CS 421: Numerical Analysis
Fall 2000
Problem Set 1

Handed out: Wed., Sep. 6.

Due: Fri., Sep. 15 in lecture.

1. How many flops (accurate to the leading term) are required to form the product LU, given an $n \times n$ lower triangular matrix L and an $n \times n$ upper triangular matrix U?

2. Let M be an $n \times n$ *elementary* unit lower triangular matrix, that is, a matrix of the form $I - \mathbf{m}\mathbf{e}_k^T$ where $\mathbf{m} \in \mathbf{R}^n$ is a vector whose first k entries are 0's and $\mathbf{e}_k$ is the kth column of the identity matrix. See p. 39 of the text for an example and more explanation. Let $P(i,j)$ be the permutation matrix that exchanges row i with row j, but leaves other rows unchanged. Assume $i > k$ and $j > k$. Show that $P(i,j)M = NP(i,j)$, where N is some other elementary lower triangular matrix. Exactly how is N related to M?

3. Let M be an $n \times n$ elementary unit lower triangular matrix $I - \mathbf{m}\mathbf{e}_k^T$ such that all entries of $\mathbf{m}$ have absolute value at most 1. Consider solving $M\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$. Show that the absolute values of entries in $\mathbf{x}$ are all no more than twice the maximum absolute value in $\mathbf{b}$, i.e.,
$$\max_i |x(i)| \leq 2 \max_i |b(i)|.$$

4. Suppose Gaussian elimination with partial pivoting applied to an $n \times n$ matrix A runs for $k-1$ outer loop iterations, but then fails (i.e., terminates because a pivot is 0) on step k. Show that $\text{rank}(A) \geq k-1$.