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1. Formalize and prove the following extension of Rice's Theorem: Every nontrivial property of *pairs* of r.e. sets is undecidable. That is, give a formal statement of the above claim along the following lines. Let $P : 2^{\Sigma^*} \times 2^{\Sigma^*} \rightarrow \{0, 1\}$ be a property on pairs of r.e. languages. Then $L = \{(i, j) \mid P(L(M_i), L(M_j))\}$ is the language induced by P . Prove that if P is non-trivial, then L is undecidable.
2. 9.3.4.
3. Consider one-tape Turing machines that are constrained not to overwrite their input tapes. They may write all they want on the blank portion of the tape to the right of the input string.
 - (a) Show that these machines accept only regular strings. *Hint.* You cannot copy the input string onto the blank portion. Why ?
 - (b) Show that in spite of this, we cannot efficiently construct a DFA for such languages. *Hint:* Show that for such a machine M we cannot decide whether $L(M)$ is infinite or empty. Conclude the result from this.
4. 9.3.8, b,c.