CS381 Fall 2001 – Homework 2
Prof Shai Ben-David

DUE: Friday, 9/21, 9:00 am

NOTE: EVERY claim you make should be supported by an explanation or a proof

1. Prove that if L_1, L_2 are regular languages, then so is:

   $L_1 \setminus L_2 = \{w \in L_1 : w \notin L_2\}$

2. Given a DFA, $M = (Q, \Sigma, q_0, \delta, F)$ and $p, q \in Q$, let $L(M, p, q) = \{w : \hat{\delta}(p, w) = q\}$ Prove/refute each of the following claims:

   (i) For every M,p,q as above and every $x, y \in \Sigma^*$, if $x \in L(M, p, q)$ and $y \in L(M, q, p)$ then $xy \in L(M, p, p)$

   (ii) For every M, p, q as above and every $x, y, z \in \Sigma^*$, if $yz \in L(M, p, q)$ then there exist some $r \in Q$ such that for every $x \in L(M, r, r)$ and every $i \in N$, $yx^i z \in L(M, p, q)$.

3. Recall that a language is called "regular" if it is computable by some DFA.

   (i) Prove that any intersection of finitely many regular languages is a regular language.

(ii) Prove that there exist a set W of regular languages so that the intersection of all languages in W is <u>not</u> regular.

(iii) **BONUS:** find a set W of regular languages such that W is infinite and yet the intersection of all the languages in W is an infinite regular language.

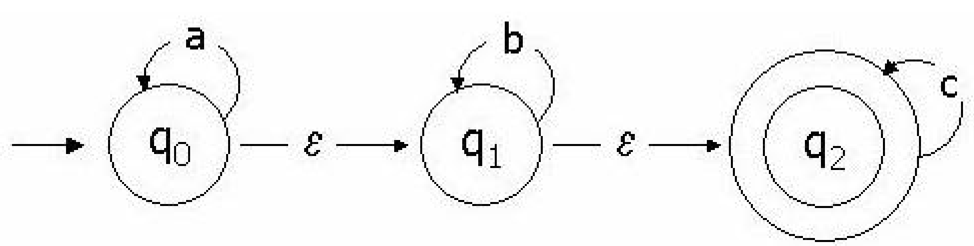
4. Find a set W consisting of infinitely many languages over {0,1} so that:

(i) Each language in W is infinite

(ii) Each language in W is regular (i.e. computable by some DFA)

(iii) For every pair of languages $L_1, L_2 \in W$, if $L_1 \neq L_2$ then $L_1 \cap L_2 = \Phi$

5. Construct a DFA, M, such that L(M)=L(N) where N is the following NFA:



(here $\Sigma = \{a, b, c\}$)

6. Construct a NFA, M, over $\Sigma = \{1,2,3,4,5\}$ such that M has only 5 states and $L(M) = \{w = \sigma_1\sigma_2 \ldots \sigma_{|w|} : \text{for all } i < j < |w|,\ \sigma_i \leq \sigma_j\}$ (that is, the numbers that are the letters in w appear in increasing order).