CS 418 Homework 4

(revised March 11, 2003)

out: Wednesday, March 5, 2003
due: Friday, March 14, 2003

Problem 1: Perspective

We would like to take a photograph of a man who is 2 meters tall and a child who is 1 meter tall in front of a house that is 6 meters high. For fun we decide to arrange for all three to appear the same height in the image.

1. If the camera is located 20 meters from the house, where should we place the two subjects (how far from the camera)?
2. What focal length is required on a 35mm camera to make the subjects 3/4 the height of the image? Note that a 35mm negative measures 36mm by 24mm.

Alfred Hitchcock's film *Vertigo* contains a famous shot in which the camera looks straight down a stairwell. Over the course of the shot, the camera zooms while simultaneously moving vertically just fast enough to maintain the size of one of the floors constant in the image. The visual effect of this is that the viewer appears to be sitting still while the bottom of the stairwell recedes.

3. Under the following assumptions, give an expression for the height of the camera above the floor at the bottom of the stairwell as a function of time for $t = 0$ to 5 sec.
   (a) The floor that appears stationary is the third floor (counting the floor of the stairwell as floor 1).
   (b) The stairwell is 3 meters across, and the floors are 3 meters apart.
   (c) The camera's zoom lens has a minimum vertical field of view of $10°$and the maximum is $60°$. It changes from one extreme to the other at a constant rate over the course of the shot.
   (d) The opening in the "fixed" floor occupies 3/4 of the frame, vertically.

Note that I have not stated whether we are zooming in or out; that is part of the problem. See Figure 1 for a rough schematic of the setup.

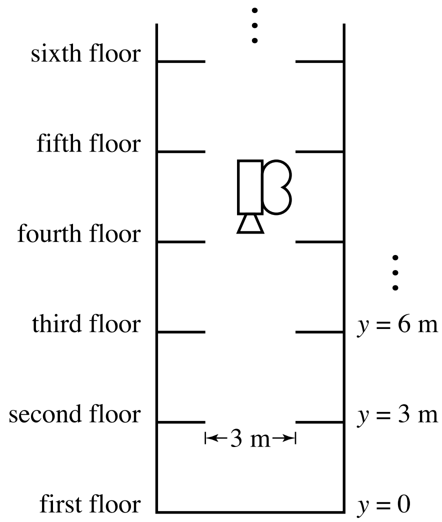
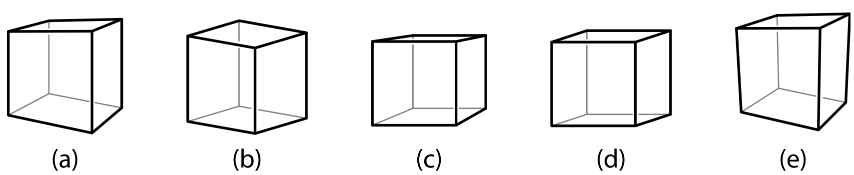


Figure 1: Schematic of setup for Problem 1-3.

Problem 2: Viewing and projective transformations



1. Match the following projections with these drawings of a cube: orthographic, oblique, one-, two-, and three-point perspective.

We saw in a previous homework that a 2D affine transformation can map any 3 non-collinear points to any other 3 points. This is not too surprising if you recognize that a 2D affine matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

has six variables in it, and each of the three point matches provides two constraints (one for x, one for y).

A 2D projective transformation is represented by the same matrix but without the requirement that first two entries in the bottom row are zero:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

This matrix has eight variables, so it ought to be able to match up four points.

2. Given the eight 2D points $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$, $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$, and $\mathbf{q}_4$, with no three of the $\mathbf{p}$s or the $\mathbf{q}$s collinear, give a 3x3 projective transformation matrix that maps $\mathbf{p}_i$ to $\mathbf{q}_i$ for $i = 1, 2, 3, 4$. All the points are represented in homogeneous coordinates (they are 3-vectors).

   Do this by first deriving a matrix to map the four points:

   $$\mathbf{a}_1 = [100]^T,$$
   $$\mathbf{a}_2 = [010]^T,$$
   $$\mathbf{a}_3 = [001]^T,$$
   $$\mathbf{a}_4 = [111]^T,$$

   to four arbitrary points $\mathbf{x}_1, \ldots, \mathbf{x}_4$. Since we are working in homogeneous coordinates, this means we are looking for a matrix M that transforms each $\mathbf{a}$ to a scalar multiple of the corresponding $\mathbf{x}$. That is, $M\mathbf{a}_i = \lambda_i \mathbf{x}_i$ for each i.

   Hint: The a, b, c form of the matrix was just for counting degrees of freedom; in solving this problem you should think of things in terms of high level linear algebra and avoid dealing with individual components.

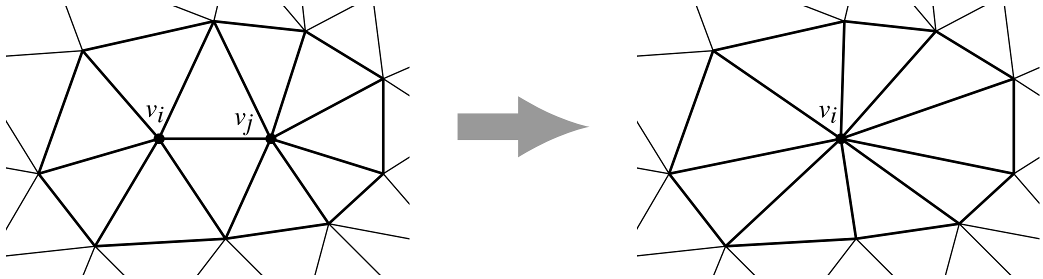
   Hint: It is useful to solve for the λ_is on the way to finding M.

   Hint: If you write out the equations leaving out $\mathbf{x}_4$ and $\mathbf{a}_4$, concatenating the column vectors into a single matrix equation, it gives you a convenient form for M.

It is OK to use matrix multiplication and inverse to construct your answers. It is also OK if your answer is not normalized to have a 1 in the lower right position.

Problem 3: Triangle mesh operations

A common editing operation on triangle meshes is the *edge collapse*. Given a particular edge in the mesh, denoted by its endpoints v_i and v_j, the operation removes one vertex, two triangles, and three edges to result in a slightly simpler triangle mesh:

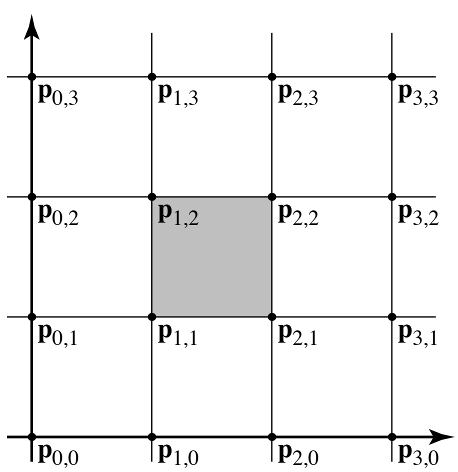


Implementing the operation on a mesh represented using an indexed triangle set is quite simple, given a manifold mesh. The two triangles that share the edge (the manifold property guarantees there are exactly two) are removed, the vertex v_j is removed, and all the remaining triangles that reference v_j are renumbered to reference v_i instead. The position of v_i is set to the midpoint of the edge being collapsed. (In practice there will be some topological data structures to be maintained also, but we will ignore those for this problem.)

This operation is easy to state, but it can sometimes cause a mesh to become non-manifold. What are the conditions under which it is safe to collapse the edge v_i–v_j in a triangle mesh? That is, if the mesh was manifold before the edge collapse operation, how can we examine the mesh and decide whether it will still be manifold after the operation? Your answer can be in the form of pseudocode that would be run before the edge collapse operation to ensure that it is safe. (Performing the collapse and then checking to see if the mesh is broken does not count.) You may assume there is a data structure available that can find all the triangles incident on a vertex.

Problem 4: Spline surfaces

A 1-by-1 unit cubic B-spline surface is defined by 16 control points $\mathbf{p}_{0,0}, \ldots, \mathbf{p}_{3,3}$ and is valid over the square $[1, 2] \times [1, 2]$:



(Note that the labels $\mathbf{p}_{i,j}$ in the diagram are to indicate the (s, t) parameter space points that correspond to the control points, which are themselves points in 3D space.)

In the same way as the B-spline curve from the last homework was a convolution of the basis function b with a list of control points, the position $\mathbf{p}(s, t)$ to which a point (s, t) in the parameter space maps in 3D is a convolution of the basis function $b(s)b(t)$ with a grid of control points:

$$\mathbf{p}(s, t) = \sum_{i=0}^{3} \sum_{j=0}^{3} b(s - i)b(t - j)\mathbf{p}_{i,j}.$$

1. Sketch a graph of this basis function.
2. Write expressions for the two tangent vectors at (s, t). Your answer should be fully expanded so that it is in terms of elementary operations.
3. Write an expression for the normal at (s, t). For this part, your expression does not need to be fully expanded out.
4. Are the tangent vectors defined by a convolution in the same way as the position is? If so, sketch the graphs of the basis functions. If not, briefly explain why.
5. Is the normal vector defined by a convolution in the same way as the position is? If so, sketch the graph of the basis function. If not, briefly explain why.

If drawing the 3D graphs in perspective is difficult, use a two-view orthographic projection (with projections along the s and t axis) instead.