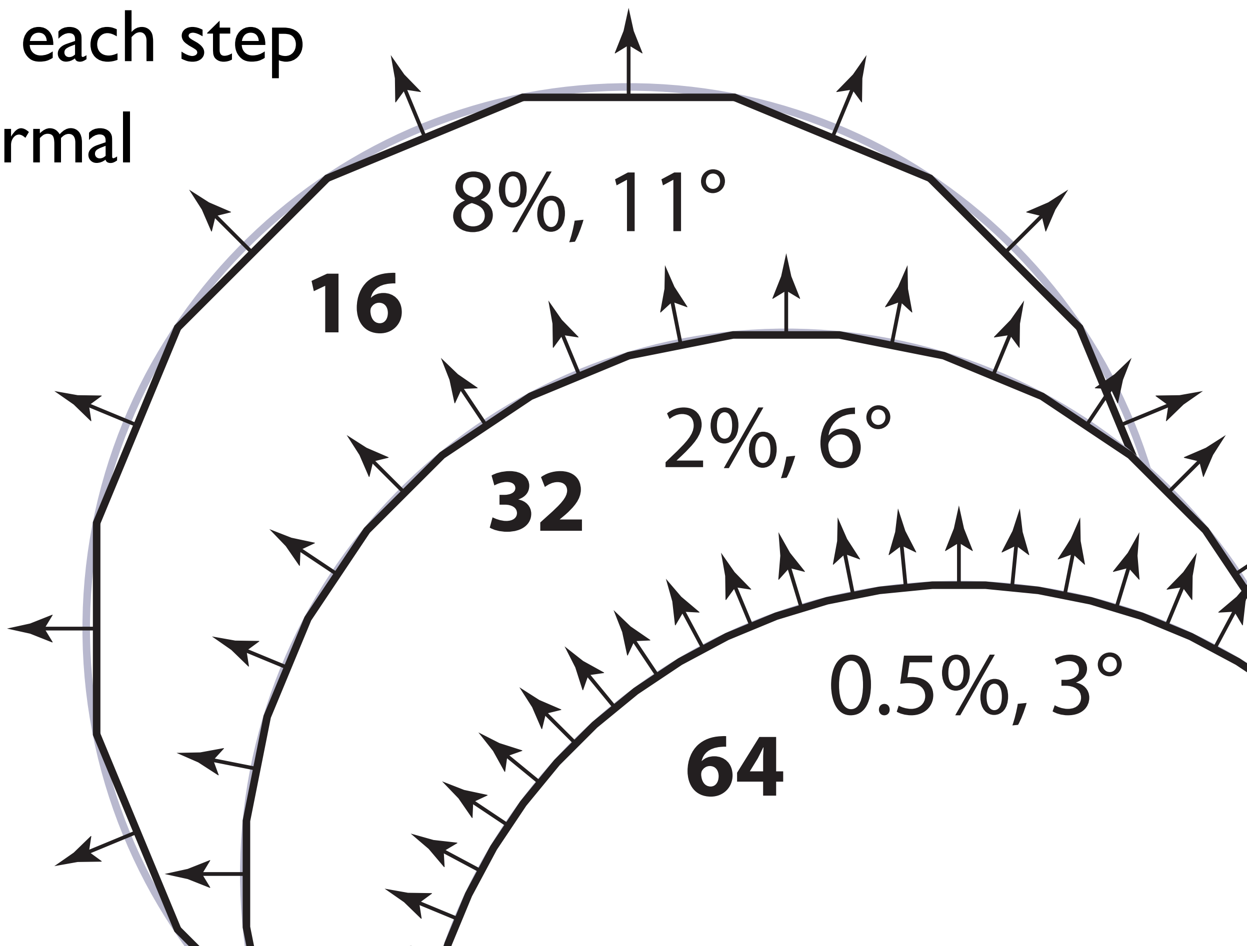


How to think about vertex normals

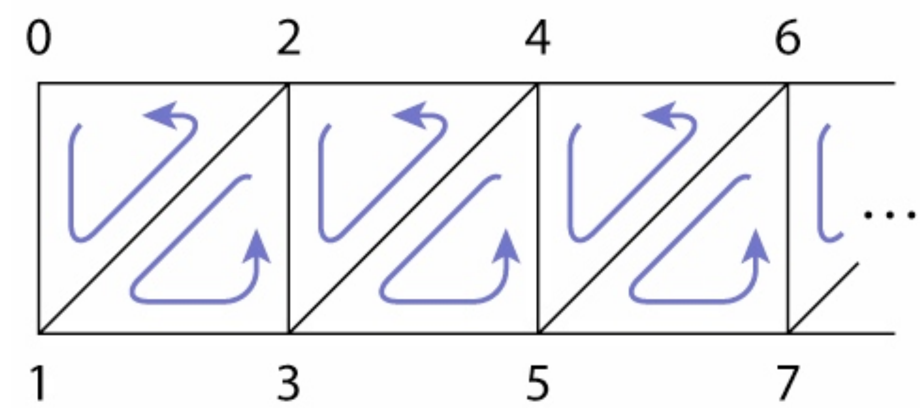
- Piecewise planar approximation converges pretty quickly to the smooth geometry as the number of triangles increases
  - for mathematicians: error is $O(h^2)$
- But the surface normals don't converge so well
  - normal is constant over each triangle, with discontinuous jumps across edges
  - for mathematicians: error is only $O(h)$
- Better: store the "real" normal at each vertex, and *interpolate* to get normals that vary gradually across triangles

Interpolated normals—2D example

- Approximating circle with increasingly many segments
- Max error in position error drops by factor of 4 at each step
- Max error in normal only drops by factor of 2

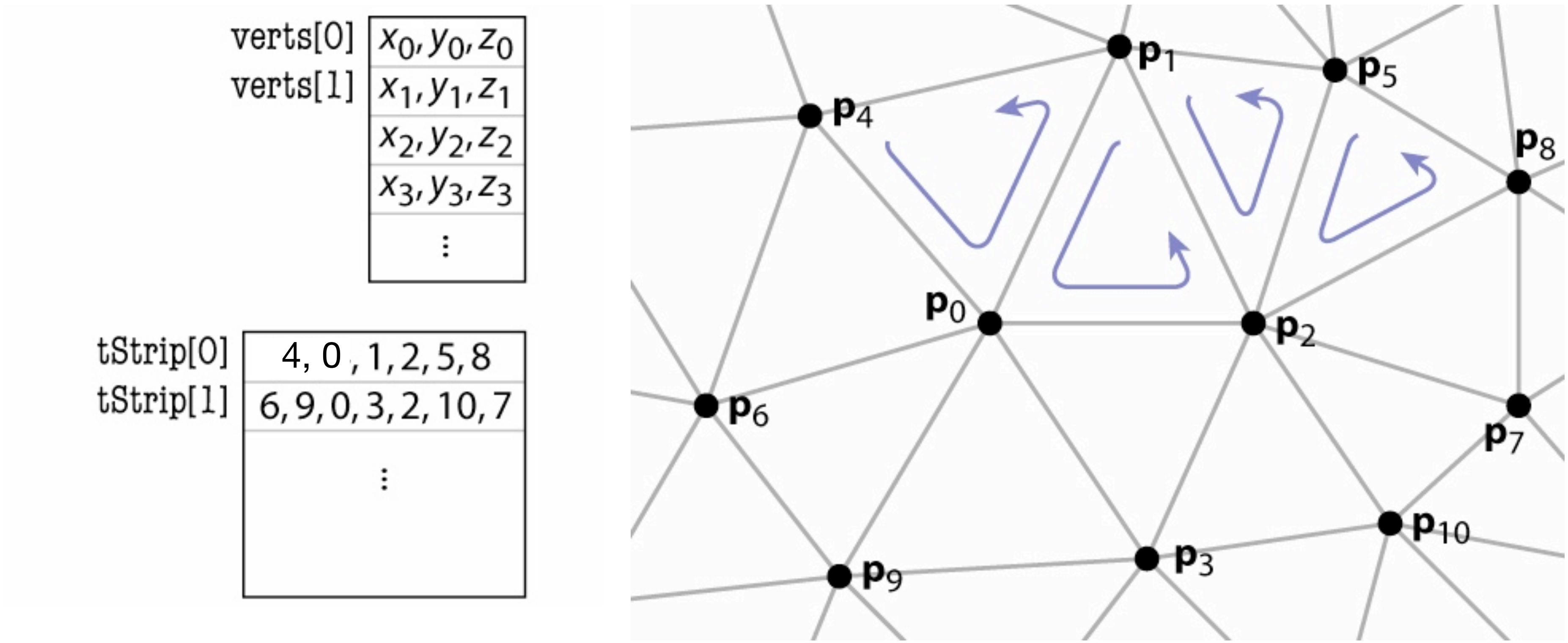


Triangle strips

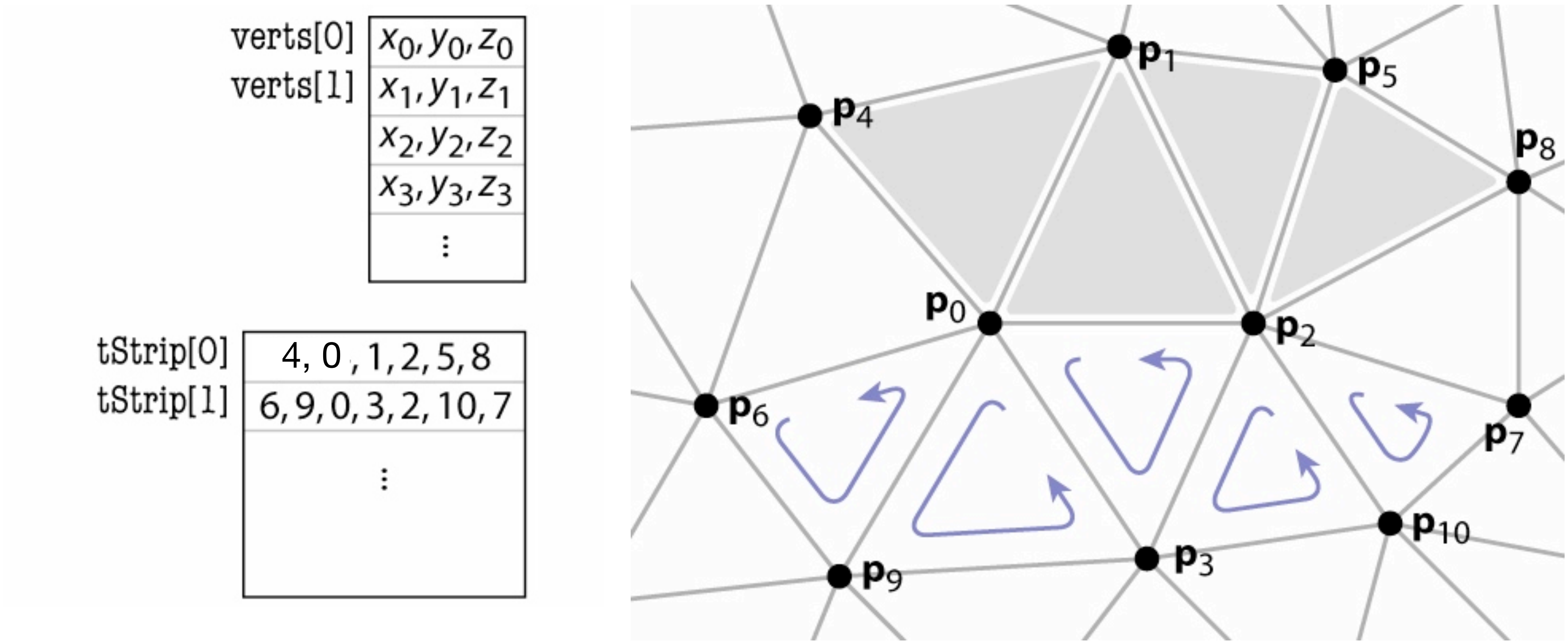


- Take advantage of the mesh property
  - each triangle is usually adjacent to the previous
  - let every vertex create a triangle by reusing the second and third vertices of the previous triangle
  - every sequence of three vertices produces a triangle (but not in the same order)
  - e. g., 0, 1, 2, 3, 4, 5, 6, 7, … leads to
    (0 1 2), (2 1 3), (2 3 4), (4 3 5), (4 5 6), (6 5 7), …
  - for long strips, this requires about one index per triangle

Triangle strips



Triangle strips

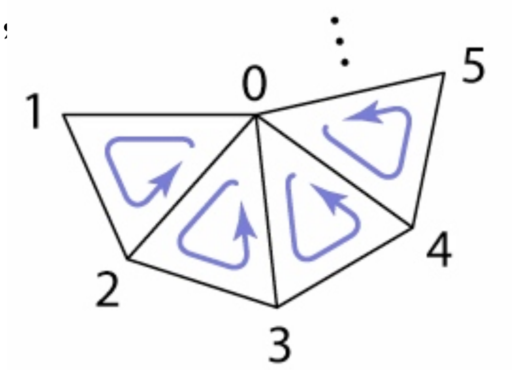


Triangle strips

- array of vertex positions
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- array of index lists
  - int[n_S][*variable*]: 2 + n indices per strip
  - on average, (1 + ε) indices per triangle (assuming long strips)
    - 2 triangles per vertex (on average)
    - about 4 bytes per triangle (on average)
- total is 20 bytes per vertex (limiting best case)
  - factor of 3.6 over separate triangles; 1.8 over indexed mesh

Triangle fans

- Same idea as triangle strips, but keep oldest rather than newest
  - every sequence of three vertices produces a triangle
  - e. g., 0, 1, 2, 3, 4, 5, … leads to
    (0 1 2), (0 2 3), (0 3 4), (0 4 5), …
  - for long fans, this requires about one index per triangle
- Memory considerations exactly the same as triangle strip

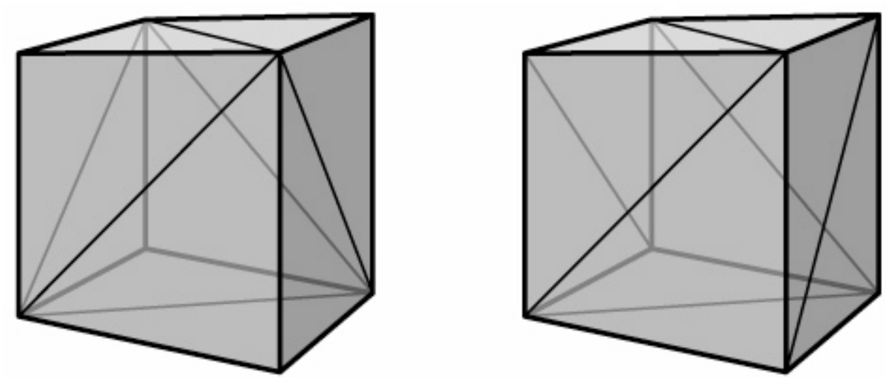


Topology vs. geometry

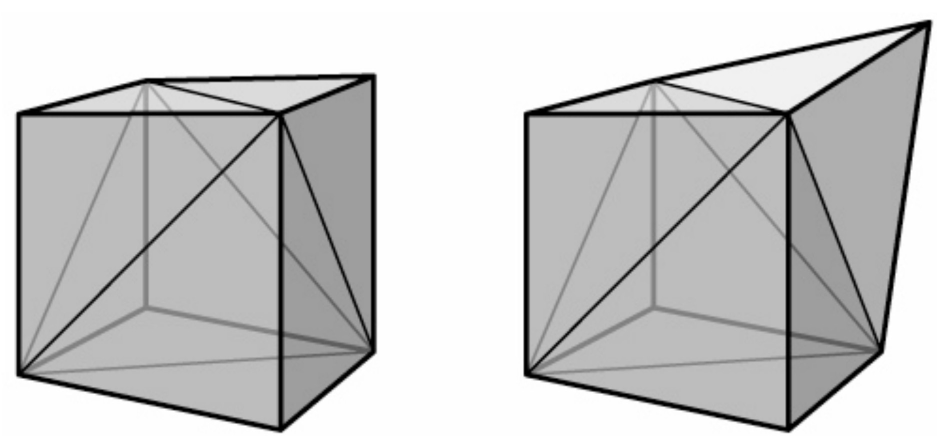
- two completely separate issues:
- **mesh topology**: how the triangles are connected (ignoring the positions entirely)
- **geometry**: where the triangles are in 3D space

Topology/geometry examples

- same geometry, different mesh topology:



- same mesh topology, different geometry:

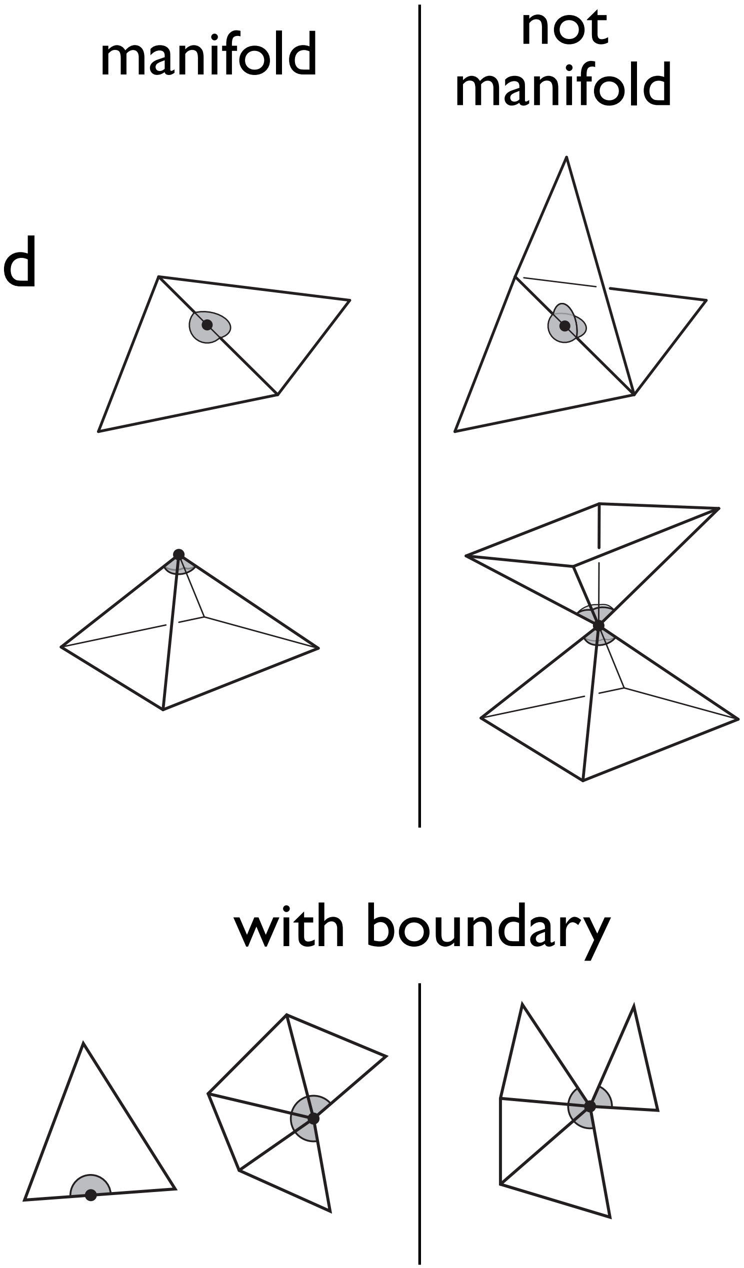


Validity of triangle meshes

- in many cases we care about the mesh being able to bound a region of space nicely
- in other cases we want triangle meshes to fulfill assumptions of algorithms that will operate on them (and may fail on malformed input)
- two completely separate issues:
  - topology: how the triangles are connected (ignoring the positions entirely)
  - geometry: where the triangles are in 3D space

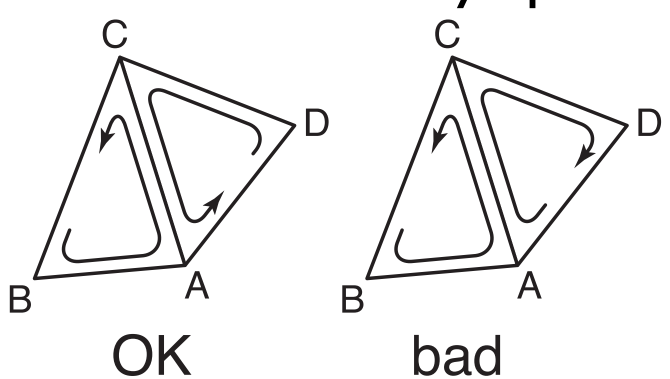
Topological validity

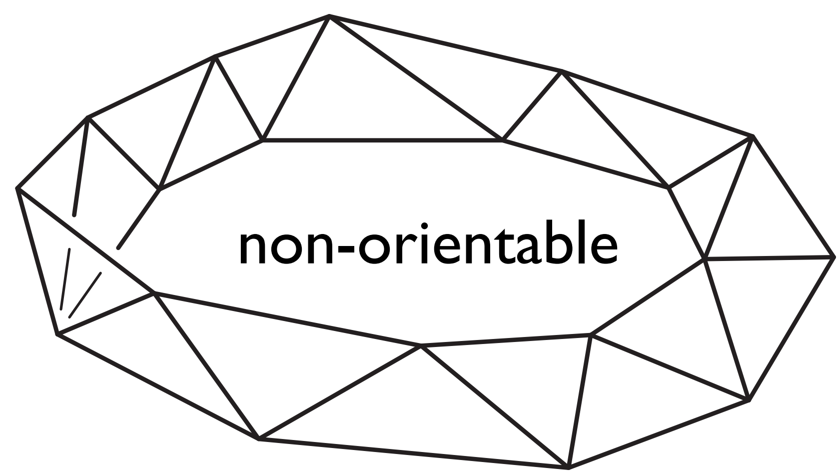
- strongest property: be a manifold
  - this means that no points should be "special"
  - interior points are fine
  - edge points: each edge must have exactly 2 triangles
  - vertex points: each vertex must have one loop of triangles
- slightly looser: manifold with boundary
  - weaken rules to allow boundaries



Topological validity

- Consistent orientation
  - Which side is the "front" or "outside" of the surface and which is the "back" or "inside?"
  - rule: you are on the outside when you see the vertices in counter-clockwise order
  - in mesh, neighboring triangles should agree about which side is the front!
  - caution: not always possible





Geometric validity

- generally want non-self-intersecting surface
- hard to guarantee in general
  - because far-apart parts of mesh might intersect