CS381 Fall 2001 **Prelim 1**
Prof Shai Ben-David **Tues, Sept 25, 2001**

You have 60 minutes to complete the exam. Please don't hesitate to ask for assistance.

NOTE: You should explain and justify any claim you make!

1. For a word $w \in \{0,1\}^*$, define a language $L_{sub(w)} = \{xwy : x,y \in \{0,1\}^*\}$. That is, $L_{sub(w)}$ is the set of all strings that contain w as a sub-string.

10 pts (i) Prove that for every pair of strings x,y,

$$L_{sub(xy)} \subseteq L_{sub(x)} \cap L_{sub(y)}$$

10 pts (ii) Prove that for x=00 and y=11, it is not the case that

$$L_{sub(xy)} \supseteq L_{sub(x)} \cap L_{sub(y)}.$$

25 pts (iii) Prove that for every $w \in \{0,1\}^*$, $L_{sub(w)}$ is a regular language.

10 pts (iv) **BONUS:** Prove that for every set K of strings (over {0,1}*),

$$\bigcap_{w \in K} L_{sub(w)} \text{ is regular.}$$

2.

25 pts (i) Describe in words what language is computed by the following NFA. Please justify any claims you make.

$$N = (Q, \Sigma, q_0, \Delta, F)$$

where: $Q = \{a, b, c\}$; $\Sigma = \{0, 1\}$; $q_0 = a$; $F = \{c\}$; and Δ is defined by:

D	**a**	**b**	**c**
0	**{a}**	**{c}**	**{b}**
1	**{a,b}**	f	f

30 pts (ii) Constuct a deterministic automaton M such that $L(M) = L(N)$ (where N is the automaton described in 4(i) above). You can describe M either by using a graph or by using a formal table for δ. **30 pts**