CS 421: Numerical Analysis
Fall 2000
Practice Prelim 1

Handed out: Fri., Sep. 15.

This was a 75-minute closed-book exam.

1. What is meant by "unstable"? Give an example of an unstable algorithm.

2. Consider performing Gaussian elimination with partial pivoting on an $n \times n$ matrix A of the form

$$A = \begin{pmatrix} B & C \\ 0 & D \end{pmatrix}$$

where each submatrix is of size $n/2 \times n/2$ (and n is assumed to be even). How many flops (accurate to the leading term) are required for GEPP on this matrix? Note that flops can be save because of the block of zeros.

3. Consider solving the linear system

$$\begin{pmatrix} 10^{-4} & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

using Gaussian elimination without pivoting. For your information, the factors are

$$L = \begin{pmatrix} 1 & 0 \\ 10^4 & 1 \end{pmatrix}, U = \begin{pmatrix} 10^{-4} & 1 \\ 0 & -9999 \end{pmatrix}.$$

During the back and forward substitution processes on these matrices, on a particular step a cancellation error will likely occur that can cause a significant change to the answer. By working through the steps, pinpoint the specific operation that could cause a serious cancellation error.

4. Given $A \in \mathbf{R}^{n \times n}$, let $H(A)$ denote the maximum absolute entry in A. Prove that

$$H(A) \leq \|A\|_1 \leq nH(A).$$

5. Let $A \in \mathbf{R}^{n \times n}$ be a rank-one matrix. (Recall that a rank-one matrix can be written as the outer product $A = \mathbf{x}\mathbf{y}^T$ of two nonzero vectors). Show that GEPP applied to A (in exact arithmetic) will terminate after at most one outer loop iteration.