

Homework 1, Problem 2

$\Rightarrow LL = \{ ab \mid a \in L \text{ and } b \in L \}$

Let $a = \epsilon$

$\{ \epsilon b \mid b \in L \} \subseteq LL$

$\{ b \mid b \in L \} \subseteq LL$

$L \subseteq LL$

$\Leftarrow L \subseteq LL$

Assume by contradiction that $\epsilon \notin L$.

Let x be the shortest string in L of length $|x|$

Then xx is the shortest string in LL .

The length of $xx = |xx| = 2|x|$

Since L is the subset of LL , these must have the same length.

$|x| = 2|x|$

But this is impossible since we are guaranteed $|x| > 0$

Therefore $\epsilon \in L$

Common Mistakes:

1. Iff necessitates that you must prove the statement two ways forwards and backwards. Many people proved the more difficult backwards part, but forgot to prove the forwards statement.

2. Many went the following route:

For any string x in L , xy is in LL , therefore $x = xy$. and y equals ϵ .

The correct statement, however is for any string x in L and yz in LL , $x = yz$

Notes:

-Formalism. The solutions ranged from very formal to pretty informal. This time we accepted all kinds of solution, but in the future we probably want more formal results.

-Empty string vs. Empty set. Some people used the notions interchangeably, though not too many people.