CS381 Fall 2001
Homework 8, Part I
Prof Shai Ben-David
Due: Friday, December 7, 2001 — 9:05 am

PLEASE NOTE: This assignment is optional in the following sense: we will drop the worst 3 assignments (instead of 2, as previously stated) when we calculate your grade.

1. Prove that for every language $L \in R$ (that is, L is a recursive language), and every non-trivial language L', there is a reduction of L to L'. (Note: L' is non-trivial if L' is neither $\emptyset$ nor Σ^*.)

2. Find a pair of languages L, L' such that $L \leq_m L'$ but $L' \not\leq_m L$ (that is, there exists a reduction of L to L' but there is no reduction of L' to L).

3. Prove or refute the following claim: For every pair of languages A, B, if there exists a reduction of A to B, then there exists a reduction from B^c to A^c (where L^c denotes the complement of a language L).

4. Define a notion of "unrestricted grammar" as follows:

$$G = (\Sigma, N, S_0, P)$$

where Σ and N are disjoint finite sets. $S_0 \in N$, and P is a finite set of "production rules" of the form $\alpha \longmapsto \beta$, where $\alpha, \beta \in (\Sigma \cup N)^*$. (This is very similar to the definition of context-free grammars, except that we do not require α to be a letter from N.) For such a grammar G and strings $\gamma, \delta \in (\Sigma \cup N)^*$ we say that $\gamma \overset{G}{\longmapsto} \delta$ if there is a rule $(\alpha \longmapsto \beta) \in P$, and strings x, y, such that $\gamma = x\alpha y$ and $\delta = x\beta y$ (that is, δ is obtained from γ by replacing the substring α by the string β). Furthermore, let $\gamma \longmapsto_\star^G \delta$ if there is a finite sequence $\gamma_1, \ldots, \gamma_n$ such that for all $i < n$, $\gamma_i \overset{G}{\longmapsto} \gamma_{i+1}$ and $\gamma \overset{G}{\longmapsto} \gamma_1$ and $\gamma_n \overset{G}{\longmapsto} \delta$. Finally, let $L(G) = \{w \in \Sigma^* : S_0 \longmapsto_\star^G w\}$.

   (a) Prove that for every such grammar G, the language $L(G)$ is recursive.

   (b) **BONUS:** Argue convincingly why for every recursive language L there exists some unrestricted grammar G such that $L = L(G)$.

NOTE: Part II of this homework, consisting of review exercises, will be on the website on Monday, Dec 3.