CS381 Fall 2001 **Homework 1**
Prof Shai Ben-David **Sept 6, 2001**

1. Which of the following statements holds for every three languages L_1, L_2, L_3?

   (i) $(L_1 \cap L_2)L_3 = L_1L_3 \cap L_2L_3$

   (ii) $L_1{}^* = L_1{}^* \between L_1{}^*$

   (iii) $(L_1 \cup L_2) \cap L_3 = L_1 \cup (L_2 \cap L_3)$

   Please prove your claims.

2. Prove that for every non-empty language L, $\varepsilon \in L$ iff $L \subseteq LL$

3. (i) Prove that if x and y are both strings over the same 1-letter alphabet, then xy=yx

   (ii) Find strings x,y over the alphabet {0,1} such that x≠y, both 0 and 1 appear in x (and y), and yet xy=yx.

   (iii) **BONUS:** Find a general (as general as you can) condition on strings such that if x,y satisfy this condition then xy=xy.

4. (i) Find an infinite set (W of Languages over {0,1} so that the following two conditions hold (simultaneously):

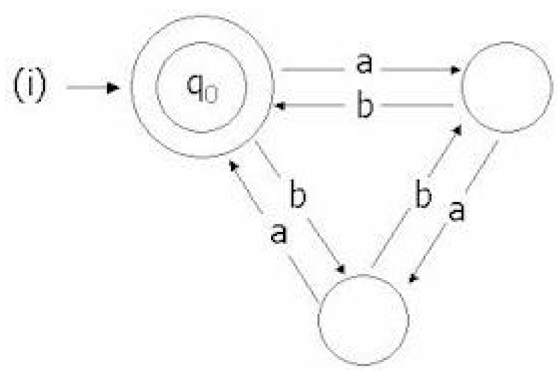
a. Every intersection of finitely many members of W is non-empty

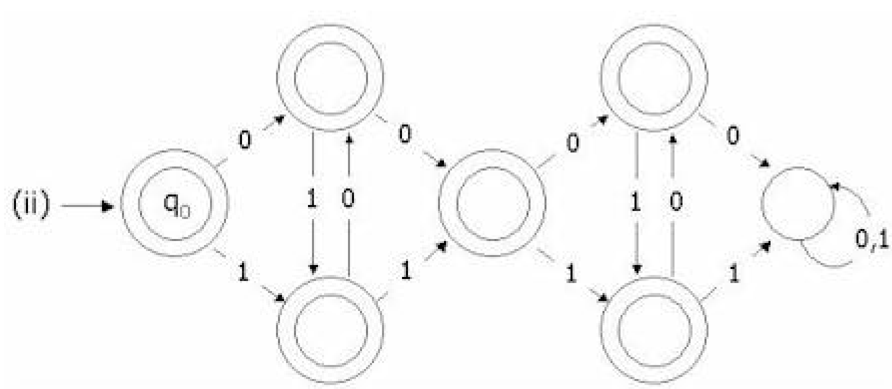
b. There is a subset of W whose intersection is empty

(ii) **BONUS:** Does there exist a set W that in addition to satisfying a & b above also satisfies:

c. Every infinite subset of W has empty intersection

5. Find what are the languages computed by each of the following automata:





Explain your claims (there's no need to <u>prove</u> them).

6. Describe automata that compute each of the following languages:

(i) $L_{5,3} = \{w \in \{0,1\}^* : |w|$ is divisible by either 3 or 5$\}$

(ii) For a given string $w \in \{0,1\}^*$, the language $\{w\}^*$.