3. (i) Prove that if x and y are both strings over the same 1-letter alphabet, then xy=yx

(ii) Find strings x,y over the alphabet {0,1} such that x≠y, both 0 and 1 appear in x (and y), and yet xy=yx.

(iii) **BONUS:** Find a general (as general as you can) condition on strings such that if x,y satisfy this condition, then xy=yx.

(i)Proof: let Σ={a} be the 1-letter alphabet.

|xy|=|x|+|y|=|y|+|x|=|yx|

so the length of xy equals to that of yx. Let m=|xy|=|yx|,then both xy and yx are consisted of m continuous a's. so

xy=yx

Note: 1. don't confuse string with set!

x and y are strings. String is not a set, it just means a finite sequence of elements in Σ. xy,yx, the cancatenation of x,y , mean putting x and y together end by end.

So if x=a,y=a, x≠{a},y≠{a},

xy≠{ε,a,aa}

2. x and y are both strings over the same 1-letter alphabet, this means x and y are consisted of a unique letter, x,y∈ Σ^* , but not x,y being a letter.

(ii) x=10, y=1010

x≠y, xy=yx=101010

(iii) x= z^m ,y= z^n , m,n<=0, z can be any string ∈ Σ^*

Note:A very common answer is y= x^n . This is right, but not general enough. This requires |y| mod |x|=0. that's not necessary. Look at this example:

X=1010, y=101010, for any n, y≠ x^n ,but xy=yx.