CS 421: Numerical Analysis
Fall 2000
Prelim 1

Handed out: Tues., Sep. 19.

This test has five questions and lasts 75 minutes. All the questions are weighted equally. Write your answers in the exam booklet. This test is closed-book and closed-note, but you may consult a 8.5-by-11 sheet of paper that you have prepared in advance.

1. Write down an example of a 2×2 ill-conditioned linear system $A\mathbf{x} = \mathbf{b}$. Write down an example of a 2×2 well-conditioned linear system.

2. A "checkerboard" matrix A has the property that $A(i, j) = 0$ whenever $i + j$ is odd. Let A, B be two $n \times n$ checkerboard matrices. How many flops, accurate to the leading term, are required for computing the product AB?

3. Let A be a unit lower triangular matrix. Consider performing plain Gaussian elimination on A. (a) Show that the factorization $A = LU$ that would be computed by plain Gaussian elimination can in fact be computed without any flops using a fairly trivial algorithm for this special case. (b) Is plain Gaussian elimination followed by forward and back substitution a stable algorithm for solving $A\mathbf{x} = \mathbf{b}$ for this special case of A? [Hint for (b): consider $\|L\|_\infty \cdot \|U\|_\infty$ versus $\|A\|_\infty$.]

4. Consider the scalar function $f(x) = 1/x$. Show that this function is well conditioned, i.e., show that a small relative perturbation to the data (that is, x) results in a small relative perturbation to the function. Your analysis should be valid for any nonzero data.

5. Prove the following inequality, which is valid for all $\mathbf{x} \in \mathbf{R}^n$:

$$\|\mathbf{x}\|_2 \leq (\|\mathbf{x}\|_1 \cdot \|\mathbf{x}\|_\infty)^{1/2}.$$