CS381 Fall 2001 – Homework 3
Prof Shai Ben-David

DUE: Friday, October 12, 9:00 am

NOTE: EVERY claim you make should be supported by an explanation or a proof

1. (i) Prove that the family of <u>non</u>-regular languages is closed under complementation (that is, if L is non-regular then so is $L^c = \{w : w \notin L\}$).

   (ii) Show that the family of non-regular languages is not closed under the union operation. That is, prove that there are non-regular L_1, L_2, such that $L_1 \cup L_2$ is regular

   (iii) **BONUS:** Prove that there is a family W of <u>infinitely many</u> non-regular languages such that $\cup\{L : L \in W\}$ is regular.

2. Prove that if L is a finite language then every DFA that computes L must have at least max $\{|w| : w \in L\}$ many states.

3. For a word $w = o_1 \ \ldots \ o_n$, let $\overline{w}$ be the reverse word $\overline{w} = o_n \ \ldots \ o_1$. Prove that $\{w\overline{w} : w \in \{0,1\}^*\}$ is not a regular language.

4. Prove that $L = \{0^k 1^n 0^n : k, n > 0\} \cup \{1^i 0^j : i, j \geq 0\}$ satisfies the requirements of the pumping lemma (that is, "there exists some $n \in N$ such that for every $w \in L$ if $|w| > n$ then there are x, y, z such that: (i) $w = xyz$; (ii) $|xy| = n + 1$; (iii) For every $i \in N$, $xy^i z \in L$"). This language is <u>not</u> regular, but we will not prove it here.

5. Prove that $\{w : \#_0(w) - \#_1(w) \equiv 1 \bmod 3\}$ is regular but, on the other hand, $\{w : |\#_0(w) - \#_1(w)| \equiv 1 \bmod 3\}$ is not regular. <u>HINT</u>: consider $0^n 1^{n+1}$ for sufficiently large n.

6. Prove that for every regular expression r there exists a regular expression t, such that: $L(r) = \{w : w \notin L(t)\}$.

7. Find a regular expression t over {0,1} such that:

$$L(t) = \{w : w \notin L(((0+1)(0+1))^*)\}.$$

8. For each of the following languages L (over $\Sigma = \{o, p, q\}$) find a regular expression r_L such that $L(r_L) = L$:

   (i) $L = \{w :$ if p occurs in w then w ends with a q$\}$

   (ii) $L = \{w :$ $\#_p(w)$ is even$\}$

   (iii) $L = \{w :$ the next-to-last letter in w is p$\}$

9. For each of the following languages L describe the equivalence classes of R_L and determine the rank of R_L:

   (i) $L = \{w \in \{0,1\}^* :$ w contains exactly two 1's.$\}$

   (ii) $L = \{0^m 1^k 0^{m+k} : m, k \in N\}$

   (iii) $L = L(ab(a+b)^*ab)$