

CS412/CS413

Introduction to Compilers

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Lecture 14: Attribute Grammars

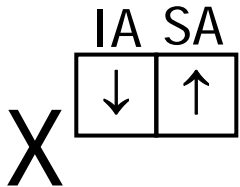
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# Attribute Grammars

- An extension of CFGs to define “semantics” of sentences in a language
- Knuth, 1968
- Intuition:
  - Decorate each parse-tree node with attributes, i.e., variables defined by equations in terms of constants and neighboring attributes in the tree
  - Evaluate the attributes like a spreadsheet evaluates cells defined by equations, i.e., order of evaluation determined automatically

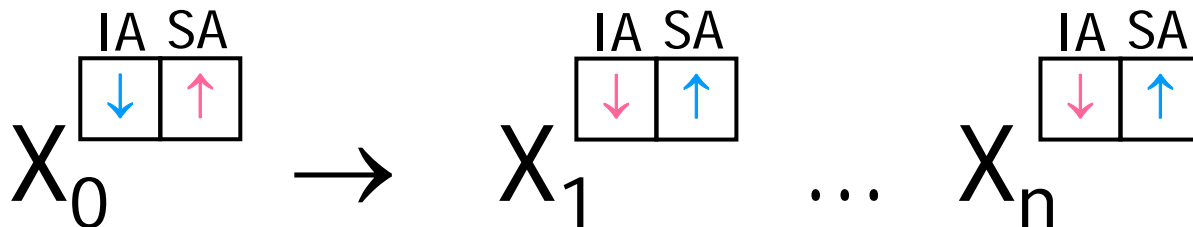
# Attributes

- Let  $G$  be a context-free grammar  $\langle V, \Sigma, S, \rightarrow \rangle$
- Associate with every  $X \in (V \cup \Sigma)$  a set of attributes  $A(X)$
- Notation. If  $a \in A(X)$ , we denote it  $X.a$
- Let each  $A(X)$  be partitioned into disjoint sets
  - synthesized attributes,  $SA(X)$
  - inherited attributes,  $IA(X)$

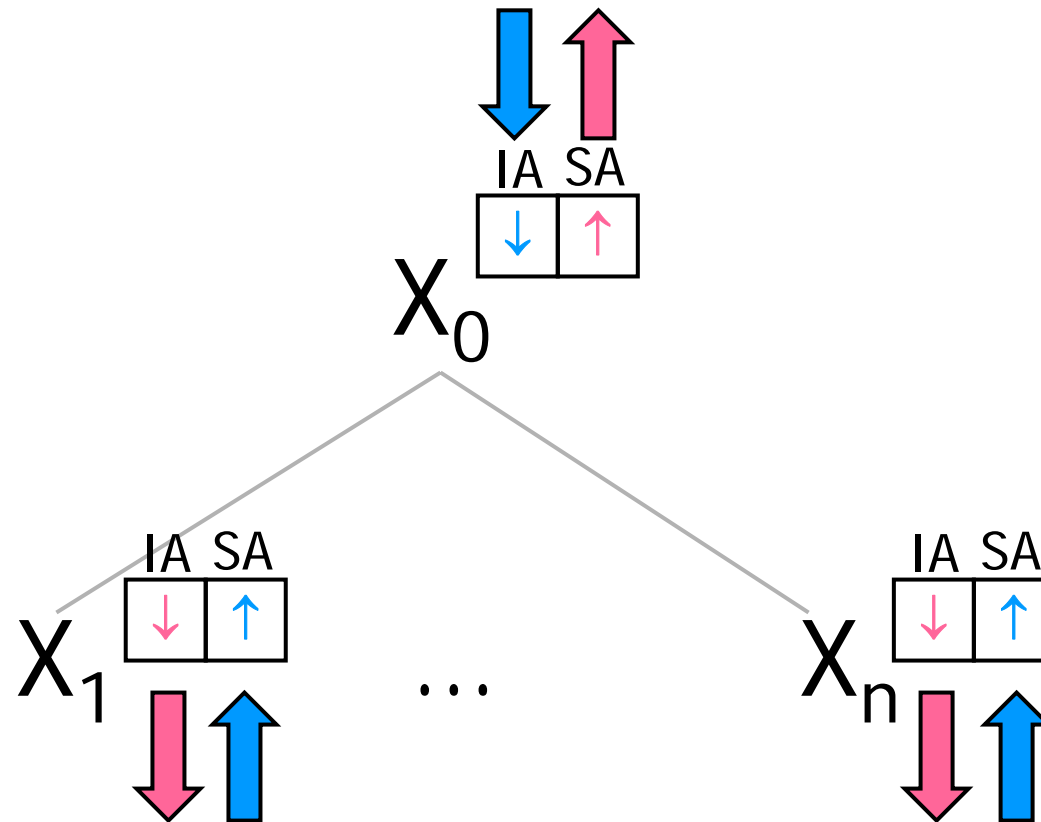


# Occurrences

- Let  $p$  be a production  $X_0 \rightarrow X_1 \dots X_n$  of  $G$
- Each  $X_i$  is a **symbol occurrence** of  $p$
- **Input( $p$ )** =  $IA(X_0) \oplus SA(X_1) \oplus \dots \oplus SA(X_n)$
- **Output( $p$ )** =  $SA(X_0) \oplus IA(X_1) \oplus \dots \oplus IA(X_n)$
- Each attribute in Input( $p$ ) or Output( $p$ ) is an **attribute occurrence** of  $p$



# Input and Output Occurrences



# Equations

- Let  $p$  be a production  $X_0 \rightarrow X_1 \dots X_n$  of  $G$
- An **attribute equation** of  $p$  defines  $a \in \text{Output}(p)$  in terms of attributes in  $\text{Input}(p) \oplus \text{Output}(p)$
- An attribute grammar is **well formed** if
  - $\text{IA}(S) = \emptyset$
  - $\text{SA}(a) = \emptyset$ , for all  $a \in \Sigma$
  - Every output attribute of every production has precisely 1 defining equation
- An attribute grammar is in **normal form** if only input attributes occur on RHS of equations

# Example

- Productions

$S \rightarrow E$

$E \rightarrow E + E$

$E \rightarrow \text{NUM}$

$E \rightarrow \text{ID}$

$E \rightarrow \mathbf{let\ ID = E\ in\ E}$

- Sample sentence

**let**  $x = 1$  **in** **let**  $y = x+1$  **in**  $x+y$

- Attributes

Inherited:  $E.\text{env}$

Synthesized:  $S.\text{value}$ ,  $E.\text{value}$ ,  $\text{NUM}.\text{value}$ ,  $\text{ID}.\text{name}$

# Example, cont.

$S \rightarrow E$

$E.\text{env} = \text{EmptyEnvironment}()$

$S.\text{value} = E.\text{value}$

$E_0 \rightarrow E_1 + E_2$

$E_1.\text{env} = E_0.\text{env}$

$E_2.\text{env} = E_0.\text{env}$

$E_0.\text{value} = E_1.\text{value} + E_2.\text{value}$

$E \rightarrow \text{NUM}$

$E.\text{value} = \text{NUM}.\text{value}$

$E \rightarrow \text{ID}$

$E.\text{value} = \text{Lookup}(\text{ID}.\text{name}, E.\text{env})$

$E_0 \rightarrow \mathbf{let\ ID = E_1\ in\ E_2}$

$E_1.\text{env} = E_0.\text{env}$

$E_2.\text{env} = \text{Insert}(\text{ID}.\text{name}, E_1.\text{value}, E_0.\text{env})$

$E_0.\text{value} = E_2.\text{value}$



# Direct Dependency Graph

- Let  $p$  be a production  $X_0 \rightarrow X_1 \dots X_n$  of  $G$
- $D_p$ , the **direct dependency graph** of  $p$ , is the directed graph  $\langle A(p), E(p) \rangle$ , where
  - Nodes:  $A(p) = \text{Input}(p) \oplus \text{Output}(p)$
  - Edges:  $E(p) = \{ \langle a_1, a_2 \rangle \mid a_2 \text{ depends on } a_1 \}$
- An attribute grammar is **locally acyclic** if for every production  $p$ ,  $D_p$  is acyclic

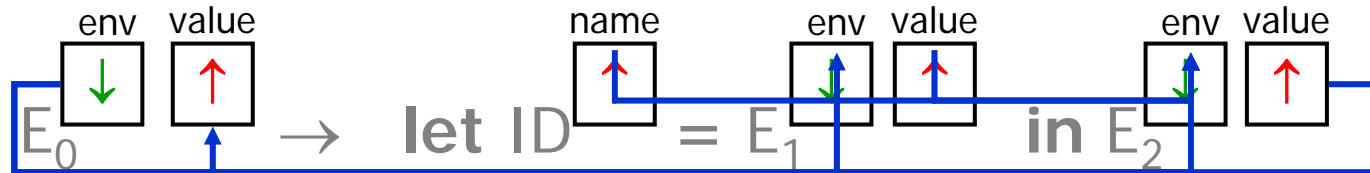
# Example, cont.

$E_0 \rightarrow \text{let ID} = E_1 \text{ in } E_2$

$E_1.\text{env} = E_0.\text{env}$

$E_2.\text{env} = \text{Insert}(\text{ID.name}, E_1.\text{value}, E_0.\text{env})$

$E_0.\text{value} = E_2.\text{value}$

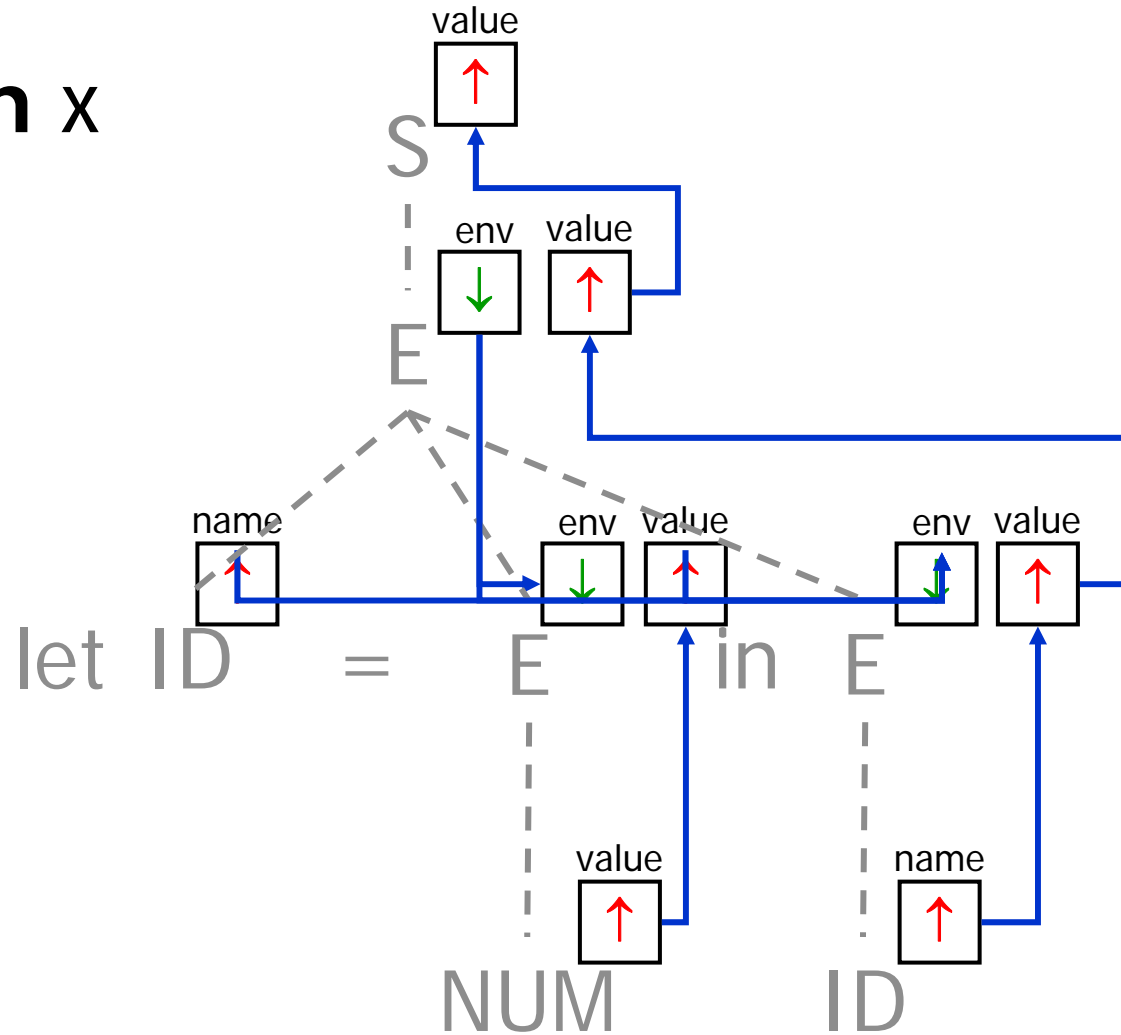


# Dependency Graph

- Let  $T$  be a derivation tree for some  $x \in L(G)$ 
  - Each subtree corresponding to production  $p$  is a **production instance** in  $T$
  - Each symbol occurrence in  $p$  is a **symbol instance** in  $T$
  - Each attribute occurrence in  $p$  is an **attribute instance** in  $T$
  - Each edge in  $D_p$  is a **dependence instance** in  $T$
- $D(T)$ , the **dependency graph** for  $T$ , has
  - Nodes: the attribute instances of  $T$
  - Edges: the dependence instances of  $T$

# Example, cont.

**let x = 1 in x**



# Noncircularity

- An attribute grammar is **noncircular** if for every derivation tree  $T$ ,  $D(T)$  is acyclic
- We are only interested in noncircular grammars

# Evaluation

- Given a derivation tree  $T$ , evaluate the attribute instances of  $T$  in **topological order** w.r.t.  $D(T)$
- **Dynamic evaluation**: Obtain the topological order using either
  - topological sort, or
  - depth first search backwards from nodes of out-degree 0
- **Static evaluation**: Analyze the grammar in advance and determine tree traversal schemes with interleaved evaluations such that for any possible derivation tree  $T$ , evaluations will be in topological order

# Topological Sort

$W := \emptyset;$

**for** each node  $n$  with  $\text{indegree}(n)=0$  **do**

$W := W \cup \{n\};$

**while**  $W \neq \emptyset$  **do**

select  $n$  from  $W$ ;

remove  $n$  from  $W$ ;

**for** each successor  $n'$  of  $n$  **do**

remove edge  $\langle n, n' \rangle$ ;

**if**  $\text{indegree}(n')=0$  **then**  $W := W \cup \{n'\}$

# S-attributed

- An attribute grammar is S-attributed iff it only has synthesized attributes.
- Evaluation: Use end-order traversal of derivation tree (e.g., during a bottom-up parse) to obtain topological evaluation order
- Yacc, Bison, and Cup only support S-attributed grammars



# L-attributed

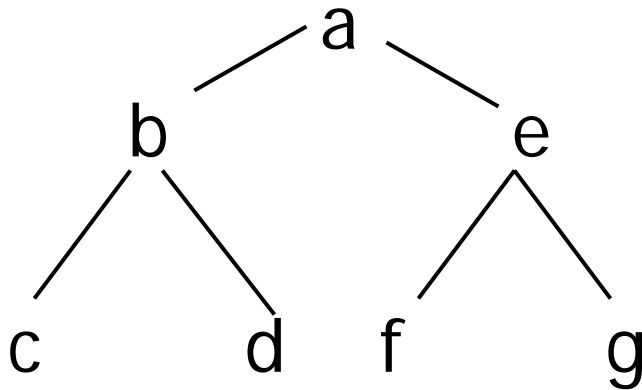
- Defined so that can be evaluated in one left-to-right pass, (e.g., during a top-down parse)
- Every RHS inherited attribute depends only on
  - LHS inherited
  - any RHS attribute to the left
- Every LHS synthesized attribute depends only on
  - LHS inherited
  - any RHS

# Alternating Pass Evaluation

- Alternate between L-attributed and R-attributed passes.
- In pass  $i$ , all attributes evaluated in previous passes are known values available for during the evaluations during pass  $i$
- An attribute grammar is **alternating pass** if there exists  $k$  alternating passes sufficient to evaluate any derivation tree  $T$

# Efficient Use of Sequential Storage

- Reverse of left-to-right endorder is right-to-left preorder (and vice-versa) so can make efficient use of sequential storage medium



Endorder: c d b f g e a

Right-to-left preorder: a e g f b d c