CS412/CS413

Introduction to Compilers

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Lecture 9: LR Parsing

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LR(k) Grammars

- LR(k) = Left-to-right scanning, Right-most derivation, k look-ahead characters

- Main cases: LR(0), LR(1), SLR(k), and LALR(1)

- Parsers for LR(0) Grammars:
  - Know whether to shift or reduce without consulting the lookahead symbol
  - Give intuition and techniques relevant for creating parsers for all grammar classes to be considered

Building LR(0) Parsing Tables

- To build the parsing table:
  - Define states of the parser
  - Build a DFA to describe the transitions between states
  - Use the DFA to build the parsing table

Viable Prefix

- γ is a viable prefix for G iff there is some derivation

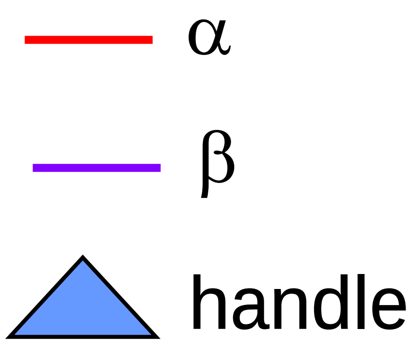
  $$S \Rightarrow^* \alpha A z \Rightarrow \alpha\beta z$$

  where γ is a prefix of $\alpha\beta$

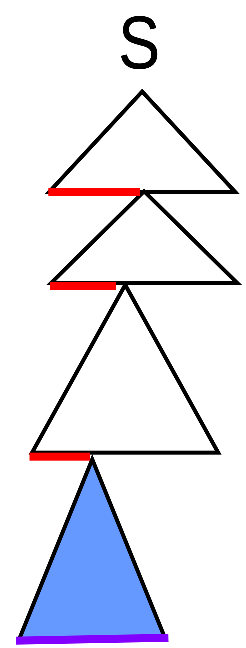
- $\{\gamma \mid \gamma$ is a viable prefix of G$\}$ is a regular language, i.e., it can be recognized by a DFA known as the Canonical LR(0) Machine

Viable Prefix (Informally)

- γ is a viable prefix for G if it is a prefix of a sentential form derived from S that does not extend past the end of the handle of the sentential form.



$$\boxed{S \Rightarrow^* \alpha A z \Rightarrow \alpha\beta z}$$



LR(0) Items

- An LR(0) item for G is a triple $\langle A, \beta_1, \beta_2 \rangle$ such that $A \rightarrow \beta_1 \beta_2$ is a production of G. The item $\langle A, \beta_1, \beta_2 \rangle$ is denoted by $[A \rightarrow \beta_1 \, . \, \beta_2]$

Validity of LR(0) Items

- The item $[A\rightarrow\beta_1 . \beta_2]$ is valid for viable prefix $\alpha\beta_1$ iff $S \Rightarrow^* \alpha A z \Rightarrow \alpha\beta_1\beta_2 z$
- Note:
  - β_1 may be ε
  - β_2 may be ε
- For any viable prefix α, let $V(\alpha)$ denote the set of LR(0) items that are valid for α.

Sets of Valid Items

- Observations
  - There are only finitely many distinct LR(0) items for a given G.
  - Thus, there are only finitely many sets of LR(0) items for G.
- Sets of valid items for viable prefixes of G will serve as the states of a DFA, i.e., the canonical LR(0) machine.

Relation $\downarrow$

- The relation $\downarrow$ on LR(0) items is defined by $I \downarrow I'$ iff $\exists$ A, B, β_1, β_2 , β_3 such that

  $I = [A \to \beta_1 . B\beta_3]$

  $I' = [B \to .\beta_2]$

- Lemma. Let I, I' be as above. If $I \in V(\alpha\beta_1)$ and $I \downarrow I'$, then $I' \in V(\alpha\beta_1)$.
  - $I \in V(\alpha\beta_1)$ implies $S \Rightarrow^* \alpha A z \Rightarrow \alpha\beta_1 B\beta_3 z$
  - Assuming G has no useless productions, $\exists y$ such that $\beta_3 \Rightarrow^* y$
  - Thus, $S \Rightarrow^* \alpha A z \Rightarrow \alpha\beta_1 B\beta_3 z \Rightarrow^* \alpha\beta_1 B y z \Rightarrow \alpha\beta_1\beta_2 y z$
  - Thus, I' (i.e., $[B \to .\beta_2]$) $\in V(\alpha\beta_1)$

Relation $\rightarrow_X$

- For any $X \in (V \cup \Sigma)$, the relation $\rightarrow_X$ is defined by $I \rightarrow_X I'$ iff $\exists$ A, β_1, β_3 such that

  $I = [A \rightarrow \beta_1 . X \beta_3]$

  $I' = [A \rightarrow \beta_1 X . \beta_3]$

- Lemma. Let I, I' be as above. If $I \in V(\alpha\beta_1)$ then $I' \in V(\alpha\beta_1 X)$.
  - $I = [A \rightarrow \beta_1 . X\beta_3] \in V(\alpha\beta_1)$ implies

    $S \Rightarrow^* \alpha A z \Rightarrow \alpha\beta_1 X \beta_3 z$

    which by definition means I' (= $[A \rightarrow \beta_1 X . \beta_3]$) $\in V(\alpha\beta_1 X)$

Technical Details

- Start symbol never appears on RHS
  - It is convenient if the start symbol never appears on the RHS of any production.
  - Given $G = \langle V, \Sigma, S, \rightarrow \rangle$, let $S' \notin V$ and

    $G' = \langle V, \Sigma, S', \rightarrow \cup \{S' \rightarrow S\} \rangle$
  - Assume that the grammars we work with have the form of G'.
- If S is a set and R is a relation, then

  $SR = \{y \mid x \in S \text{ and } \langle x,y \rangle \in R\}$

  SR is called S mapped by R

V(ε), the base case

- Let S' be the start symbol of G. Then
  - $V(\varepsilon) = \{\ [S' \rightarrow .S\ ]\ \}\downarrow^*$

  (i.e., the "initial item" of G $\{[S' \rightarrow .S\ ]\}$ mapped by the reflexive transitive closure of the $\downarrow$ relation.)
- If Q is a set of items, we call $Q\downarrow^*$ the closure(Q).

$V(\alpha X)$, the inductive case

- For any α and X, $V(\alpha X) = V(\alpha)\rightarrow_X\downarrow^*$
- For any set Q of items, we call $Q\rightarrow_X\downarrow^*$ the X-successor of Q, or Goto(Q,X).

Canonical LR(0) Machine

- States: Sets of valid items
- Transition function: Goto, as defined above.
- Algorithm: To compute all sets of valid items

  STATES := $V(\varepsilon)$

  while $\exists$ Q $\in$ STATES, X$\in$(V$\cup\Sigma$) such that

  Goto(Q,X) $\notin$ STATES

  do STATES := STATES $\cup$ { Goto(Q,X) }

- Clearly, this terminates, as STATES is bounded above by the Powerset(LR(0) items)

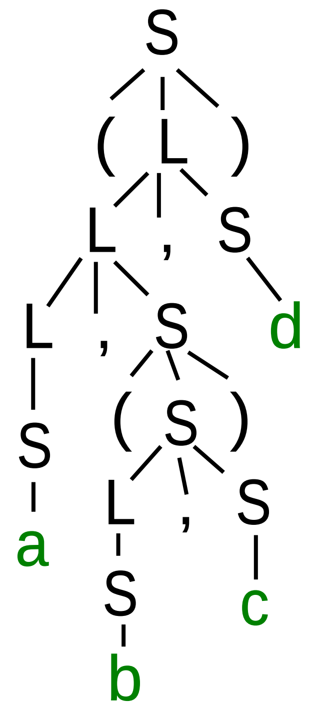
LR(0) Grammar

- Nested lists:

  $S \rightarrow (L) \mid id$
  $L \rightarrow S \mid L,S$

- Sample strings
  - (a,b,c)
  - ((a,b),(c,d),(e,f))
  - (a,(b,c,d),((f,g)))

Parse tree for
(a, (b,c), d)



Start State

Grammar
$S \to (L) \mid id$
$L \to S \mid L, S$

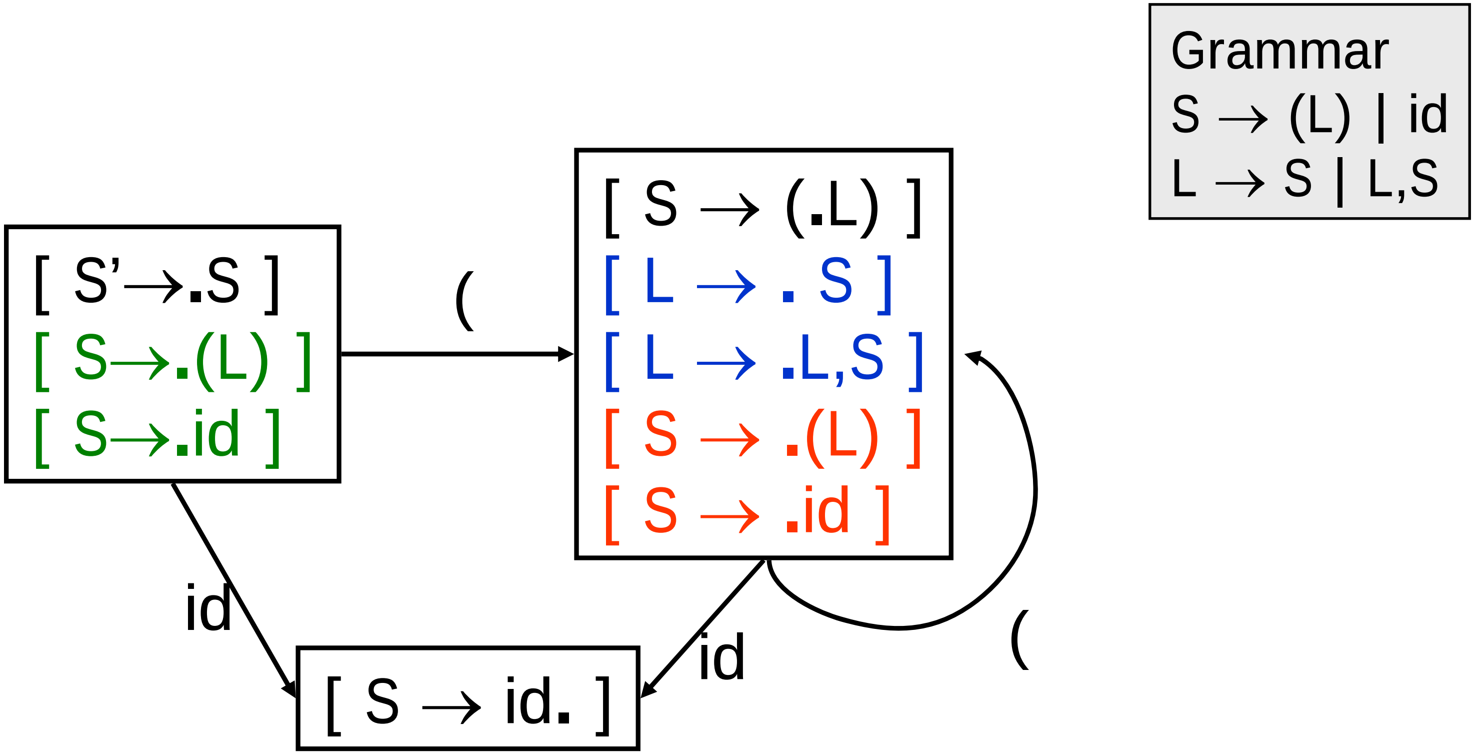
- Start state
  - $V(\varepsilon) = \{ [S' \to .S] \}\downarrow^*$
    $= \{ [S' \to .S] [S \to .(L)], [S \to .id] \}$
- Closure of a parser state Q:
  - Start with Closure(Q) := Q
  - Then for each item in Q:

    $A \to \alpha.B\beta$

    add the items for all the productions $B \to \gamma$ to the closure of Q:

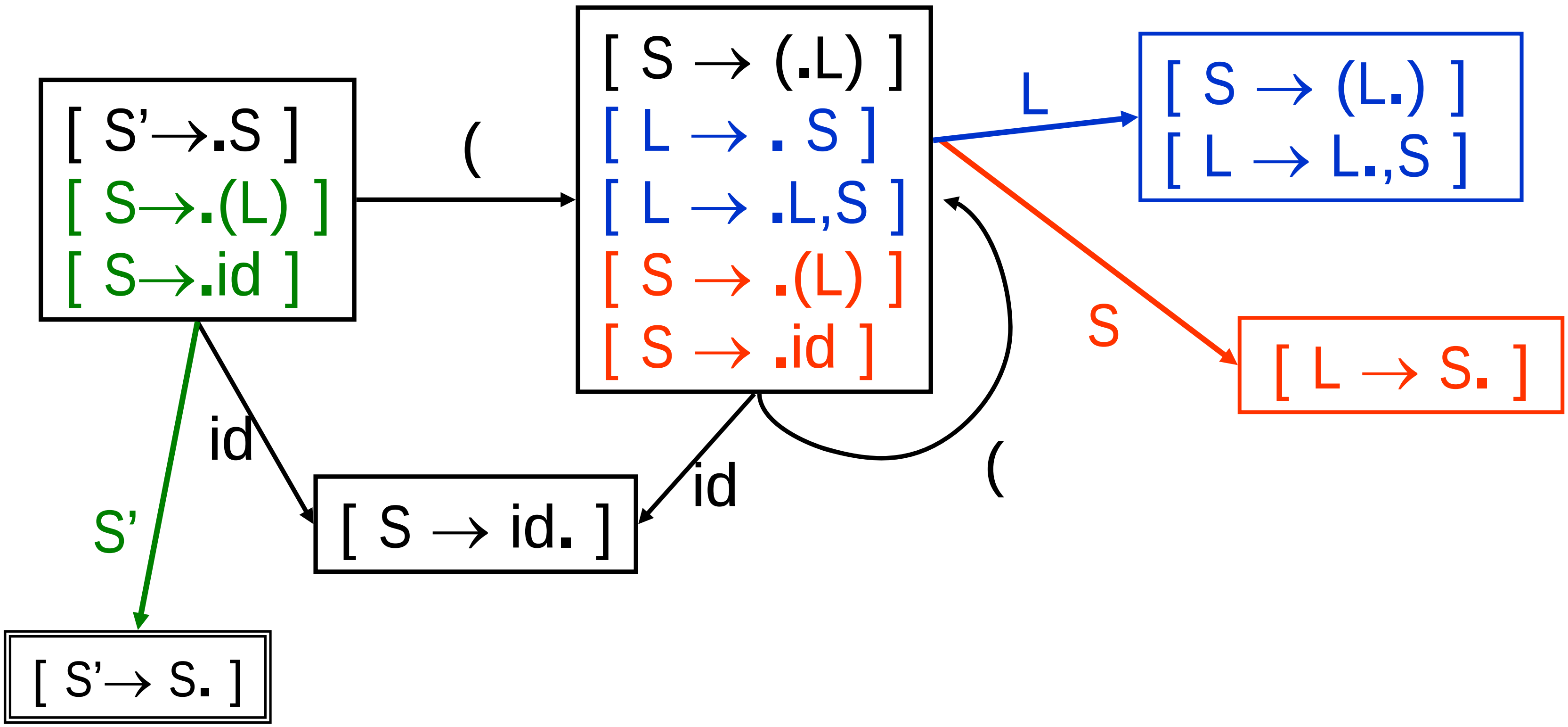
    $B \to .\gamma$

Goto: Terminal Symbols



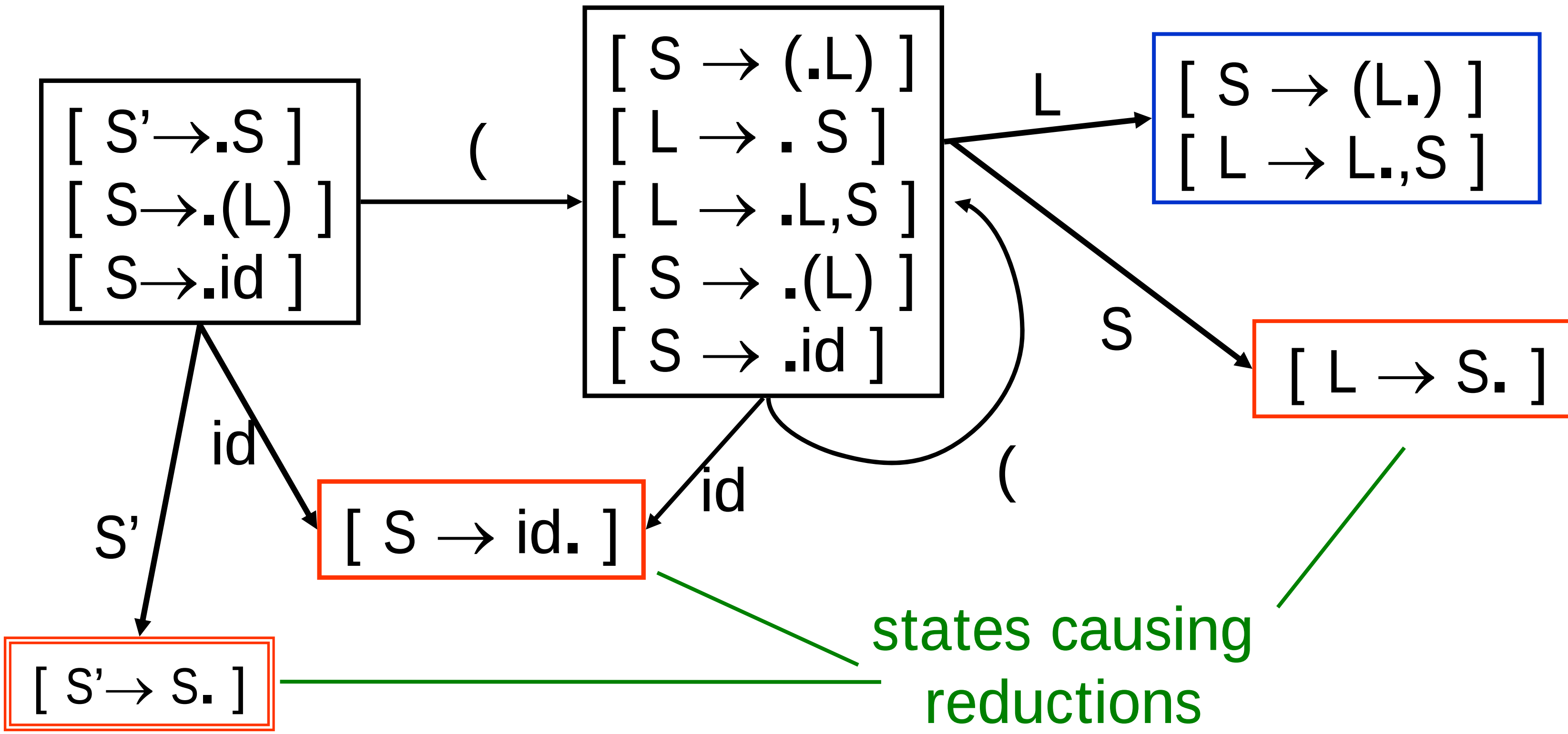
In new state, include all items that have appropriate input symbol just after dot, advance dot in those items, and take closure.

Goto: Nonterminal Symbols

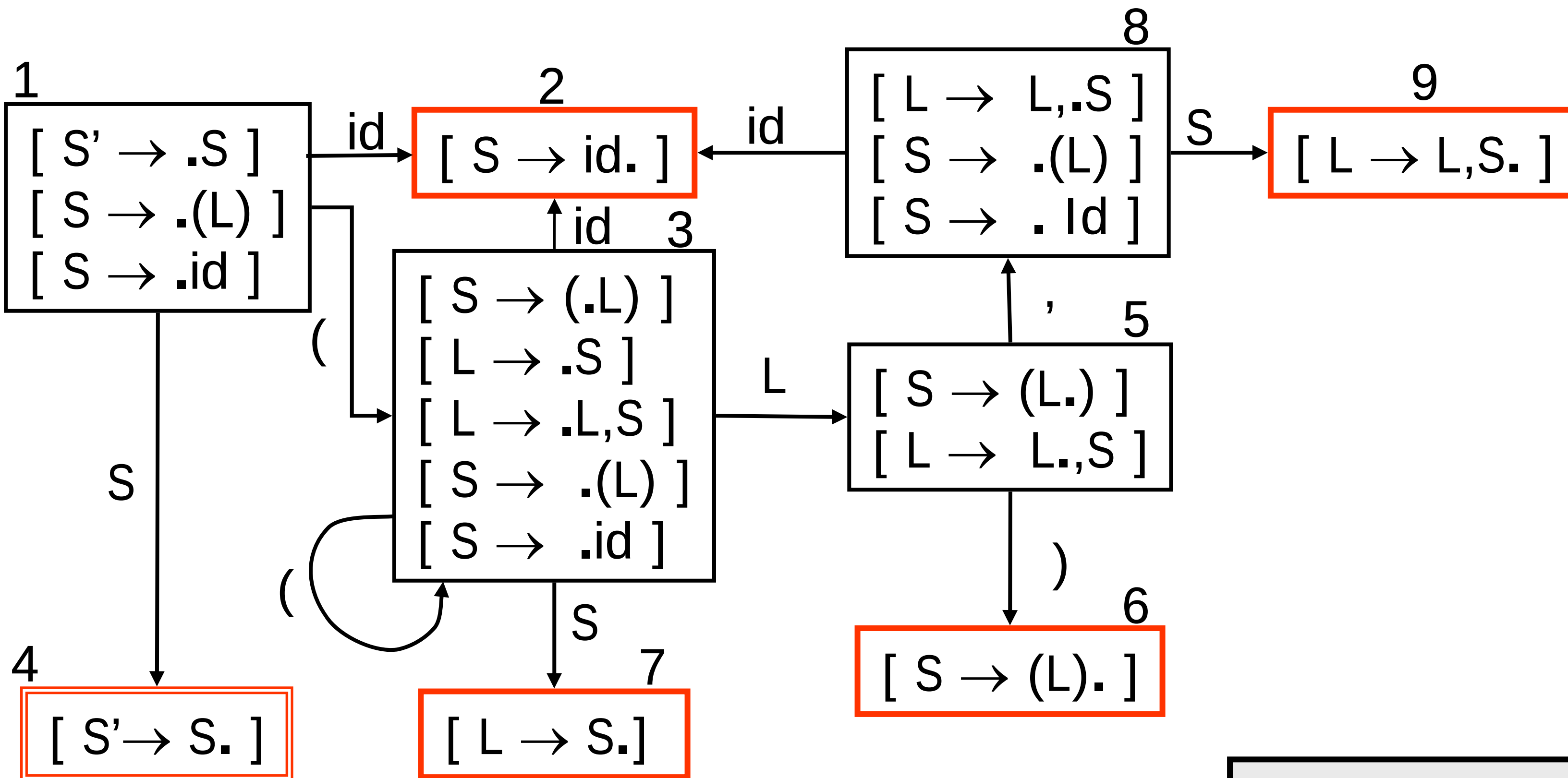


(same algorithm for transitions on nonterminals)

Reduce States



Full LR(0) Machine



Grammar:
S → (L) | id
L → S | L, S

Parsing Example: ((a),b)

Grammar:
$S \rightarrow (L) \mid id$
$L \rightarrow S \mid L, S$

derivation	stack	input	action
((a),b) $\Leftarrow$	1	((a),b)	shift, goto 3
((a),b) $\Leftarrow$	13	(a),b)	shift, goto 3
((a),b) $\Leftarrow$	133	a),b)	shift, goto 2
((a),b) $\Leftarrow$	1332	),b)	reduce $S \rightarrow id$
((S),b) $\Leftarrow$	1337	),b)	reduce $L \rightarrow S$
((L),b) $\Leftarrow$	1335	),b)	shift, goto 6
((L),b) $\Leftarrow$	13356	,b)	reduce $S \rightarrow (L)$
(S,b) $\Leftarrow$	137	,b)	reduce $L \rightarrow S$
(L,b) $\Leftarrow$	135	,b)	shift, goto 8
(L,b) $\Leftarrow$	1358	b)	shift, goto 9
(L,b) $\Leftarrow$	13582	)	reduce $S \rightarrow id$
(L,S) $\Leftarrow$	13589	)	reduce $L \rightarrow L , S$
(L) $\Leftarrow$	135	)	shift, goto 6
(L) $\Leftarrow$	1356		reduce $S \rightarrow (L)$
S	14		done

Reductions

- On reducing $B \rightarrow \beta$ with stack $\alpha\beta_2$:
  - pop $|\beta|$ states off stack
  - This reveals topmost state Q, which contains an item [$A \rightarrow \beta_1 . B\beta_3$]
  - push state Goto(Q,B) onto the stack

LR(0) Parsing Table

	(	)	id	,	ε	S	L
1	s3		s2			g4	
2	S→id	S→id	S→id	S→id	S→id		
3	s3		s2			g7	g5
4					accept		
5		s6		s8			
6	S→(L)	S→(L)	S→(L)	S→(L)	S→(L)		
7	L→S	L→S	L→S	L→S	L→S		
8	s3		s2			g9	
9	L→L,S	L→L,S	L→L,S	L→L,S	L→L,S		

LR(0) Summary

- LR(0) parsing recipe:

  Start with an LR(0) grammar

  Compute LR(0) states and build DFA:

  Build the LR(0) parsing table from the DFA

LR(0) Limitations

- An LR(0) machine only works if each state with a reduce action has only one possible reduce action and no shift action
- With more complex grammars, construction gives states with shift/reduce or reduce/reduce conflicts
- Need to use look-ahead to choose

ok	shift /reduce	reduce / reduce
[L $\rightarrow$ L,S.]	[L $\rightarrow$ L,S.] [S $\rightarrow$ S.,L]	[L $\rightarrow$ S,L.] [L $\rightarrow$ S.]

A Non-LR(0) Grammar

- Grammar for addition of numbers:

  $S \rightarrow S + E \mid E$

  $E \rightarrow \text{num} \mid (S)$

- Left-associative is LR(0)

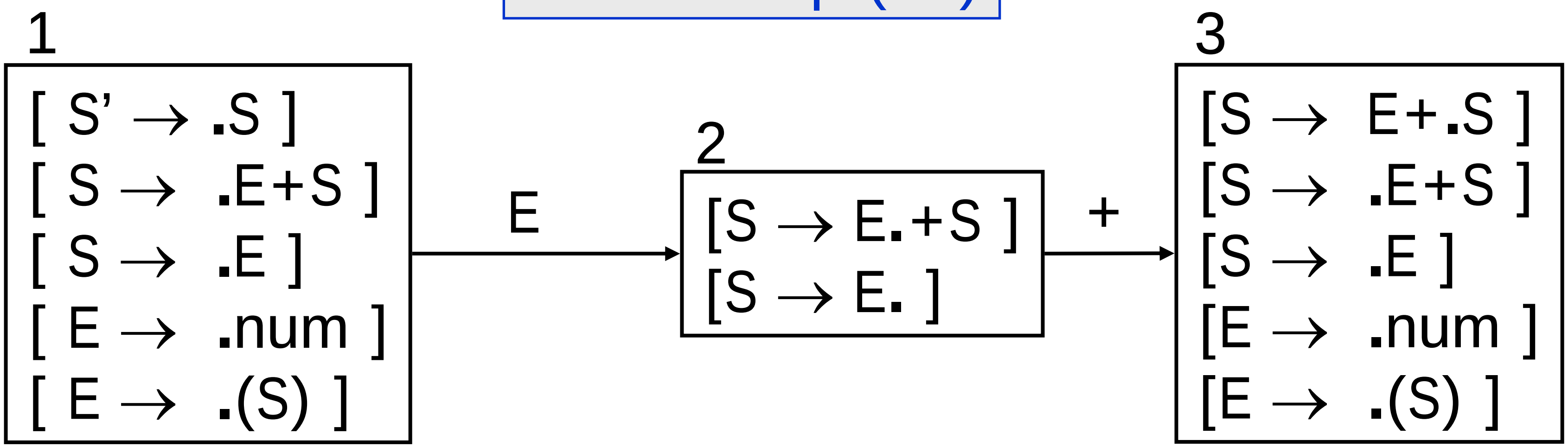
- Right-associative version is not LR(0)

  $S \rightarrow E + S \mid E$

  $E \rightarrow \text{num} \mid (S)$

LR(0) Parsing Table

Grammar
S → E + S | E
E → num | (S)



What to do in state 2?

	+	ε	E
1			2
2	s3/S→E	S→E	

SLR(k)

- Use the LR(0) machine states as rows of table
- Let Q be a state and u be a lookahead string
  - Action(Q,u) = <u>shift Goto(Q,b)</u>

    if Q contains an item of the form $[A \rightarrow \beta_1 . b\beta_3]$, with $u \in \text{FIRST}_k(b\beta_3 \text{FOLLOW}_k(A))$
  - Action(Q,u) = <u>accept</u>

    if $Q = \{ [S' \rightarrow S\ ] \}$ and $u = \varepsilon$
  - Action(Q,u) = <u>reduce</u> i

    if Q contains the item $[A \rightarrow \beta .]$, where $A \rightarrow \beta$ is the i<u>th</u> production of G and $u \in \text{FOLLOW}_k(A)$
  - Action(Q,u) = <u>error</u> otherwise
- G is SLR(k) iff the Action function given above is single-valued for all Q and u, i.e, there are no shift-reduce or reduce-reduce conflicts.

Next Time

- Learn about other kinds of LR parsing:
  - SLR = improved LR(0)
  - LR(1) = 1 character lookahead
  - LALR(1) = Look-Ahead LR(1)

- Basic ideas are the same as for LR(0)
  - Parser states with LR items
  - DFA with transitions between parser states
  - Parser table with shift/reduce/goto actions