CS381 Fall 2001 – Prelim Practice Exercises
Prof Shai Ben-David

1. Given a language $L \subseteq \Sigma^*$ let L_{init} = {w: there exists some $x \in \Sigma^*$ so that $wx \in L$} (that is, the set of all initial segments of words in L).

   (i) Prove that if two strings x,y are R_L-equivalent then they are also $R_{L_{init}}$-equivalent

   (ii) Prove that if L is regular then so is L_{init}.

2. For each of the following languages L, find a set of strings S_L that contains exactly one string from every equivalence class of R_L:

   (i) $L = (01 + 101)^*$

   (ii) $L = \{a^n b^{2n} : n \in N\}$

3. Recall that a language L is called "boring" if for every $l \in N$ there exists some $k \in N$ such that all the strings whose lengths are between k and k+l belong to L.

   Prove that if L is a boring CFL then for some $n \in N$ every string w of length greater than n belongs to L.

4. Construct a grammar G such that $L(G) = \{0^n 1^{2n} : n \in N\}$. Prove that this is indeed the language that your grammar generates.

5. Prove that $\{0^n 1^{2^n} : n \in N\}$ is <u>not</u> a CFL.